RETRIAL QUEUEING SYSTEM WITH TWO MODES OF FAILURE

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Abstract: *Markovian arrival general service retrial queueing system, where the server is subject to two modes of failure, partial or total is analyzed in this paper. It is assumed that the server fails only in working phase. When total failure occurs in the system the repair of the server starts instantaneously, but in the case of partial failure the repair starts only after the completion of work in hand. Life of the server in working and in partial mode follows exponential distribution with different rates. Retrial times, service times and repair times follow general distribution. Various performance measures are derived in equilibrium state and numerical results are presented.*

Keywords: Orbit Size, Retrial Queue, Steady State Distribution.

1. INTRODUCTION

Retrial queues have the characteristic that the customers arriving in the system and finding the system full leaves the system, after some random period make a new attempt to get service. Between trials the customer is said to be in orbit. Retrial queue are widely used in telephonic problems and computer and communication network. Queues with server subject to breakdowns and repairs are often encountered in many practical applications such as in computer, manufacturing systems and communication networks. As the system performance deteriorates seriously by server breakdowns and the limitation of repair capacity, the study of queues with server breakdowns and repairs is important. Retrial queues with server failures and repairs are introduced by Aissani[2] and Kulkarni and Choi[3]. The other notable works related to this model are Sherman and Kharoufeh[7] Maraghi and Madan[6] Abbas and Aïssani [1] and Kalyanaraman and Seenivasan[4]

If the server is a machine the breakdown may be subject to two modes of failure, partial or total. When the service facility is in partial failure mode, it gives service with a lower rate than in normal operating conditions. Madan [5], and Sridharan and Jayashree [8] have analyzed classical queueing system where the server is subject to two modes of failure. These types of service interruption are common in industry, factories, telephone booths, computer systems etc. In this paper an M/G/1 retrial queueing system with two modes of failure is considered.

2. DESCRIPTION OF THE MODEL

Consider a single server queueing system in which new customer arrive in a Poisson process with rate λ. If an arriving customer finds the server idle, it begins service immediately and leaves the system after service completion. Otherwise, if the server is found busy or down the customer joins the retrial group to try again after random intervals for their requests. Successive interretrial times of any customer are governed by an arbitrary probability distribution function A(x) with corresponding density function a(x) and Laplace Stieltjes transform A*(s). When the server is working in normal mode it may fail totally or partially. The time until total failure (partial failure) is exponentially distributed with rate α_1 (α_2). The service times of customers in normal working mode (partial mode) are generally distributed with distribution function $B_1(x)$ ($B_2(x)$), density function $b_1(x)$ ($b_2(x)$) and Laplace Stieltjes function $B_1^*(s)$ ($B_2^*(s)$). When the total failure occurs in the system the repair starts immediately, but in the occasion of partial failure the repair of the system starts only after the completion of work in hand. The repair times follow general distribution with distribution function H(x), density function h(x) and Laplace Stieltjes function H*(s) and first two moments h_1 and h_2.

The stochastic behaviour of this retrial queueing system can be described by the Markov process $\{N(t), t \geq 0\} = \{C(t), X(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t), t \geq 0\}$ where C(t) denotes the server state 0, 1, 2 or 3 according as the server being idle, providing service in normal working mode, providing service in partial working mode or in failure mode respectively and X(t) corresponds to the number of customers in the orbit at time t. If C(t) = 0 and X(t) > 0, then $\xi_0(t)$ represents the elapsed retrial time at time t. If C(t) = 1 and X(t) ≥ 0 then $\xi_1(t)$ corresponds to the elapsed time of the customer being provided normal service. If C(t) = 2 and X(t) ≥ 0, then $\xi_2(t)$ represents the elapsed time of the customer being provided partial mode service and if C(t) = 3 and X(t) ≥ 0, then $\xi_3(t)$ represents the elapsed repair time of the server at time t. The functions $\eta(x)$, $\mu_1(x)$, $\mu_2(x)$ and $\beta(x)$ are the conditional completion rates for repeated attempts, normal mode service, partial mode service and for repair respectively.

Then $\eta(x) = \dfrac{a(x)}{1 - A(x)}$; $\mu_1(x) = \dfrac{b_1(x)}{1 - B_1(x)}$;

$$\mu_2(x) = \frac{b_2(x)}{1 - B_2(x)} \quad \text{and} \quad \beta(x) = \frac{h(x)}{1 - H(x)}.$$

Steady State Distribution

In this section the steady state distributions for the model under consideration are derived. For the process $\{N(t), t \geq 0\}$ define the probability,

$I_0(t) = P\{C(t) = 0, X(t) = 0\}$

$I_n(x, t)\,dx = P\{C(t) = 0, X(t) = n, x \leq \xi_0(t) < x + dx\}$,
$t \geq 0, x \geq 0$ and $n \geq 1$

$W_n(x, t)\,dx = P\{C(t) = 1, X(t) = n, x \leq \xi_1(t) < x + dx\}$,
$t \geq 0, x \geq 0$ and $n \geq 0$

$S_n(x, t)\,dx = P\{C(t) = 2, X(t) = n, x \le \xi_2(t) < x + dx\},$
$t \ge 0, x \ge 0$ and $n \ge 0$

$V_n(x, t)\,dx = P\{C(t) = 3, X(t) = n, x \le \xi_3(t) < x + dx\},$
$t \ge 0, x \ge 0$ and $n \ge 0$

By supplementary variable technique, the system of equations that governs the model under equilibrium state are given below,

$$\lambda I_0 = \int_0^\infty \mu_1(x) W_0(x)\,dx + \int_0^\infty \beta(x) F_0(x)\,dx$$

$$\frac{d\,I_n(x)}{dx} = -(\lambda + \eta(x))\,I_n(x), \qquad n \ge 1$$

$$\frac{d\,I_n(x)}{dx} = -(\lambda + \alpha_1 + \alpha_2 + \mu_1(x))\,W_n(x) + \lambda(1 - \delta_{0n})\,W_{n-1}(x), \quad n \ge 0$$

$$\frac{d\,S_n(x)}{dx} = -(\lambda + \alpha_3 + \mu_2(x))\,S_n(x) + \lambda(1 - \delta_{0n})\,S_{n-1}(x), \quad n \ge 0$$

$$\frac{d\,F_n(x)}{dx} = -(\lambda + \beta(x))\,F_n(x) + \lambda(1 - \delta_{0n})\,F_{n-1}(x), \quad n \ge 0$$

with boundary conditions

$$I_n(0) = \int_0^\infty \mu_1(x) W_n(x)\,dx + \int_0^\infty \beta(x) F_n(x)\,dx, \quad n \ge 1$$

$$W_0(0) = \int_0^\infty \eta(x)\,I_1(x)\,dx + \lambda I_0$$

$$W_n(0) = \int_0^\infty \eta(x)\,I_{n+1}(x)\,dx + \lambda \int_0^\infty I_n(x)\,dx, \quad n \ge 1$$

$$S_n(0) = \alpha_2 \int_0^\infty W_n(x)\,dx, \qquad n \ge 0$$

$$F_0(0) = \int_0^\infty \mu_2(x)\, S_0(x)\, dx$$

$$F_n(0) = \int_0^\infty \mu_2(x)\, S_n(x)\, dx + \alpha_1 \int_0^\infty W_{n-1}(x)\, dx + \alpha_3 \int_0^\infty S_{n-1}(x)\, dx, \quad n \geq 1$$

Define the probability generating function
$P(., z) = \Sigma\, p_n(.)\, z^n$ for any probability $p_n(.)$.
Let $g_1(z) = \lambda(1 - z) + \alpha_1 + \alpha_2$ and
$g_2(z) = \lambda(1 - z) + \alpha_3$.

The steady state distributions of $N(t), t \geq 0\}$ are given by

$I(x, z) = I(0, z)\, e^{-\lambda x}\, [1 - A(x)]$

$W(x, z) = W(0, z)\, e^{-g_1(z)\, x}\, [1 - B_1(x)]$

$S(x, z) = S(0, z)\, e^{-g_2(z)\, x}\, [1 - B_2(x)]$

$F(x, z) = F(0, z)\, e^{-\lambda(1-z)\, x}\, [1 - H(x)]$

$W(0, z) = I_0\, \lambda\, (1 - z)\, A^*(\lambda)\, g_1(z)\, g_2(z) / D(z)$

$I(0, z) = I_0\, \lambda\, z\, [1 - B_1^*(g_1(z))]\, \{g_1(z)\, g_2(z) - H^*(\lambda(1-z)[g_2(z)[z\alpha_1 + \alpha_2 B_2^*(g_2(z))] + z\, \alpha_2\, \alpha_3\, (1 - B_2^*(g_2(z)))]\} / D(z)$

$S(0, z) = I_0\, \alpha_2\, \lambda\, (1-z) A^*((\lambda) g_2(z)(1 - B_1^*(g_1(z))) / D(z)$

$F(0, z) = I_0\, \lambda(1-z) A^*(\lambda)(1 - B_1^*(g_1(z))\, [g_2(z)\, (z\, \alpha_1 + \alpha_2\, B_2^*(g_2(z))) + z\, \alpha_2\, \alpha_3\, (1 - B_2^*(g_2(z)))] / D(z)$

$D(z) = [(1-z) A^*(\lambda) + z]\{g_1(z) g_2(z)\, B_1^*(g_1(z)) + H^*(\lambda(1-z))[1 - B_1^*(g_1(z))][\alpha_2\, B_2^*(g_2(z))\, (g_2(z) - z\, \alpha_3) + \alpha_2\, \alpha_3\, z + \alpha_1\, z\, g_2(z)]\} - z\, g_1(z)\, g_2(z)$

Define the partial generating function

$$\psi(z) = \int_0^\infty \psi(x, z)\, dz$$ for any generating function $\psi(x, z)$. Then

$I(z) = I_0(1 - A^*(\lambda) z[1 - B_1^*(g_1(z))]\{g_1(z) g_2(z) - H^*(\lambda(1-z))$
$[\alpha_2 B_2^*(g_2(z))(g_2(z) z \alpha_3) + \alpha_2 \alpha_3 z + \alpha_1 z g_2(z)]\}$

$W(z) = I_0 \lambda A^*(\lambda)(1-z)\, g_2(z)\, (1 - B_1^*(g_1(z))) / D(z)$

$S(z) = I_0 \lambda A^*(\lambda) \alpha_2 (1-z)(1 - B_1^*(g_1(z)))(1 - B_2^*(g_2(z))) / D(z)$

$F(z) = I_0 A^*(\lambda)[1 - B_1^*(g_1(z))][1 - H^*(\lambda(1-z))]$
$[\alpha_2 g_2(z) B_2^*(g_2(z)) + \alpha_2 \alpha_3 z(1 - B_2^*(g_2(z))) + \alpha_1 z\, g_2(z)]$

Using the normalizing condition I_0 is obtained as,

$I_0 = \{[1-B_1^*(\alpha_1+\alpha_2)][\lambda\alpha_2 B_2^*(\alpha_3)-\lambda(\alpha_2+\alpha_3)-\lambda\,\alpha_3(\alpha_1+\alpha_2)\,h_1-\alpha_3\,(\alpha_1+\alpha_2)\,(1-B_2^*(\alpha_3)]+\alpha_3(\alpha_1+\alpha_2)A^*(\lambda)\}/\,[A^*(\lambda)\,\alpha_3\,D_1]$

where

$D_1 = (\alpha_1+\alpha_2)B_1^*(\alpha_1+\alpha_2)+\alpha_2 B_2^*(\alpha_3)(1-B_1^*(\alpha_1+\alpha_2))$

The probability generating function K(z) for the number of customers in the system is

$$K(z) = I_0 + I(z) + z\,W(z) + z\,S(z) + F(z)$$
$$= I_0 A^*(\lambda)(1-z)g_2(z)\{g_1(z)B_1^*(g_1(z))+\ \alpha_2 B_2^*(g_2(z))[1-B_1^*(g_1(z))]\}/D(z)$$

The probability generating function H(z) for the number of customers in the orbit is

$$H(z) = I_0 + I(z) + W(z) + S(z) + F(z)$$
$$= I_0 A^*(\lambda)(1-z)\{g_2(z)[\lambda(1-z)+\alpha_1 B_1^*(g_1(z))] + \lambda(1-z)\,\alpha_2 + \alpha_2\,\alpha_3\,[B_2^*(g_2(z))\,[1-B_1^*(g_1(z))]\ + B_1^*(g_1(z))]\}\,/\,D(z)$$

3. PERFORMANCE MEASURES

In this section some performance measures for the system under steady state are derived.

1. The probability that the server is idle during the retrial time is given by
   $I(1) = (-A^*(\lambda))[1-B_1^*(\alpha_1+\alpha_2)]\,[\lambda(\alpha_2+\alpha_3)+\lambda\alpha_3(\alpha_1+\ \alpha_2)\,h_1-\lambda\,\alpha_2\,B_2^*(\alpha_3)+\alpha_3(\alpha_1+\alpha_2)-\alpha_2\,\alpha_3\,B_2^*(\alpha_3)]\,/\,[\alpha_3\,A^*(\lambda)\,D_1]$.

2. The probability that the server is busy in normal working mode is given by
   $W(1) = \lambda[1-B_1^*(\alpha_1+\alpha_2)]\,/\,D_1$.

3. The probability that the server is busy with partial failure mode is given by
   $S(1) = \lambda\alpha_2[1-B_1^*(\alpha_1+\alpha_2)][1-B_2^*(\alpha_3)]/(\alpha_3 D_1)$

4. The probability that the server is in failure mode is given by
   $F(1) = \lambda\,(\alpha_1+\alpha_2)\,[1-B_1^*(\alpha_1+\alpha_2)]\,h_1\,/\,D_1$
   Write,
   $T_1 = [1-B_1^*(\alpha_1+\alpha_2)]\,[\lambda\,\alpha_2\,B_2^*(\alpha_3)-\lambda\ \ (\alpha_2+\alpha_3)-\lambda\,\alpha_3\,(\alpha_1+\alpha_2)\,h_1-\alpha_3\,(\alpha_1+\alpha_2)+\alpha_2\,\alpha_3\,B_2^*(\alpha_3)]+\alpha_3\,(\alpha_1+\alpha_2)\,A^*(\lambda)$

   $T_2 = [1-A^*(\lambda)]\,\{[1-B_1^*(\alpha_1+\alpha_2)]\,[\alpha_1\,\alpha_3+\alpha_2\,\alpha_3\,(1-B_2^*(\alpha_3))-\alpha_2\,\lambda\,B_2^*(\alpha_3)+\lambda\,h_1\,\alpha_3\,(\alpha_1+\alpha_2)]-\lambda\,(\alpha_2+\alpha_3)\,B_1^*(\alpha_1+\alpha_2)-\lambda\,\alpha_1\}+\dfrac{\lambda^2}{2}\,\alpha_3\,(\alpha_1+\alpha_2)\,[h_2\,(1-B_1^*(\alpha_1+\alpha_2))-2\,h_1\,B_1^{*\prime}(\alpha_1+\alpha_2)]+\lambda\,h_1\,[1-B_1^*(\alpha_1+\alpha_2)-\lambda\,B_1^{*\prime}(\alpha_1+\alpha_2)][\alpha_1\,\alpha_3+\alpha_2\,\alpha_3\,(1-B_2^*(\alpha_3))-\lambda\,(\alpha_1+\alpha_2\,B_2^*(\alpha_3))]-\lambda\,(1-B_1^*(\alpha_1+\alpha_2))\,[\alpha_1+\alpha_2\,\lambda\,B_2^{*\prime}(\alpha_3)-\alpha_2\,\alpha_3\,B_2^{*\prime}(\alpha_3)]$.

$N_1 = \lambda \{\alpha_3 [[\alpha_1 + \alpha_2 (1 - B_2^*(\alpha_3))]B_1^{*\prime}(\alpha_1 + \alpha_2) + \alpha_2 B_2^{*\prime}(\alpha_3)(1 - B_1^*(\alpha_1 + \alpha_2)) - B_1^*(\alpha_1 + \alpha_2)] - D_1$

$N_2 = \alpha_3 \{\alpha_1 B_1^*(\alpha_1 + \alpha_2) + \alpha_2 [B_1^*(\alpha_1 + \alpha_2) + B_2^*(\alpha_3) (1 - B_1^*(\alpha_1 + \alpha_2))]\}$

$N_3 = \lambda \{\alpha_2 \alpha_3 [B_2^{*\prime}(\alpha_3) (1 - B_1^*(\alpha_1 + \alpha_2)) + B_1^{*\prime}(\alpha_1 + \alpha_2) (1 - B_2^*(\alpha_3))] - \alpha_1 B_1^*(\alpha_1 + \alpha_2) + \alpha_1 \alpha_3 B_1^{*\prime}(\alpha_1 + \alpha_2) - (\alpha_2 + \alpha_3)\}$

The mean number of customers in the system is

$$L_s = T_2 / T_1 + N_1 / (\alpha_3 D_1)$$

The mean number of customers in the orbit is,

$$L_q = (N_3 + N_2 T_2 / T_1) / (\alpha_3 D_1)$$

4. REFERENCES

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