ANOM PROCEDURE FOR EXPONENTIAL, GAMMA VARIATE

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Abstract: A measurable quality characteristic is assumed to follow exponential/gamma distribution. The constants for the limits of the control chart for individual observations are developed for these two models. The principle is adopted to work out the decision lines useful for executing the well known "Analysis of Means (ANOM)" procedure. The preferability of the proposed ANOM decision lines over that of Ott (1967) is illustrated by some examples.

Keywords: ANOM, Confidence Interval, Exponential, Gamma, Control Charts for Individual observations.

1. INTRODUCTION

In classical statistical inference confidence intervals for unknown parameters of a statistical population is an inferential procedure. An application of confidence interval usually the 99.73% confidence interval when the variate follows normal distribution is the origin of the well-known Shewart control charts. Construction of control charts using the theory of confidence intervals when the variate follows Inverse Gaussian distribution is considered by Edgemen (1989). The justification of Edgemen (1989) is that the appeal to the central limit theorem for non-normal process control charts is not possible as sample size in the control chart analysis is usually 10 or less. Moreover quality variate such as product life is often better modeled by the probability distribution of a positive valued random variable rather than a normal distribution.

If normal distribution is considered as a central model for any classical inferential procedure similar place can be attributed to exponential distribution in life testing and reliability studies. It is the only model exemplifying constant failure rate of a product which has life and eventual failure or death. On the other hand we know that ageing of the product is a primary criterion that contributes to its failure. Hence models that represent ageing phenomenon of a product are also equally desirable in problems of quality control and reliability. Among many such models gamma distribution with shape 2 is an ageing model also called an increasing failure rate model. From a different version it is a generalization of exponential distribution also. Kantam and Sriram (2001) developed control charts to be used when the process variate follows a gamma distribution. Similar study is made in Kantam *et al* (2006).In this paper we made an attempt to develop control chart for individual observations representing a quality characteristics modeled by exponential/gamma distribution. Some times such situations are of practical importance as exemplified in Montegomery (1997, P.221). The development of control chart constants is given in Section 2. Application of these constants for the well-known Analysis of Means is described in Section 3 with illustrations.

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2. CONTROL CHART FOR INDIVIDUAL OBSERVATIONS

Sometimes we may come across situations wherein the sample size used for monitoring a process is just one. That is a sample consists of an individual unit. Some examples of these situations are as follows (Montgomery, 1997; P.221).

a. Automated inspection and measurement technology is used, and every unit manufactured is analyzed.

b. The production rate is very low, and it is inconvenient to allow samples sizes of n>1 to accumulate before analysis.

c. Repeat measurements on the process differ only because of laboratory or analysis error, as in many chemical process.

d. In process plants, such as papermaking, measurements on some parameter such as coating thickness across the roll will differ very little and produce a standard deviation that is much too small if the objective is to control coating thickness along the roll.

In such situations the control chart for individual observation is necessary. Generally the control procedure uses the moving range of two successive observations to estimate the process variability where the moving range is defined as $\left|x_i - x_{i-1}\right|$, i=1, 2, ..n.

Care should be exercised in interpreting patterns on the moving range chart because the moving ranges are correlated whereas the individual measurements on the chart of individual observations are assumed to be uncorrelated. If this has been the existing practice of dealing with a control chart of individual observation, we adopt a slightly different procedure to overcome problem of correlation in developing the control chart. Our study confines to only exponential/gamma populations.

If 'x' is a random variable on which individual measurements of quality data are available we have to find two limits L and U such that

$$P\{L < x < U\} = 0.9973 \qquad (2.1)$$

If z corresponds to the standard variate, from the c.d.f of z we can have L^*, U^* such that

$$P\{L^* < z < U^*\} = 0.9973 \qquad (2.2)$$

Since $z=x/\sigma$ it follows that $L=\sigma L^*$ and $U=\sigma U^*$. Having known L^*, U^* from standard c.d.f of the population (equitailed situation), Equation (5.1) becomes

$$P\{\sigma L^* < x < \sigma U^*\} = 0.9973 \qquad (2.3)$$

If the scale parameter 'σ' is unknown we have to estimate it to get the estimated control limits for x. In this section we estimate 'σ' using statistics-Sample mean, Sample median, Sample range, Inter quartile range.

i). Exponential Model

The constants L^*, U^* of Equation (2.2) for an exponential model with equitailed probabilities are $L^*= 0.00135$, $U^*= 6.60765$ the estimates of 'σ' as listed above are $\hat{\sigma}_1 = \bar{x}$, $\hat{\sigma}_2 = m$, $\hat{\sigma}_3 = R$, $\hat{\sigma}_4 = IQR$.

From Equation (2.3) the control limits with $\hat{\sigma}_1$ are $\left[L^*\bar{x}, U^*\bar{x}\right]$ where L^*, U^* are given above. When $\hat{\sigma}_2$ is used

$$\hat{\sigma}_2 = X_{(k)} \text{ when the sample size is 2k+1}$$

$$= \frac{X_{(k)} + X_{(k+1)}}{2} \text{ when the sample size is 2k.}$$

Then the control limits are $\left[\frac{L^*M}{\alpha_{(k)}}, \frac{U^*M}{\alpha_{(k)}}\right]$ or $\left[\frac{2L^*M}{\alpha_{(k)}+\alpha_{(k+1)}}, \frac{2U^*M}{\alpha_{(k)}+\alpha_{(k+1)}}\right]$ according as the sample size is 2k+1 or 2k, where $\alpha_{(i)}$ is expected value of i^{th} order statistic in a standard exponential given in Sarhan and Greenberg (1962). When $\hat{\sigma}_3$ is used the control limits are $\left[\frac{L^*R}{\alpha_{(n)}-\alpha_{(1)}}, \frac{U^*R}{\alpha_{(n)}-\alpha_{(1)}}\right]$. When σ is estimated by $\hat{\sigma}_4$ the control limits are $\left[\frac{L^*IQR}{E(Z_{Q_3}-Z_{Q_1})}, \frac{U^*IQR}{E(Z_{Q_3}-Z_{Q_1})}\right]$ where Z_{Q_3}, Z_{Q_1} are respectively the upper and lower quartiles of the sample. The control limits using $\hat{\sigma}_1$ are simple in form and do not require any specific tabulation. The limits based on $\hat{\sigma}_2, \hat{\sigma}_3, \hat{\sigma}_4$ need the theory of order statistics in samples from exponential model. As given in Sarhan and Greenberg (1962) for n=2(1)10 the control limit constants are evaluated by us and are given in Table 2.1.

II) Gamma Model

The constants L', U' similar to those of Equation (2.2) for a gamma model with equitailed probabilities are $L' = 0.053$, $U' = 8.9$. The estimates of σ as listed in earlier lines now, are $\hat{\sigma}_1 = \frac{\bar{x}}{2}$, $\hat{\sigma}_2 = m$, $\hat{\sigma}_3 = R$, $\hat{\sigma}_4 = IQR$. From Equation (2.3) the control limits with $\hat{\sigma}_1$ are $\left[\frac{L'\bar{x}}{2}, \frac{U'\bar{x}}{2}\right]$ where L', U' are given above.

When $\hat{\sigma}_2$ is used

$$\hat{\sigma}_2 = X_{(k)} \text{ when the sample size is 2k+1}$$

$$= \frac{X_{(k)} + X_{(k+1)}}{2} \text{ when the sample size is 2k.}$$

Then the control limits are $\left[\frac{L'M}{\alpha_{(k)}}, \frac{U'M}{\alpha_{(k)}}\right] or \left[\frac{2L'M}{\alpha_{(k)} + \alpha_{(k+1)}}, \frac{2U'M}{\alpha_{(k)} + \alpha_{(k+1)}}\right]$ according as the sample size is 2k+1 or 2k, where $\alpha_{(i)}$ is expected value of i^{th} order statistic in a standard gamma model given in Gupta (1960). When $\hat{\sigma}_3$ is used the control limits are $\left[\frac{L'R}{\alpha_{(n)} - \alpha_{(1)}}, \frac{U'R}{\alpha_{(n)} - \alpha_{(1)}}\right]$. When σ is estimated by $\hat{\sigma}_4$ the control limits are $\left[\frac{L'IQR}{E(Z_{Q_3} - Z_{Q_1})}, \frac{U'IQR}{E(Z_{Q_3} - Z_{Q_1})}\right]$ where Z_{Q_3}, Z_{Q_1} are respectively the upper and lower quartiles of the sample. The control limits using $\hat{\sigma}_1$ are simple in form and do not require any specific tabulation. The limits based on $\hat{\sigma}_2, \hat{\sigma}_3, \hat{\sigma}_4$ need the theory of order statistics in samples from gamma model. As given in Gupta (1960) for n=2(1)10 the control limit constants are evaluated by us and are given in Table 2.2.

The constants given in Tables 2.1, 2.2 are control limits for individual observations of a control chart under exponential and gamma models. As explained above different pairs of control limits are based on different linear unbiased estimators of σ. Hence a relative comparison would be desirable for the pairs of constants. Here we have taken the variance of the width of each control band using the variances and covariances of order statistics in exponential and gamma models. These are given in Tables 2.3 and 2.4. respectively, which reveal that the variance of the control band is uniformly least when σ is estimated using the sample mean. This is quite expected because sample mean is the BLUE. The next best is either sample median or sample range in exponential according as the sample size is even or odd. On the otherhand the second place is occupied by sample median uniformly for gamma distribution. We therefore suggest that though $\bar{x}$ is unbeatable, as an alternative to $\bar{x}$ one can go for sample range or sample median to estimate σ and get the estimated control limits.

3. ANALYSIS OF MEANS

The Shewart control chart is a common tool of statistical quality control for many practitioners. When these charts indicate the presence of an assignable cause (of non-random variability) are adjustment of the process is made if the remedy is known. Otherwise the suspected presence of assignable cause is regarded to be an indication of heterogeneity of the sub group statistic for which the control chart is developed. For instance the statistic is sample mean, this leads to heterogeneity of process mean indicating departures from target mean. Such an analysis is generally carried out with the help of the well-known analysis of variance. Ott (1967) developed a procedure called analysis of means (ANOM) to divide a collection of a given number of sub group means into categories such that means within a category are homogeneous and those between categories are heterogeneous, under the assumption that the probability model of the variate is normal. We have already noticed that any statistical method if needs to be applied for an non-normal data separate evaluation is essential. In this section we make an attempt to develop the ANOM procedure of Ott (1967) when the data variate is suppose to follow exponential/gamma distribution.

Suppose $\bar{x}_1, \bar{x}_2, \ldots\ldots \bar{x}_k$ are arithmetic means of k subgroups of size 'n' each drawn from an exponential model. If these subgroup means are used to develop control charts to assess whether the population from which these subgroups are drawn is operating with admissible quality variations depending on the basic population model we may use the control chart constants developed by us or the popular Shewart constants given in any SQC textbook. Generally we say that the process is in control if all the subgroup means fall within the control limits. Otherwise we say the process lacks control. If α is the level of significance of the above decisions we can have the following probability statements.

$$P\{LCL < \bar{x}_i, \forall i = 1 to k < UCL\} = 1 - \alpha \qquad (3.1)$$

Using the notion of independent subgroups Equation (3.1) becomes

$$P\{LCL < \bar{x}_i < UCL\} = (1 - \alpha)^{1/k} \qquad (3.2)$$

With equitailed probability for each subgroup mean we can find two constants say L^* and U^* such that

$$P\{\bar{x}_i < L^*\} = P\{\bar{x}_i < U^*\} = \frac{1-(1-\alpha)^k}{2} \qquad (3.3)$$

In the case of normal population L^* and U^* satisfy $U^* = -L^*$. For the skewed populations like exponential/gamma we have to calculate L^*, U^* separately from the sampling distribution of $\bar{x}_i$. Accordingly these depend on the subgroup size 'n' and the number of subgroups 'k'. Since sampling distribution of $\bar{x}$ on the base of sample of size 'n' can be obtained from $\Gamma(n)$, $\Gamma(2n)$ according as the population is exponential or gamma, with the help of incomplete gamma function tables of Pearson(1922), L^*, U^* can be tabulated. These are given in Tables 3.1, 3.2, 3.3, 3.4, 3.5, 3.6. Because the sampling distributions are based on $\Gamma(n)\, and\, \Gamma(2n)$ the even sizes in the case of exponential population model would get repeated at some sizes for gamma population also.

A control chart for averages giving 'In Control' conclusion indicates that all the subgroup means though vary among themselves are homogeneous in some sense. This is exactly the null hypotheses in an analysis of variance technique. Hence the constants of Tables 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 can be used as an alternative to analysis of variance technique. For a normal population one can use the Tables of Ott (1967). For an exponential or gamma population our Tables can be used. We therefore present below some examples for which the goodness of fit of an appropriate model is assessed with Q-Q plot technique (strength of linearity between observed and theoretical quantiles of a model) and tested the homogeneity of means involved in each case.

Example 1: Wadsworth (1986, P.605)

Consider the following data of 20 observations on "A manufactures of metal products that suspect variations in iron content of raw material supplied by five suppliers. Five ingots were randomly selected from each of the five suppliers. The following table contains the data for the iron determinations on each ingots in percent by weight.

Supplier				
1	2	3	4	5
3.46	3.59	3.51	3.38	3.29
3.48	3.46	3.64	3.40	3.46
3.56	3.42	3.46	3.37	3.37
3.39	3.49	3.52	3.46	3.32
3.40	3.50	3.49	3.39	3.38

Treating these 20 observations as a single sample, we have ranked it and got the correlation coefficient between the sample quantile and the corresponding population quantile under three populations namely normal, exponential, gamma (2).Similarly two more examples are borrowed from Satya Prasad (1987) and assessed the goodness of fit of the three models in the same manner.

Example 2: Three brands of batteries are under study. It is suspected that the life (in weeks) of the three brands is different. Five batteries of each brand are tested with the following results.

Weeks of life

Brand 1	Brand 2	Brand 3
100	76	108
96	80	100
92	75	96
96	84	98
92	82	100

Test whether the lives of these brands of batteries are different at 5% level.

Example 3: Four catalysts that may affect the concentration of one component in a three-component liquid mixture are being investigated. The following concentrations are obtained.

Catalyst			
1	2	3	4
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57.0	55.4	50.0
55.8	55.3	54.9	51.7

Test whether the four catalysts have the same affect on the concentration at 5% level.

The goodness of fit as revealed by Q-Q plot (correlation coefficients) are summarized in the following table.

Example/Population	**Exponential**	**Gamma(2)**	**Normal**
Example1	0.963419	0.978498	0.206750
Example2	0.949730	0.974366	0.414920
Example3	0.881030	0.919905	0.444710

Above Table shows that gamma (2) is the best among the three choices. However we have calculated the decision limits for all the populations and verified homogeneity of means. Our conclusions are summarized in the following tables.

	Normal				
	(LDL,UDL)	**No. of counts**			
		In	p=in/k	Out	q=Out/k
Example1 n=5,k=5,α=0.05	[3.517,3.379]	3	0.6	2	0.4
Example2 n=5,k=3,α=0.05	[87.82,95.52]	2	0.7	1	0.3
Example3 n=4,k=4,,α=0.05	[26.14,82.84]	2	0.5	2	0.5

	Exponential				
	(LDL,UDL)	No. of counts			
		In	p=in/k	Out	q=Out/k
Example1 n=5,k=5,α=0.05	[0.7401,8.6656]	5	1	0	0
Example2 n=5,k=3,α=0.05	[22.1881,217.4174]	3	1	0	0
Example3 n=4,k=4,,α=0.05	[9.6078,145.1743]	4	1	0	0

	Gamma(2)				
	(LDL,UDL)	No. of counts			
		In	**p=in/k**	**Out**	q=Out/k
Example1 n=5,k=5,α=0.05	[1.28114,6.8836]	5	1	0	0
Example2 n=5,k=3,α=0.05	[36.72392,217.41]	3	1	0	0
Example3 n=4,k=4,,α=0.05	[18.1574,114.057]	4	1	0	0

TABLE 2.1

Control Chart Constants for Individual Observations of Exponential Distribution

n	$\hat{\sigma}_1$		$\hat{\sigma}_2$		$\hat{\sigma}_3$		$\hat{\sigma}_4$	
	LCL	**UCL**	**LCL**	**UCL**	**LCL**	**UCL**	**LCL**	**UCL**
2	0.00135	6.60765	0.00135	6.60765	0.00135	6.60765	0.00135	6.60765
3	0.00135	6.60765	0.00162	7.92918	0.00090	4.40510	0.00090	4.40510
4	0.00135	6.60765	0.00162	7.92918	0.00074	3.60418	0.00090	4.40510
5	0.00135	6.60765	0.00172	8.43530	0.00065	3.17168	0.00162	7.92921
6	0.00135	6.60765	0.00172	8.43530	0.00059	2.89386	0.00125	6.09937
7	0.00135	6.60765	0.00177	8.69972	0.00055	2.69699	0.00105	5.14879
8	0.00135	6.60765	0.00177	8.69972	0.00052	2.54859	0.00105	5.14879
9	0.00135	6.60765	0.00181	8.86178	0.00049	2.43119	0.00142	6.95542
10	0.00135	6.60765	0.00181	8.86178	0.00048	2.33571	0.00123	6.04626

TABLE 2.2
Control Chart Constants for Individual Observations of Gamma (2)Distribution

n	$\hat{\sigma}_1$		$\hat{\sigma}_2$		$\hat{\sigma}_3$		$\hat{\sigma}_4$	
	LCL	UCL	LCL	UCL	LCL	UCL	LCL	UCL
2	0.02650	4.45000	0.02650	4.45000	0.03533	5.93333	0.03533	5.93333
3	0.02650	4.45000	0.02905	4.87920	0.02355	3.95556	0.02355	3.95556
4	0.02650	4.45000	0.02905	4.87920	0.01932	3.24525	0.02513	4.21926
5	0.02650	4.45000	0.02991	5.02314	0.01706	2.86525	0.04116	6.91200
6	0.02650	4.45000	0.02991	5.02314	0.01562	2.62289	0.03171	5.32558
7	0.02650	4.45000	0.03034	5.09442	0.01460	2.45212	0.02683	4.50567
8	0.02650	4.45000	0.03034	5.09442	0.01384	2.32379	0.02791	4.68734
9	0.02650	4.45000	0.03059	5.13682	0.01324	2.22293	0.03582	6.01502
10	0.02650	4.45000	0.03059	5.13682	0.01275	2.14101	0.03119	5.23809

TABLE 2.3
EXPONENTIAL DISTRIBUTION
VARIANCE OF WIDTH FOR CONTROL BAND IN CONTROL CHARTS

CHART STATISTIC				
n	$\bar{x}$	**Median**	**Range**	**IQR**
2	10.9108	21.8216	43.6432	43.6432
3	4.8492	90.7785	24.2461	24.2461
4	2.7277	14.8387	17.6743	24.2465
5	1.7457	60.7727	14.3154	22.6946
6	1.2123	11.2417	12.2522	15.7528
7	0.8907	45.5983	10.8423	12.2853
8	0.6819	9.0358	9.8158	12.2853
9	0.5388	36.4707	9.0244	11.6726
10	0.4364	7.5516	8.3961	9.5689

TABLE 2.4
GAMMA (2) DISTRIBUTION
VARIANCE OF WIDTH FOR CONTROL BAND IN CONTROL CHARTS

CHART STATISTIC				
n	$\bar{x}$	**Median**	**Range**	**IQR**
2	9.78367	19.56730	60.8762	60.87620
3	4.34830	18.94848	32.1024	32.10270
4	2.44592	12.49523	22.6963	34.84913
5	1.56538	12.39136	18.0234	32.50798
6	1.08707	9.22681	15.2200	21.80056
7	0.79867	9.19114	13.3378	16.57555
8	0.61148	7.31709	11.9837	17.37692
9	0.48314	7.30076	10.9573	16.52759
10	0.39135	6.06289	10.1510	13.26580

TABLE 3.1
EXPONENTIAL DISTRIBUTION
CONSTANTS FOR ANALYSIS OF MEANS (1-α=0.95)

n\k	1	2	3	4	5	6	8	10	15	20	30	40	60
2	(0.12109)	(0.12113)	(0.06771)	(0.05864)	(0.05224)	(0.04804)	(0.04103)	(0.03657)	(0.02986)	(0.02569)	(0.01984)	(0.01796)	(0.01436)
	(2.78582)	(3.18312)	(3.42000)	(3.59911)	(3.70272)	(3.79526)	(3.96724)	(4.09261)	(4.31533)	(4.47958)	(4.76143)	(4.86333)	(5.10875)
3	(0.20342)	(0.15539)	(0.13206)	(0.12102)	(0.11229)	(0.10272)	(0.08832)	(0.08032)	(0.06972)	(0.06427)	(0.05783)	(0.04863)	(0.03112)
	(2.40896)	(2.70260)	(2.87643)	(2.99151)	(3.08321)	(3.15058)	(3.27526)	(3.36611)	(3.52625)	(3.64494)	(3.84771)	(3.92027)	(4.09532)
4	(0.27034)	(0.21807)	(0.19398)	(0.17633)	(0.16525)	(0.15858)	(0.14684)	(0.13451)	(0.11818)	(0.10977)	(0.09949)	(0.09060)	(0.07455)
	(2.19226)	(2.43124)	(2.57178)	(2.66436)	(2.73783)	(2.79175)	(2.89189)	(2.96487)	(3.09303)	(3.18782)	(3.34825)	(3.40669)	(3.54566)
5	(0.32337)	(0.27103)	(0.24204)	(0.22761)	(0.21464)	(0.20498)	(0.19045)	(0.18237)	(0.16333)	(0.15144)	(0.13741)	(0.13340)	(0.11453)
	(2.04878)	(2.25284)	(2.37174)	(2.45091)	(2.51322)	(2.55909)	(2.64359)	(2.70559)	(2.81342)	(2.89355)	(3.02902)	(3.07768)	(3.19474)
6	(0.36691)	(0.31172)	(0.28620)	(0.26717)	(0.25507)	(0.24779)	(0.23042)	(0.21949)	(0.20505)	(0.19035)	(0.17178)	(0.16715)	(0.15111)
	(1.94538)	(2.12435)	(2.23001)	(2.29899)	(2.35367)	(2.39381)	(2.46797)	(2.52148)	(2.61568)	(2.68554)	(2.80368)	(2.84612)	(2.94738)
7	(0.40071)	(0.34777)	(0.31950)	(0.30434)	(0.29015)	(0.28088)	(0.28693)	(0.25486)	(0.23637)	(0.22685)	(0.20362)	(0.19781)	(0.18472)
	(1.86627)	(2.02785)	(2.12181)	(2.18391)	(2.23247)	(2.26824)	(2.33506)	(2.38260)	(2.46763)	(2.52921)	(2.63479)	(2.67263)	(2.76243)
8	(0.43092)	(0.40413)	(0.35163)	(0.33323)	(0.32169)	(0.31257)	(0.29549)	(0.28600)	(0.26590)	(0.25425)	(0.23323)	(0.22623)	(0.21359)
	(1.80285)	(1.95083)	(2.03709)	(2.09327)	(2.13806)	(2.17071)	(2.23050)	(2.27454)	(2.35135)	(2.40700)	(2.50263)	(2.53695)	(2.61818)
9	(0.45627)	(0.40480)	(0.37703)	(0.36087)	(0.34727)	(0.33910)	(0.32261)	(0.31137)	(0.29370)	(0.27995)	(0.26074)	(0.25262)	(0.23795)
	(1.75207)	(1.88849)	(1.96726)	(2.01988)	(2.06065)	(2.09089)	(2.14646)	(2.18659)	(2.25701)	(2.30857)	(2.39583)	(2.42737)	(2.50195)
10	(0.47897)	(0.42732)	(0.40061)	(0.38447)	(0.37156)	(0.36218)	(0.34808)	(0.33531)	(0.31820)	(0.30401)	(0.28543)	(0.27709)	(0.26056)
	(1.70854)	(1.83583)	(1.91021)	(1.95816)	(1.99640)	(2.02408)	(2.07625)	(2.11327)	(2.17857)	(2.22688)	(2.30734)	(2.33665)	(2.40591)

TABLE 3.2
EXPONENTIAL DISTRIBUTION
CONSTANTS FOR ANALYSIS OF MEANS (1-α=0.99)

n\k	1	2	3	4	5	6	8	10	15	20	30	40	60
2	(0.05169)	(0.03620)	(0.02928)	(0.02544)	(0.02262)	(0.02086)	(0.01777)	(0.01590)	(0.01287)	(0.01111)	(0.00843)	(0.00758)	(0.00602)
	(3.71511)	(4.10369)	(4.33500)	(4.48863)	(4.61111)	(4.70687)	(4.88720)	(4.98875)	(5.21167)	(2.38000)	(5.65750)	(5.73500)	(5.99950)
3	(0.11101)	(0.07966)	(0.06897)	(0.06394)	(0.06079)	(0.05889)	(0.04762)	(0.03811)	(0.02554)	(0.01906)	(0.01143)	(0.00953)	(0.00609)
	(3.09223)	(3.37426)	(3.54115)	(3.65153)	(3.73973)	(3.80775)	(3.92612)	(4.01037)	(4.16904)	(4.28912)	(4.54259)	(4.54259)	(4.72886)
4	(0.16435)	(0.13349)	(0.11702)	(0.10927)	(0.10441)	(0.10149)	(0.08969)	(0.08094)	(0.06947)	(0.06349)	(0.05655)	(0.05481)	(0.05161)
	(2.74473)	(2.97142)	(3.10424)	(3.19261)	(3.26313)	(3.31783)	(3.41189)	(3.47827)	(3.60370)	(2.69851)	(3.85541)	(3.89870)	(4.04645)
5	(0.21333)	(0.18172)	(0.16169)	(0.15073)	(0.14385)	(0.13972)	(0.13234)	(0.12205)	(0.10857)	(0.10155)	(0.09337)	(0.09133)	(0.07909)
	(2.51957)	(2.71096)	(2.82311)	(2.89762)	(2.95651)	(3.00249)	(3.08159)	(3.13725)	(3.24348)	(3.32293)	(3.45392)	(3.48996)	(3.61342)
6	(0.25408)	(0.21861)	(0.20391)	(0.18941)	(0.18031)	(0.17484)	(0.16668)	(0.16010)	(0.14398)	(0.13557)	(0.12579)	(0.12335)	(0.10801)
	(2.35886)	(2.52596)	(2.62453)	(2.68903)	(2.74053)	(2.78042)	(2.84958)	(2.89737)	(2.98976)	(3.05846)	(3.17267)	(3.20441)	(3.31033)
7	(0.28889)	(0.25372)	(0.24395)	(0.22575)	(0.21432)	(0.20745)	(0.19721)	(0.19148)	(0.17635)	(0.16648)	(0.15501)	(0.15214)	(0.13665)
	(2.23757)	(2.38712)	(2.47527)	(2.53224)	(2.57875)	(2.61421)	(2.67570)	(2.71836)	(2.80005)	(2.86189)	(2.96300)	(2.99060)	(3.08561)
8	(0.32076)	(0.28518)	(0.26429)	(0.25355)	(0.24610)	(0.23784)	(0.22552)	(0.21862)	(0.20604)	(0.19475)	(0.18162)	(0.17834)	(0.16332)
	(2.14242)	(2.27846)	(2.35804)	(2.41019)	(2.45213)	(2.48428)	(2.53971)	(2.57835)	(2.65212)	(2.70836)	(2.79924)	(2.82440)	(2.91008)
9	(0.34617)	(0.31045)	(0.29180)	(0.27912)	(0.27116)	(0.26609)	(0.25179)	(0.24379)	(0.23329)	(0.22069)	(0.20605)	(0.20239)	(0.18793)
	(2.06443)	(2.19004)	(2.26293)	(2.31142)	(2.34975)	(2.37931)	(2.42986)	(2.46534)	(2.53298)	(2.58461)	(2.66738)	(2.69110)	(2.76864)
10	(0.37029)	(0.33425)	(0.31698)	(0.30307)	(0.29396)	(0.28849)	(0.27616)	(0.26714)	(0.25534)	(0.24451)	(0.22854)	(0.22455)	(0.21053)
	(2.00035)	(2.11633)	(2.18425)	(2.22944)	(2.26486)	(2.29234)	(2.33893)	(2.37184)	(2.43458)	(2.48238)	(2.55905)	(2.58103)	(2.65272)

TABLE 3.3
EXPONENTIAL DISTRIBUTION
CONSTANTS FOR ANALYSIS OF MEANS (1-α=0.9973)

n\k	1	2	3	4	5	6	8	10	15	20	30	40	60
2	(0.02635)	(0.01846)	(0.01513)	(0.01288)	(0.01149)	(0.01066)	(0.00901)	(0.00789)	(0.00638)	(0.00509)	(0.00305)	(0.00246)	(0.00165)
	(4.45000)	(4.83000)	(5.05750)	(5.21167)	(5.33750)	(5.42500)	(5.59000)	(5.69500)	(5.93500)	(6.07250)	(6.25400)	(6.37505)	(6.53000)
3	(0.06514)	(0.05121)	(0.03414)	(0.02563)	(0.02048)	(0.01737)	(0.00736)	(0.01024)	(0.00686)	(0.00512)	(0.00307)	(0.00255)	(0.00174)
	(3.62393)	(3.89697)	(4.05941)	(4.16904)	(4.25906)	(4.32128)	(4.43848)	(4.51372)	(4.68375)	(4.78046)	(4.90830)	(4.99455)	(5.10377)
4	(0.11111)	(0.09300)	(0.07733)	(0.06810)	(0.06482)	(0.06196)	(0.05773)	(0.05543)	(0.05234)	(0.05074)	(0.03565)	(0.02966)	(0.02025)
	(3.17103)	(3.38795)	(3.51721)	(3.60370)	(3.67568)	(3.72506)	(3.81771)	(3.87706)	(4.01099)	(4.08719)	(4.18851)	(4.25484)	(4.34151)
5	(0.15333)	(0.13484)	(0.11780)	(0.10863)	(0.10309)	(0.09974)	(0.09476)	(0.09206)	(0.08384)	(0.07345)	(0.06125)	(0.05818)	(0.05335)
	(2.87932)	(3.06204)	(3.16993)	(3.24348)	(3.30278)	(3.34428)	(3.42216)	(3.47181)	(3.58341)	(3.64747)	(3.73220)	(3.78802)	(3.85919)
6	(0.19285)	(0.16838)	(0.15502)	(0.14405)	(0.13743)	(0.13342)	(0.12746)	(0.13452)	(0.11211)	(0.10312)	(0.09256)	(0.08990)	(0.08572)
	(2.67332)	(2.83229)	(2.91439)	(2.98976)	(3.04186)	(3.07766)	(3.14422)	(3.18739)	(3.28517)	(3.33970)	(3.41313)	(3.46132)	(3.52355)
7	(0.22836)	(0.19935)	(0.18911)	(0.17644)	(0.16866)	(0.16396)	(0.15696)	(0.15317)	(0.14069)	(0.13182)	(0.12141)	(0.11878)	(0.11466)
	(2.51860)	(2.66033)	(2.74423)	(2.80005)	(2.84659)	(2.87815)	(2.93823)	(2.97640)	(3.06198)	(3.11133)	(3.17532)	(3.21832)	(3.27380)
8	(0.25609)	(0.22809)	(0.21578)	(0.20614)	(0.19724)	(0.19186)	(0.18385)	(0.17952)	(0.16186)	(0.15839)	(0.14773)	(0.14505)	(0.13894)
	(2.39721)	(2.52592)	(2.60184)	(2.65212)	(2.69422)	(2.72253)	(2.77737)	(2.81182)	(2.88946)	(2.93289)	(2.99194)	(3.03028)	(3.07957)
9	(0.28213)	(0.25478)	(0.24049)	(0.23336)	(0.22347)	(0.21747)	(0.20854)	(0.20370)	(0.19219)	(0.18289)	(0.17186)	(0.16910)	(0.15979)
	(2.30000)	(2.41740)	(2.48698)	(2.53297)	(2.57151)	(2.59771)	(2.64776)	(2.67924)	(2.74403)	(2.79057)	(2.84389)	(2.87927)	(2.92438)
10	(0.30651)	(0.27952)	(0.26342)	(0.25539)	(0.24754)	(0.24099)	(0.23126)	(0.22599)	(0.21490)	(0.20532)	(0.19407)	(0.19124)	(0.18022)
	(2.21791)	(2.32756)	(2.39201)	(2.43458)	(2.47018)	(2.49459)	(2.54067)	(2.56977)	(2.63563)	(2.67286)	(2.72147)	(2.75395)	(2.79614)

TABLE 3.4
GAMMA (2) DISTRIBUTION
CONSTANTS FOR ANALYSIS OF MEANS (1-α=0.95)

n\k	1	2	3	4	5	6	8	10	15	20	30	40	60
2	(0.27033)	(0.21807)	(0.19398)	(0.17633)	(0.16525)	(0.15859)	(0.14684)	(0.13451)	(0.11818)	(0.10977)	(0.09949)	(0.09060)	(0.07455)
	(2.19226)	(2.43124)	(2.57178)	(2.66436)	(2.73783)	(2.79175)	(2.89189)	(2.96487)	(3.09303)	(3.18782)	(3.34825)	(3.40669)	(3.54566)
3	(0.36691)	(0.31172)	(0.28620)	(0.26717)	(0.25507)	(0.24779)	(0.23042)	(0.21949)	(0.20505)	(0.19035)	(0.17178)	(0.16715)	(0.15111)
	(1.94538)	(2.12435)	(2.23001)	(2.29899)	(2.35367)	(2.39381)	(2.46797)	(2.52148)	(2.61568)	(2.68554)	(2.80368)	(2.84612)	(2.94738)
4	(0.43092)	(0.40413)	(0.35163)	(0.33323)	(0.32169)	(0.31257)	(0.29549)	(0.28600)	(0.26590)	(0.25425)	(0.23323)	(0.22623)	(0.21359)
	(1.80285)	(1.95083)	(2.03709)	(2.09327)	(2.13806)	(2.17071)	(2.23050)	(2.27454)	(2.35135)	(2.40700)	(2.50263)	(2.53695)	(2.61818)
5	(0.47897)	(0.42732)	(0.40061)	(0.38447)	(0.37156)	(0.36218)	(0.34808)	(0.33531)	(0.31820)	(0.30401)	(0.28543)	(0.27709)	(0.26056)
	(1.70854)	(1.83582)	(1.91021)	(1.95816)	(1.99640)	(2.02408)	(2.07625)	(2.11327)	(2.17857)	(2.22688)	(2.30734)	(2.33665)	(2.40591)
6	(0.51634)	(0.46686)	(0.44047)	(0.42421)	(0.41179)	(0.40433)	(0.38705)	(0.37739)	(0.35772)	(0.34689)	(0.32471)	(0.31908)	(0.30101)
	(1.64049)	(1.75356)	(1.81851)	(1.86191)	(1.89556)	(1.92031)	(1.96529)	(1.99839)	(2.05601)	(2.09827)	(2.16923)	(2.19449)	(2.25559)
7	(0.54585)	(0.49827)	(0.47297)	(0.45761)	(0.44494)	(0.43639)	(0.42128)	(0.41022)	(0.39221)	(0.37989)	(0.35886)	(0.35257)	(0.33577)
	(1.58839)	(1.69053)	(1.74980)	(1.78804)	(1.81829)	(1.84075)	(1.88202)	(1.91160)	(1.96343)	(2.00095)	(2.06509)	(2.08777)	(2.14242)
8	(0.57118)	(0.52557)	(0.50094)	(0.48465)	(0.47379)	(0.46451)	(0.45056)	(0.43892)	(0.42194)	(0.40878)	(0.38846)	(0.38183)	(0.36567)
	(1.54651)	(1.64066)	(1.69472)	(1.73006)	(1.75802)	(1.77814)	(1.81591)	(1.84286)	(1.89028)	(1.92430)	(1.98311)	(2.00377)	(2.05345)
9	(0.59232)	(0.54752)	(0.52360)	(0.50833)	(0.49743)	(0.48897)	(0.47450)	(0.46385)	(0.44742)	(0.43403)	(0.41413)	(0.40751)	(0.39155)
	(1.51239)	(1.59956)	(1.64958)	(1.68287)	(1.70862)	(1.72736)	(1.76199)	(1.78689)	(1.83080)	(1.86218)	(1.91653)	(1.93553)	(1.98122)
10	(0.61024)	(0.56688)	(0.54361)	(0.51954)	(0.51805)	(0.51017)	(0.49572)	(0.48551)	(0.46933)	(0.45623)	(0.43661)	(0.43025)	(0.41428)
	(1.48365)	(1.56515)	(1.61230)	(1.64326)	(1.66732)	(1.68487)	(1.71710)	(1.74035)	(1.78133)	(1.81061	(1.86110)	(1.87866)	(1.92144)

TABLE 3.5
GAMMA (2) DISTRIBUTION
CONSTANTS FOR ANALYSIS OF MEANS (1-α=0.99)

n\k	1	2	3	4	5	6	8	10	15	20	30	40	60
2	(0.16435)	(0.13349)	(0.11702)	(0.10927)	(0.10441)	(0.10149)	(0.08969)	(0.08094)	(0.06947)	(0.06349)	(0.05655)	(0.05481)	(0.05161)
	(2.74473)	(2.97142)	(3.10424)	(3.19261)	(3.26313)	(3.31783)	(3.41189)	(3.47827)	(3.60370)	(2.69851)	(3.85541)	(3.89870)	(4.04645)
3	(0.25408)	(0.21861)	(0.20391)	(0.18941)	(0.18031)	(0.17484)	(0.16668)	(0.16010)	(0.14398)	(0.13557)	(0.12579)	(0.12335)	(0.10801)
	(2.35886)	(2.52596)	(2.62453)	(2.68903)	(2.74053)	(2.78042)	(2.84958)	(2.89737)	(2.98976)	(3.05846)	(3.17267)	(3.20441)	(3.31033)
4	(0.32076)	(0.28518)	(0.26429)	(0.25355)	(0.24610)	(0.23784)	(0.22552)	(0.21862)	(0.20604)	(0.19475)	(0.18162)	(0.17834)	(0.16332)
	(2.14242)	(2.27846)	(2.35804)	(2.41019)	(2.45213)	(2.48428)	(2.53971)	(2.57835)	(2.65212)	(2.70836)	(2.79924)	(2.82440)	(2.91008)
5	(0.37029)	(0.33425)	(0.31698)	(0.30307)	(0.29396)	(0.28849)	(0.27616)	(0.26714)	(0.25534)	(0.24451)	(0.22854)	(0.22455)	(0.21053)
	(2.00035)	(2.11633)	(2.18425)	(2.22944)	(2.26486)	(2.29234)	(2.33893)	(2.37184)	(2.43458)	(2.48238)	(2.55905)	(2.58103)	(2.65272)
6	(0.41078)	(0.37661)	(0.35622)	(0.34611)	(0.33506)	(0.32841)	(0.31851)	(0.30876)	(0.29485)	(0.28639)	(0.26857)	(0.26412)	(0.25054)
	(1.89874)	(2.00141)	(2.06129)	(2.10042)	(2.13146)	(2.15579)	(2.19687)	(2.22584)	(2.28058)	(2.32248)	(2.38982)	(2.40879)	(2.47178)
7	(0.44378)	(0.40932)	(0.39051)	(0.37915)	(0.37046)	(0.36302)	(0.35192)	(0.34422)	(0.32906)	(0.32116)	(0.30303)	(0.29838)	(0.28469)
	(1.82150)	(1.91417)	(1.96797)	(2.00279)	(2.03066)	(2.05269)	(2.08986)	(2.11580)	(2.16472)	(2.20231)	(2.26271)	(2.27939)	(2.33586)
8	(0.47253)	(0.43794)	(0.42012)	(0.40798)	(0.44563)	(0.39282)	(0.38116)	(0.37433)	(0.35879)	(0.35069)	(0.33299)	(0.32839)	(0.31431)
	(1.76082)	(1.84512)	(1.89432)	(1.92616)	(1.95174)	(1.97163)	(2.00567)	(2.02926)	(2.07389)	(2.10794)	(2.16307)	(2.17786)	(2.22912)
9	(0.49655)	(0.46284)	(0.44557)	(0.43323)	(0.42548)	(0.41849)	(0.40684)	(0.40001)	(0.38484)	(0.37676)	(0.35940)	(0.35506)	(0.34045)
	(1.71113)	(1.78894)	(1.83448)	(1.86404)	(1.88760)	(1.90594)	(1.93729)	(1.95899)	(2.00042)	(2.03142)	(2.08228)	(2.09591)	(2.14303)
10	(0.51716)	(0.48452)	(0.46750)	(0.45545)	(0.44788)	(0.44079)	(0.42961)	(0.42222)	(0.40798)	(0.39895)	(0.38312)	(0.37825)	(0.36406)
	(1.66963)	(1.74225)	(1.78476)	(1.81221)	(1.83412)	(1.85143)	(1.88034)	(1.90049)	(1.93946)	(1.96774)	(2.01506)	(2.02841)	(2.07219)

TABLE 3.6 GAMMA (2) DISTRIBUTION
CONSTANTS FOR ANALYSIS OF MEANS (1-α=0.9973)

n\k	1	2	3	4	5	6	8	10	15	20	30	40	60
2	(0.11111)	(0.09300)	(0.07733)	(0.06810)	(0.06482)	(0.06196)	(0.05773)	(0.05543)	(0.05234)	(0.05074)	(0.03565)	(0.02966)	(0.02025)
	(3.17100)	(3.38795)	(3.51721)	(3.60370)	(3.67568)	(3.72506)	(3.81771)	(3.87706)	(4.01099)	(4.08719)	(4.18851)	(4.25484)	(4.34151)
3	(0.19280)	(0.16838)	(0.15502)	(0.14405)	(0.13743)	(0.13342)	(0.12746)	(0.13452)	(0.11211)	(0.10312)	(0.09256)	(0.08990)	(0.08572)
	(2.67330)	(2.83229)	(2.91439)	(2.98976)	(3.04186)	(3.07766)	(3.14422)	(3.18739)	(3.28517)	(3.33970)	(3.41313)	(3.46132)	(3.52355)
4	(0.25610)	(0.22809)	(0.21578)	(0.20614)	(0.19724)	(0.19186)	(0.18385)	(0.17952)	(0.16186)	(0.15839)	(0.14773)	(0.14505)	(0.13895)
	(2.39720)	(2.52592)	(2.60184)	(2.65212)	(2.69422)	(2.72253)	(2.77737)	(2.81182)	(2.88946)	(2.93289)	(2.99194)	(3.03028)	(3.07957)
5	(0.30650)	(0.27952)	(0.26342)	(0.25539)	(0.24754)	(0.24099)	(0.23126)	(0.22599)	(0.21490)	(0.20532)	(0.19407)	(0.19124)	(0.18022)
	(2.21790)	(2.32756)	(2.39201)	(2.43458)	(2.47018)	(2.49459)	(2.54067)	(2.56977)	(2.63563)	(2.67286)	(2.72147)	(2.75395)	(2.79614)
6	(0.34860)	(0.32057)	(0.30438)	(0.29492)	(0.28919)	(0.28247)	(0.27160)	(0.26572)	(0.25500)	(0.24522)	(0.23375)	(0.23070)	(0.21809)
	(2.09800)	(2.18665)	(2.24325)	(2.28058)	(2.31167)	(2.33329)	(2.37332)	(2.39856)	(2.45609)	(2.48869)	(2.53202)	(2.56039)	(2.59618)
7	(0.38180)	(0.35424)	(0.33945)	(0.32914)	(0.32291)	(0.31753)	(0.30619)	(0.30005)	(0.28905)	(0.27495)	(0.26829)	(0.26281)	(0.25172)
	(1.99450)	(2.08039)	(2.13128)	(2.16472)	(2.19252)	(2.21210)	(2.24756)	(2.27028)	(2.30889)	(2.35071)	(2.38978)	(2.41504)	(2.44745)
8	(0.41090)	(0.38359)	(0.36943)	(0.35887)	(0.35248)	(0.34732)	(0.33611)	(0.33005)	(0.31839)	(0.30945)	(0.29756)	(0.29142)	(0.28176)
	(1.91850)	(1.99692)	(2.04337)	(2.07389)	(2.09909)	(2.11701)	(2.14909)	(2.17017)	(2.21704)	(2.24332)	(2.27793)	(2.30057)	(2.33043)
9	(0.42390)	(0.40927)	(0.39523)	(0.38491)	(0.37868)	(0.37294)	(0.36235)	(0.35662)	(0.34414)	(0.33606)	(0.32251)	(0.31727)	(0.30903)
	(1.80460)	(1.92931)	(1.97217)	(2.00042)	(2.02370)	(2.04003)	(2.06979)	(2.08909)	(2.13212)	(2.15633)	(2.18816)	(2.20937)	(2.23629)
10	(0.45830)	(0.43194)	(0.41773)	(0.40805)	(0.40195)	(0.39546)	(0.38581)	(0.38059)	(0.36730)	(0.36021)	(0.34546)	(0.34108)	(0.33265)
	(1.80570)	(1.87321)	(1.91304)	(1.93946)	(1.96111)	(1.97607)	(2.00397)	(2.02174)	(2.06139)	(2.08393)	(2.11396)	(2.13357)	(2.15866)

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