BULK ARRIVAL TWO PHASE RETRIAL QUEUEING SYSTEM WITH ORBITAL SEARCH, IMPATIENT CUSTOMERS AND DIFFERENT TYPES OF SERVER VACATIONS

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Abstract: Single server batch arrival two phase service retrial queue with orbital search is analysed in this chapter. Customer may balk or renege at particular times. The server operates under two vacation mechanisms. During essential service the server may leave the system to attend an emergency call and after completing the vacation, the server continues the service for the same customer. Upon completion of a optional service, the server follows Bernoulli schedule with multiple vacation policy. Performance measures are derived. Numerical results are calculated.

Key Words - Emergency vacation, Multiple vacation, Retrial queue, Two phase service.

1. INTRODUCTION

Retrial queueing system are characterized by the fact that arriving customer who finds the server busy is to leave the service area and repeat his demand after some time called retrial time. Between trials, the blocked customer joins a pool of unsatisfied customers called orbit. For example, web access, telecommunication networks, computer systems, packet switching networks, collision avoidance star local area networks, etc.

Krishnakumar and Arivudainambi [1] have analyzed a single server queue with Bernoulli vacation schedules and general retrial times, Krishnakumar et.al. [2] have introduced an M/G/1 retrial queueing systems with two phase service and preemptive resume. The models with impatient customers were considered by [3, 4]. The concept of orbital search was discussed by Krishnammorthy et al. [3]. Although some aspects have been discussed separately on queueing systems with bulk arrival, vacations, repeated attempts, two phase service and orbital search, no work have been found with all the features. To fill up to this gap, in this article an attempt has been made to study a two phase retrial $M^X/G/1$ retrial queue with two dfifferent types of vacations with impatient customers and orbital search.

2. MODEL DESCRIPTION

Consider a single server retrial queueing system with two phases of service essential and optional. Customers arrive in batches according to a Poisson process with rateλ. The batch size Y is a random variable with $P(Y=k) = C_k$, $k = 1, 2, \ldots$ Let $C(z) = \sum_{k=1}^{\infty} C_k z^k$ be the generating function of the batch size distribution with first two moments $\overline{C}_1$ and $\overline{C}_2$. If the server is busy or in vacation the customers join

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the orbit with probability p. Retrial customers may renege with probability 1 – q. The retrial time of the customer is generally distributed with distribution function A(x), density function a(x), Laplace transform A*(s) and the hazard rate function $\eta(x)$.

The essential service is needed to all arriving customers, the service time has general distribution. Its distribution function, density function, Laplace transform, the first two moments and the hazard rate function are respectively, $B_1(x)$, $b_1(x)$, $B_1^*(s)$, μ_{11}, μ_{12}, and $\mu_1(x)$. During the essential service the server may take the emergency vacation distributed as exponentially with rate β. When the server is in emergency vacation, the interrupted customer either remains in the service position with probability r_1 or leaves the service area and keeps returning at times exponentially distributed with mean $1 / \theta$. After completion of the emergency vacation, the server waits for the interrupted customer. This time is known as reserved time. The emergency vacation time has the distribution function H(x), density function h(x), Laplace transform H*(s), the first two moments h_1 and h_2 and hazard rate function $\gamma(x)$. As soon as the essential service is completed, then the customer may leave the system with probability τ or opt for optional service. If the customer leaves the system, the server remains idle with probability ρ_1 or searches for customers in the orbit (if any) with probability $1 - \rho_1$. The optional service time follows an arbitrary distribution, and its distribution function, density function, Laplace transform, the first two moments and the hazard rate functions are respectively, $B_2(x)$, $b_2(x)$, $B_2^*(s)$, μ_{21}, μ_{22}, $\mu_2(x)$.

After completing the optional service, the server takes a vacation with probability $1-\alpha$, the or waites for the next customer else with probability α. During the waiting time the server may search for customers in the orbit (if any) with probability $1 - \rho_2$, or remains idle with probability ρ_2. The vacation time follows an arbitrary distribution, and its distribution function, density function, Laplace transform, first two moments and the hazard rate functions are F(x), f(x), F*(s), v_1, v_2 and $\phi(x)$, respectively. At the end of the vacation period if the orbit is not empty, the server searches for customers in the orbit with probability $1 - \rho_3$, or waits to serve a customer with probability ρ_3. Otherwise the server begins another vacation immediately and continues in this manner until the server finds alteast one customer in the orbit upon returning from a vacation.

At time t, let N(t) be the number of customers in the retrial queue, W(t) the elapsed retrial time of the customer in the retrial queue, X(t) the elapsed service time of the customer in service, Y(t) the elapsed vacation time of the server and R(t) the elapsed reserved time of the server. Define the following state probabilities :

$I_n(t, w)\,dw$ is the joint probability that at time t, there are n customers in the retrial orbit, the server is idle, and the elapsed retrial time of a customer in the retrial queue is between w and w + dw, where $n \geq 1$.

$P_n^1(t, x)\,dx$ $(P_n^2(t, x)\,dx)$ is the joint probability that at time t there are n customers in the retrial orbit, and a customer is being served in the essential (optional) service with the elapsed service time between x and $x + dx$, where $n \geq 0$.

$E_{0,n}(t, x, y)\,dx\,dy$ $(E_{1,n}(t, x, y)\,dx\,dy)$ is the joint probability that at time t there are n customers in the retrial orbit, the server is in vacation, the elapsed service time for the customer under essential service is x, the interrupted customer remains in the service position (not in service position) and the elapsed vacation time is between y and $y+dy$, where $n \geq 0$.

$R_n(t, x, r)\,dx\,dr$ is the joint probability that at time t there are n customers in the retrial orbit, the elapsed service time for the customer under essential service is equal to x, with the elapsed reserve time between r and $r + dr$, where $n \geq 0$.

$V_n(t, y)\,dy$ is the joint probability that at time t there are n customers in the retrial orbit, the server is in vacation and the elapsed vacation time is between y and $y + dy$, where

$n \geq 0$.

3. STABILITY CONDITION

The inequality $\lambda p \bar{C}_1 [\mu_{11}(1 + \beta(\frac{1-r_1}{\theta} + h_1)) + (1 - \tau)\,\mu_{21} + (1-\tau)(1-\alpha)\,v_1] < 1 - (1 - A^*(\lambda))\,(\bar{C}_1 - 1 + q)\,(\rho_1\,\tau + (1 - \tau)\,(\rho_2\,\alpha + \rho_3\,(1 - \alpha)))$ is the necessary and sufficient condition for the system to be stable.

4. STEADY STATE DISTRIBUTIONS

The steady state equations corresponding to this modal are

$$\frac{dI_n(w)}{dw} = -(\lambda + \eta(w))\,I_n(w), \qquad n \geq 1 \tag{1}$$

$$\frac{dP_n^1(x)}{dx} = -(p\lambda + \mu_1(x) + \beta)\,P_n^1(x) + \int_0^\infty E_{0,n}(x, y)\,\gamma(y)\,dy + \theta \int_0^\infty R_n(x,r)dr + p\lambda(1-\delta_{0n}) \sum_{k=1}^{n} C_k\,P_{n-k}^1(x), n\geq 0 \tag{2}$$

$$\frac{dP_n^2(x)}{dx} = -(p\lambda+\mu_2(x))\,P_n^2(x) p\lambda(1-\delta_{0n}) \sum_{k=1}^{n} C_k\,P_{n-k}^2(x),\ n\geq 0 \tag{3}$$

$$\frac{\partial E_{i,n}(x,\ y)}{\partial y} = -(p\lambda + \gamma(y))E_{i,n}(x,y)+p\lambda\,(1-\delta_{0n})\sum_{k=1}^{n} C_k\, E_{i,n-k}(x,\ y), \qquad n \geq 0\ ;\ i = 0,\ 1 \tag{4}$$

$$\frac{\partial R_n(x,\ r)}{\partial r} = -(p\lambda + \theta)\,R_n(x,\ r) + p\lambda\,(1-\delta_{0n})\sum_{k=1}^{n} C_k\, R_{n-k}(x,\ r), \qquad n \geq 0 \tag{5}$$

$$\frac{d\,V_n(y)}{dy} = -(p\lambda + \phi(y))\,V_n(y) + p\lambda\,(1-\delta_{0n})\sum_{k=1}^{n} C_k\, V_{n-k}(y), \quad n \geq 0 \tag{6}$$

The steady state boundary conditions are

$$I_n(0) = \rho_1\,\tau\int_0^\infty P_n^1(x)\,\mu_1(x)\,dx + \rho_2\,\alpha\int_0^\infty P_n^2(x)\,\mu_2(x)\,dx + \rho_3\int_0^\infty V_n(y)\ \phi(y)\,dy, \qquad n \geq \tag{7}$$

$$P_0^1(0) = \int_0^\infty I_1(w)\,\eta(w)dw + \lambda C_1(1-q)\int_0^\infty I_1(w)\,dw + (1-\rho_3)\int_0^\infty V_1(y)\,\phi(y)dy + \tau(1-\rho_1)\int_0^\infty P_1^1(x)\,\mu_1(x)dx + \alpha(1-\rho_2)\int_0^\infty P_1^2(x)\,\mu_2(x)\,dx \tag{8}$$

$$P_n^1(0) = \int_0^\infty I_{n+1}(w)\,\eta(w)\,dw + q\,\lambda\sum_{k=1}^{n} C_k\int_0^\infty I_{n-k+1}(w)\,dw + \lambda(1-q)\sum_{k=1}^{n+1}\int_0^\infty I_{n-k+2}(w)\,dw + \tau(1-\rho_1)\int_0^\infty P_{n+1}^1(x)\,\mu_1(x)dx + \alpha(1-\rho_2)\int_0^\infty P_{n+1}^2(x)\,\mu_2(x)\,dx$$
$$+ (1-\rho_3)\int_0^\infty V_{n+1}(y)\,\phi(y)\,dy, \quad n \geq 1 \tag{9}$$

$$P_n^2(0) = (1-\tau)\int_0^\infty P_n^1(x)\,\mu_1(x)\,dx, \ n \geq 0 \tag{10}$$

$$E_{0,n}(x,\ 0) = r_1\,\beta\,P_n^1(x), \qquad n \geq 0 \tag{11}$$

$$E_{1,n}(x,\ 0) = (1-r_1)\,\beta\,P_n^1(x), \ n \geq 0 \tag{12}$$

$$R_n(x,\ 0) = \int_0^\infty E_{1,n}(x,y)\,\gamma(y)\,dy, \ n \geq 0 \tag{13}$$

$$V_0(0) = \int_0^\infty V_0(y)\,\phi(y)\,dy + \tau\int_0^\infty P_0^1(x)\,\mu_1(x)\,dx + \int_0^\infty P_0^2(x)\,\mu_2(x)\,dx, \tag{14}$$

$$V_n(0) = (1-\alpha)\int_0^\infty P_n^2(x)\,\mu_2(x)\,dx, \ n \geq 1 \tag{15}$$

Solving equations (1) – (15) we get,

The partial probability generating function of the orbit size when the server is idle is given by

$$I(z) = \frac{z\,M\,T_3(z)\,(1 - A^*(\lambda))\,V_0(0)}{\lambda\,D(z)}$$

The partial generating function of the orbit size when the server is busy in essential service and in optional service are respectively

$$P^1(z) = \frac{M\,T_1(z)\,[1 - K(z)]\,V_0(0)}{D(z)\,G(p\lambda(1 - C(z)))} \quad \text{and}$$

$$P^2(z) = \frac{M\,(1 - \tau)\,T_1(z)\;K(z)\,[1 - B_2^*(p\lambda\,(1 - C(z)))]\,V_0(0)}{p\lambda\,(1 - C(z))\,D(z)}$$

The partial generating function of the orbit size when the server is in emergency vacation and the customer in service remains in the service position and not in service position are respectively

$$E_0(z) = \frac{r_1\,\beta\,M\,T_1(z)\,[1 - K(z)]\,[1 - H^*(p\lambda\,(1 - C(z)))]\,V_0(0)}{p\lambda\,(1 - C(z))\,D(z)\,G\,(p\lambda\,(1 - C(z)))}$$

and

$$E_1(z) = \frac{(1 - r_1)\,\beta\,M\,T_1(z)\,[1 - K(z)]\,[1 - H^*(p\lambda\,(1 - C(z)))]\,V_0(0)}{p\lambda\,(1 - C(z))\,D(z)\,G\,(p\lambda\,(1 - C(z)))}$$

The partial probability generating function of the orbit size when the server is reserved is,

$$R(z) = \frac{(1 - r_1)\,\beta\,M\,T_1(z)\;H^*(p\lambda\,(1 - C(z)))\,[1 - K(z)]\,V_0(0)}{D(z)\,G\,(p\lambda\,(1 - C(z)))\,[p\lambda\,(1 - C(z)) + \theta]}$$

The partial probability generating function of the orbit size when the server is in multiple vacation is given by

$$V(z) = \frac{M\;T_4(z)\,[1 - F^*(p\lambda\,(1 - C(z)))]\,V_0(0)}{p\lambda\,(1 - C(z))\,D(z)}$$

Using the normalizing condition $I(1)+P^1(1)+P^2(1)+E_0(1)+E_1(1)+R(1)+V(1)=1$, the expression for $V_0(0)$ is obtained as

$$V_0(0)= \frac{[\tau + (1-\tau) B_2^*(p\lambda)]\, T_1}{N_1\, [\tau + (1-\tau) B_2^*(p\lambda)\, (\alpha + (1-\alpha) F^*(p\lambda))]}$$

where

$$G(x) = x + \beta - \beta \left(\frac{r_1 x + \theta}{x + \theta}\right) H^*(x),$$

$$K(z) = B^*(G(p\lambda (1 - C(z))))$$

$$M = \frac{[\tau + (1-\tau) B_2^*(p\lambda)\, (\alpha + (1-\alpha) F^*(p\lambda))]}{[\tau + (1-\tau) B_2^*(p\lambda)]}$$

$$T_1(z) = T_2(z)[1-\rho_3 F^*(p\lambda(1-C(z)))]-z(1-\rho_3) F^*(p\lambda(1 - C(z)))$$

$$T_2(z) = z A^*(\lambda) + C(z) (1 - A^*(\lambda)) (1 - q + qz)$$

$$D(z) = zK(z)[\tau (1-\rho_1)+(1-\tau) B_2^* (p\lambda (1-C(z)))(\alpha(1-\rho_2)+(1-\alpha)(1-\rho_3)F^*(p\lambda(1-C(z))))]+K(z)T_2(z)[\rho_1\tau+(1-\tau) B_2^* (p\lambda(1-C(z)))(\rho_2\alpha+\rho_3(1-\alpha)F^*(p\lambda(1-C(z))))]-z^2$$

$$T_3(z) = [1-\rho_3F^*(p\lambda(1-C(z)))][z-K(z)[\tau+(1-\tau) B_2^* (p\lambda(1-C(z))) (\alpha+(1-\alpha)F^*(p\lambda(1-C(z))))]]+K(z)[1-F^*(p\lambda(1-C(z)))] [\rho_1\tau+(1-\tau) B_2^* (p\lambda(1-C(z)))(\alpha\rho_2+(1-\alpha)\rho_3 F^*(p\lambda(1C(z))))]$$

$$T_4(z)=zK(z)[\tau(1-\rho_1)+\alpha(1-\tau)(1-\rho_2) B_2^* (p\lambda(1-C(z)))] +K(z)T_2(z)[\rho_1\tau+(1-\tau) B_2^* (p\lambda(1- C(z)))(1-\alpha+\alpha\rho_2)] - z^2$$

$$T_1 = 1-K'(1)-p\lambda \overline{C}_1(1-\tau)[\mu_{21}+(1-\alpha)v_1]-(1- A^*(\lambda))$$

$$(\overline{C}_1-1+q)(\rho_1\tau+(1-\tau)(\rho_2\, \alpha+\rho_3\, (1- \alpha)))$$

$$N_1 = (1-A^*((\lambda)(1-\rho_3)([1-q-\overline{C}_1(1-\rho)][K'(1)+\lambda(1-\tau)\mu_{21}] - p\overline{C}_1)/(\lambda p\overline{C}_1)+v_1(1-K'(1)+p\overline{C}_1K'(1)+(1-A^*(\lambda))(1-q-\overline{C}_1(1-\rho))(\rho_1\tau+(1- \tau)\, (1 - \alpha + \alpha\, \rho_2)))$$

$$K'(1) = p\, \lambda\, \overline{C}_1\, \mu_{11}\, (1 + \beta \left(\frac{1-r_1}{\theta} + h_1\right))$$

5. PERFORMANCE MEASURES

The steady state probability that the server is idle during the retrial time is

$I_1 = I(1) = (1 - A^*(\lambda))\, N_2 / (\lambda N_1)$

The steady state probability that the server busy is given by

$P_1 = P^1(1) + P^2(1) = N_3\,[\mu_{11}+(1-\tau)\,\mu_{21}] / N_1$

The steady state probability that the server is in emergency vacation is

$E_1 = E_0(1) + E_1(1) = \beta\, \mu_{11}\, h_1\, N_3 / N_1$ The steady state probability that the server is in reserved time is

$R_1 = (1 - r_1)\, \beta\, \mu_{11}\, N_3 / (\theta N_1)$

The steady state probability that the server is in multiple vacation is

$V_1 = v_1 N_4 / N_1$

where

$N_2 = p\lambda \overline{C}_1 v_1[\rho_1\tau+(1-\tau)(\alpha\rho_2+(1-\alpha))]-(1-\rho_3)(1- K'(1) - (1-\tau)\, p\,\lambda\, \overline{C}_1\, \mu_{21})$

$N_3 = p\,\lambda \overline{C}_1\, v_1 -(1-\rho_3)\,(1 - A^*(\lambda))+(\overline{C}_1 - 1 + q)$

$N_4 = 1 - K'(1) - (1-\tau)\, p\,\lambda\, \overline{C}_1\, \mu_{21} - (1 - A^*(\lambda))(\overline{C}_1 - 1 + q)\,(\rho_1\,\tau + (1-\tau)\,(1 - \alpha + \alpha\,\rho_2))$

The probability generating function of the number of customers in the retrial queue and the system is respectively given by

$P_q(z) = I(z)+P^1(z)+ P^2(z)+E_0(z)+E_1(z)+R(z) + V(z)$

$=V_0(0)M\{[1-K_2(z)(\tau+(1-\tau)B_2^*(p\lambda(1-C(z))))]\,T_1(z)+pz(1-C(z))(1-A^*(\lambda))T_3(z)+[1-F^*(p\lambda(1- C(z)))]\,T_4(z)\} / [p\lambda\,(1 - C(z))\,D(z)]$

$P_s(z) =I(z)+z[P^1(z)+P^2(z)+E_0(z)+E_1(z)+R(z)]+V(z)$

$=V_0(0)M\{z[1-K_2(z)(\tau+(1-\tau)B_2^*(p\lambda(1-C(z))))]\,T_1(z)+pz(1-C(z))(1-A^*(\lambda))T_3(z)+[1-F^*(p\lambda(1-C(z)))]T_4(z)\}/[p\lambda(1- C(z))\,D(z)]$

The steady state availability of the server as is

$$A = \sum_{n=1}^{\infty} \int_0^{\infty} I_n(w)\, dw + \sum_{n=0}^{\infty} \sum_{i=1}^{2} \int_0^{\infty} P_n^i(x)\, dx + \sum_{n=0}^{\infty} \int_0^{\infty}\int_0^{\infty} R_n(x, r)\, dx\, dr$$

$$= (1-A^*(\lambda))N_2 /(\lambda N_1)+N_3(\mu_{11}+(1-\tau)\mu_{21})/N_1 + (1 - r_1)\, \beta\, \mu_{11}\, N_3 / (\theta N_1)$$

6. NUMERICAL RESULTS

Assume that the distributions of essential service time, optional service time, retrial time, emergency vacation time and multiple vacation times distributions are exponential with rate respectively μ_1, μ_2, η, γ and ϕ.

Table 1 presents the performance measures I_1, P_1 and V_1 for the fixed parameters $(\lambda, \eta, \mu_1, \mu_2, \gamma, \phi, \beta, r, \rho_1, \rho_2, \rho_3, \tau, \theta, \alpha, C_1, C_2, p, q) = (1, 6, 30, 35, 3, 1, 5, 0.5, 0.5, 0.5, 0.5, 0.5, 4, 0.4, 0.6, 0.4, 0.8, 0.8)$. It is observed that I_1 and P_1 decrease and V_1 increases for the increasing values of $1 - \rho_i$, $i = 1, 2, 3$. From Table 2, we see that for the increasing values μ_1 and μ_2, the values of I_1 and P_1 decrease whereas V_1 increases for the case $(\lambda, \eta, \mu_1, \mu_2, \gamma, \phi, \beta, r_1, \rho_1, \rho_2, \rho_3, \tau, \theta, \alpha, C_1, C_2, p, q) = (2, 6, 30, 35, 3, 1, 5, 0.5, 0.5, 0.5, 0.5, 0.5, 4, 0.4, 0.6, 0.4, 0.8, 0.8)$. For $\lambda = 1$, $\mu_1 = 10$, $\mu_2 = 15$, $\gamma = 3$, $\phi = 1$, $\beta = 5$, $r_1 = 0.5$, $\rho_1 = \rho_2 = \rho_3 = 1$, $\tau = 0.5$, $p = 0.6$ and $q = 0.6$ the performance measures are calculated by varying η.

The combined effect of (μ_1, λ), $(\eta, 1 - \alpha)$, (η, β) and (μ_1, β) on the availability of the server A are displayed in figures (a) – (d).

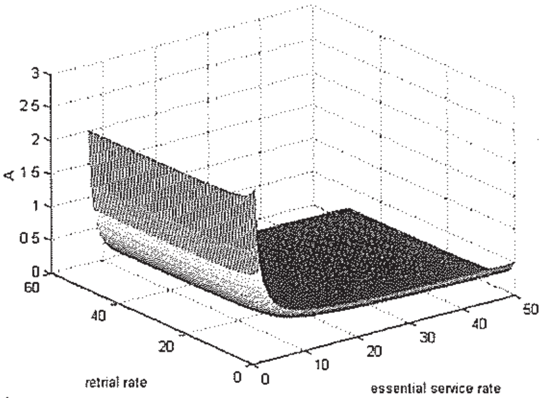

Figure (a)

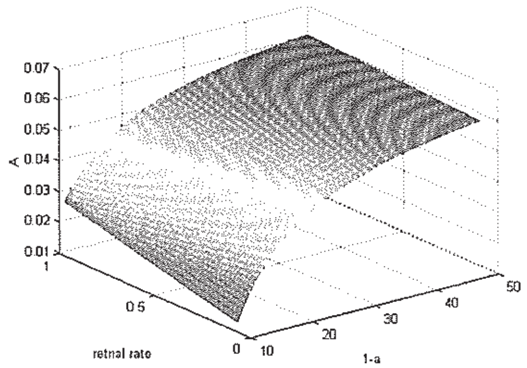

Figure (b)

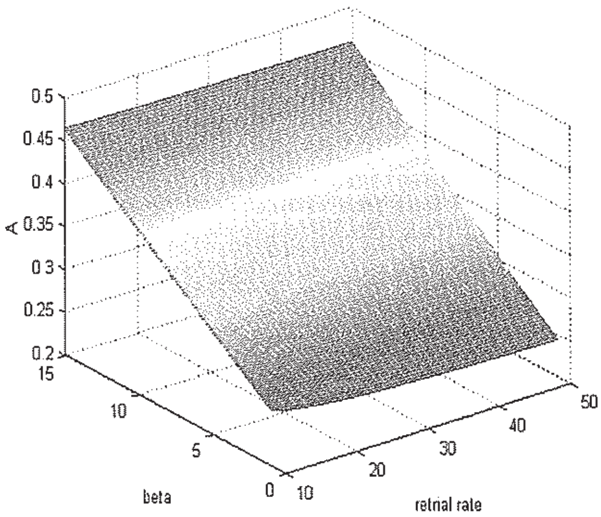

Figure (c)

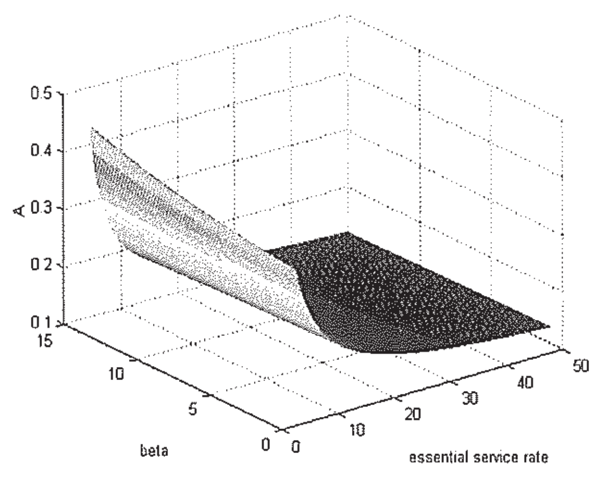

Figure (d)

Figure (a) A versus (μ_1, η) for a queue with (λ, μ_2, γ, ϕ, β, r_1, ρ_1, ρ_2, ρ_3, τ, θ, α, C_1, C_2, p, q) = (1, 5, 3, 1, 5, 0.5, 0.5, 0.5, 0.8, 0.5, 4, 0.5, 0.6, 0.4, 0.8, 0.8)

Figure (b) A versus (η, $1-\alpha$) for a queue with (λ, μ_1, μ_2, γ, ϕ, β, r_1, ρ_1, ρ_2, ρ_3, τ, θ, C_1, C_2, p, q) = (1, 20, 25, 3, 5, 5, 0.5, 0.6, 0.5, 0.8, 0.5, 4, 0.6, 0.4, 0.5, 0.5)

Figure (c) A versus (η, β) for a queue with (λ, μ_1, μ_2, γ, ϕ, r_1, ρ_1, ρ_2, ρ_3, τ, θ, α, C_1, C_2, p, q) = (1, 10, 5, 5, 1, 0.5, 0.5, 0.5, 0.5, 0.5, 4, 0.5, 0.6, 0.4, 0.8, 0.8)

Figure (d) A versus (μ_1, β) for a queue with (λ, η, μ_2, γ, ϕ, r_1, ρ_1, ρ_2, ρ_3, τ, θ, α, C_1, C_2, p, q) = (1, 3, 15, 1, 1,0.5, 0.5, 0.5, 0.5, 0.5, 4, 0.5, 0.6, 0.4, 0.8, 0.8)

Table 1. Performance measures Vs. 1 – ρ_i

1 – ρ_i	i = 1			i = 2			i = 3		
	I_1	P_1	V_1	I_1	P_1	V_1	I_1	P_1	V_1
0.0	0.2183	0.1050	0.4771	0.1734	0.1047	0.5230	0.1790	0.1049	0.5187
0.1	0.2033	0.1049	0.4925	0.1674	0.1046	0.5291	0.1723	0.1048	0.5233
0.2	0.1883	0.1048	0.5077	0.1615	0.1046	0.5351	0.1654	0.1048	0.5320
0.3	0.1734	0.1047	0.5230	0.1555	0.1045	0.5412	0.1584	0.1047	0.5389
0.4	0.1585	0.1046	0.5382	0.1496	0.1045	0.5473	0.1511	0.1046	0.5461
0.5	0.1436	0.1045	0.5534	0.1436	0.1045	0.5534	0.1436	0.1045	0.5534
0.6	0.1288	0.1043	0.5685	0.1377	0.1044	0.5594	0.1360	0.1044	0.5609
0.7	0.1140	0.1042	0.5836	0.1318	0.1044	0.5655	0.1281	0.1042	0.5686
0.8	0.0993	0.1041	0.5987	0.1259	0.1043	0.5715	0.1200	0.1041	0.5765
0.9	0.0845	0.1040	0.6138	0.1199	0.1043	0.5776	0.1117	0.1040	0.5846
1.0	0.0698	0.1039	0.6288	0.1140	0.1042	0.5836	0.1032	0.1039	0.5930

Table 2. Performance measures Vs. Service rate

μ_i	i = 1			i = 2		
	I_1	P_1	V_1	I_1	P_1	V_1
10	0.0623	0.1227	0.5230	0.0485	0.0922	0.7590
15	0.0540	0.0882	0.6603	0.0471	0.0737	0.761
20	0.0498	0.0706	0.7304	0.0464	0.0645	0.7889
25	0.0472	0.0599	0.7730	0.0459	0.0590	0.7948
30	0.0454	0.0527	0.8016	0.0456	0.0553	0.7988
35	0.042	0.0475	0.8222	0.0454	0.0527	0.8016
40	0.0433	0.0436	0.8376	0.0453	0.0507	0.8038
45	0.0425	0.0406	0.8479	0.0452	0.0492	0.8054
50	0.0419	0.0391	0.8594	0.0451	0.0479	0.8068

Table 3. Performance measures Vs. Retrial rate

μ_i	i = 1			i = 2		
	I_1	P_1	V_1	I_1	P_1	V_1
10	0.0623	0.1227	0.5230	0.0485	0.0922	0.7590
15	0.0540	0.0882	0.6603	0.0471	0.0737	0.761
20	0.0498	0.0706	0.7304	0.0464	0.0645	0.7889
25	0.0472	0.0599	0.7730	0.0459	0.0590	0.7948
30	0.0454	0.0527	0.8016	0.0456	0.0553	0.7988
35	0.042	0.0475	0.8222	0.0454	0.0527	0.8016
40	0.0433	0.0436	0.8376	0.0453	0.0507	0.8038
45	0.0425	0.0406	0.8479	0.0452	0.0492	0.8054
50	0.0419	0.0391	0.8594	0.0451	0.0479	0.8068

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