RELIABILITY TEST PLANS: EXPONENTIATED EXPONENTIAL DISTRIBUTION

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Abstract: Sampling plans in which items that are put to test, to collect the life of the items in order to decide upon accepting or rejecting a submitted lot, are called Reliability test plans. The basic probability model of the life of the product is specified as the well-known Exponentiated Exponential Distribution. For a given producer risk, sample size, termination number and waiting time to terminate the test plan are computed. The preferability of the test plan over similar plans existing in the literature is established with respect to cost and time of the experiment.

Key Words: Exponentiated Exponential distribution, Reliability Test Plan, Producers risk, Acceptance Sample number.

1. INTRODUCTION

In reliability studies quite often we come cross systems of components rather than components themselves. i.e., a system arranged as a combination of various components with each component having a probabilistic life pattern of its own. Generally such systems are classified as series or parallel systems, or mixture of both. It is well known that the parallel system is one which survives as long as atleast one of its components survives. If the individual components are governed by a common probability model given by a cumulative distribution function F(.) then for a parallel system of 'k' components the failure probability is equal to the probability of failure of all components which in turn is $[F(.)]^k$, provided the failures are independent it can be seen that for k a natural number $[F(.)]^k$ represents another cumulative distribution function. Such a cumulative distribution function is named as exponentiated version F (.). For Example: If F (.) is exponential $[F(.)]^k$ is exponentiated exponential, If F (.) is Weibull $[F(.)]^k$ is exponentiated Weibull and so on. **Mudholkar and Srivastwa (1993)**, considered exponentiated Weibull distribution as a model suitable for approximating bath tub failure rate life time data, for testing goodness of fit of the Weibull and negative exponential models as sub-hypothesis. **Mudholkar** ***et al.*** **(1995)** have used the exponentiated Weibull model as a probability distribution to re-analyze five classical data sets on bus motor failures borrowed from **Davis(1952)** which are typical of data in repair, reuse situation. **Data of Efron (1988)** pertaining to a head & neck cancer clinical trial is also used to model by exponentiated Weibull family. **Jiang & Murthy (1999)** developed a graphical approach to decide weather a given data set can be adequately modeled by exponentiated Weibull family.

A life test is an experiment that is conducted to determine weather or not a product needs the specified requirements for average life. Generally in such a test fixed number of products taken as sample out of a submitted lot of those products. To decide upon acceptance or otherwise of the lot on the basis of the observed life times of sampled test procedure requires a specification of sample size, a terminating rule to arrive at a decision, the criterion that defines, the preferability or otherwise of the lot and above all the risks associated with the decisions. Since exponential distribution the CFR model the central distribution in reliability studies, **Epstein (1954)** developed reliability test plans for exponential distribution. **Gupta & Groll (1961)** constructed similar sampling plans based Gamma life test sample data. **Goode & Kao (1961)** constructed sampling plans based on Weibull distribution. **Kantam & Roasiah (1998)** suggested acceptance sampling plans based on life test when the failure density model of the product is half-logistic distribution. Kantam ***et al.*** (2001) studied Acceptance Sampling based on life tests when the failure density model of the products is a log logistic distribution. **Rosaiah *et al.* (2006)** suggested reliability test plans for exponentiated log logistic distribution. The Reliability test plans based on gamma variate was suggested by **Kantam & SriRam (2010).**

In this paper we present a different approach to the construction of sampling plans for exponentiated exponential distribution following **Braverman (1981, Ch II)** by considering the exponentiated exponential distribution a the failure density governing the life times of the product in the submitted lot and made an attempt to construct the necessary test plan that can be used to decide upon accepting or otherwise of the submitted lot of products such test plan is suggested **Kantam *et al.* (2006)** for log logistic distribution. **Rosaiah *et al.* (2008)** suggested similar plans Inverse Rayleigh distribution, the operating characteristic of such plan are also presented. The necessary theory of the present plans is given in Section 2, the operating characteristic are given in Section 3, and comparative study is presented in Section 4

2. RELIABILITY TEST PLAN:

Let a lot of products of indefinitely large size be submitted for sampling inspection and decision to reject or accept. Let us assume that the pdf of life of a product is a Exponentiated Exponential distribution with scale parameter σ, whose pdf f(x, σ), cdf F(x, σ) are given in equations.

$$f(x,\sigma) = 2e^{-\frac{x}{\sigma}}\left(1-e^{-\frac{x}{\sigma}}\right) \tag{2.1}$$

$$F(x,\sigma) = \left(1-e^{-\frac{x}{\sigma}}\right)^2 \tag{2.2}$$

Let α be the probability rejecting the submitted lot ie., truly good in some sense known as Producer's risk. Naturally 'α' should be as small as possible. We can think of the decision making into different ways. i. Let σ_0 be a specified value of σ representing the mean life of the product and 't_0' be preassigned time at which the life testing experiment of sample products is designed to be terminated. Hence 't_0' may be called "Terminating time". **Gupta & Groll (1961),** suggested the minimum sample size required 'n' and an acceptance number c, such that if c or less failures occur out of 'n', before the time 't_0', the lot would be accepted with a probability (1-

α). This approach is basically counting number of failure out of n, with in the terminating time 't_o' and hence the life testing experiment would be stopped as soon as the time 't_o' is reached or $(c+1)^{st}$ failure is realized whichever is earlier. This were tabulated in Tables 2.1 and 2.2 for exponentiated exponential distribution.

ii. Alternatively one can think of another reliability test plan. Let 'n' stand for the number of sampled items to be inspected. Let 'r' be natural number such that if 'r' failures are realized before the termination time 't_o' the lot would be rejected, that is the experiment is stopped as soon as r^{th} failure is reached or termination time 't_o' is reached whichever is earlier, in this sense 'r' is called termination number. The size naturally depends on cost considerations and expected waiting time to reach a decision. Large sample sizes may decrease expected waiting time but increase cost of experimentation. As a balance between these two aspects let us consider the sample size as a multiple of termination number. We know that the probability of 'r' failures out of 'n' tested items is given by $\binom{n}{r}p^r q^{n-r}$ where P= F(x, σ) the cdf of the exponentiated exponential distribution, hence acceptance probability of lot is

$$P_a = \sum_{i=0}^{r-1}\binom{n}{r}p^r q^{n-r} \quad (2.3)$$

For specified Producer's risk say α, termination number r, sample size 'n' as a multiple of 'r' we can write the above equation as

$$\sum_{i=0}^{r-1}\binom{kr}{r}p^r q^{n-r} = 1-\alpha \quad (2.4)$$

Using the Cumulative probability of Binomial distribution the above equation can be solved for p. Equating F(x,σ) to p we can get the value of x/ σ corresponding to p, that is x/ σ is the solution of

$$F(x,\sigma) = \left(1 - e^{-\frac{x}{\sigma}}\right)^2 \quad (2.5)$$

3. OPERATING CHARACTERISTIC CURVE:

If the true but unknown life of the product deviates from the specified life of the product, it should result in a considerable change in the probability of acceptance of the lot based on the Sampling plan. Hence the probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called Operating characteristic function of the sampling plan, hence the operating characteristic lies between 0 and 1. Specifically if F(T/σ) is the cdf of the life time random variable of the product, σ_0 corresponds to specified life, we can write

$$F\left(\frac{T}{\sigma}\right) = F\left(\frac{T}{\sigma}, \frac{\sigma_0}{\sigma}\right) \quad (3.1)$$

where σ corresponds to true but unknown average life. The ratio σ_0/σ in the R.H.S of above equation can be taken as a measure of changes between true and specified lives. For instance $\sigma_0/\sigma < 1$ implies true mean life is more than the declared life leading to more acceptance probability or less failure risk. Similarly σ_0/σ more than 1 implies less acceptance probability or more failure risk. Hence giving a set of hypothetical values say σ_0/σ = 0.1(0.1)0.9 we can have the corresponding acceptance probabilities of acceptance given by equation (2.3) for a Sampling plan forms the O.C. Curve of the plan. Here we have selected some plans and O.C. values of these plans are given in tables 3.1 & 3.2. and the graph between σ_0/σ, the probability of acceptance given by Equation(3.1) for a sampling plan forms the O.C.curve of the plans and were given by Figures 3.1 to 3.4.

4. NUMERICAL EXAMPLE:

Considered the following ordered failure times of **Sinha(1986),** the hundred electronic tubes of a certain type were tested (without replacement) given in terms of hours from starting of the testing of the components upon the time at which a failure of the component occurs. This data can be regarded as an ordered sample of size n=10 with observations.

{(x_i, i=1,2,..10)} i.e., {40, 50, 80, 90, 120, 190, 220, 290, 350, 460}

On the basis of the approach –(i) at the required average life time be 600 hours in testing time be t = 600 hrs , this leads to sample size n=10 ,with a corresponding ratio of $t_0/\sigma_0 = 1$ and the acceptance number c = 1, which are obtained from Table 2.1 for 1-α = 0.95. Therefore, the sampling plan of the above sample data is { n = 10, c = 1, $t_0/\sigma_0 = 1$ }. Based on the observations, we have to decide whether to accept the product or reject it, we accept the product only, if the number of failures before 600 hours less than or equal to1. From the given ordered sample we notice that the earliest failure of the electronic tubes is at 40 hours, which are less than 600 hours.

On the basis approach –(ii) for given ordered sample of size 10 and termination number 2, at a risk probability of 0.05 the value of t_0/σ_0 from Table 2.3 in t_0/σ_0 = 0.21292.i.e., the termination time is 0.21292*600=127.752 ~ 128hours. The second plan means –" The number of failures earlier than 128 hours of the experiment should not be more than 2". From the given ordered data, we notice 2 failures at 40 & 50 hours, which is not earlier than the termination time 128 hours. From the above approaches (i) & (ii) the decision on the 1st approach can be reached at the 600th and that in the 2nd approach the decision is taken at 128th hour.

Hence the waiting time due to 2nd approach to come to a decision is less than that of 1st approach. Hence the second approach is preferred. These tables show that for the same α, sample size acceptance number, termination time of second approach is much earlier than in the first approach. Resulting in a considerable saving in the waiting time to come to a decision for any specified value of 'σ'. We have tabulated from the above equations 2.1, 2.2, 2.3 the values of x/σ for α_i =0.05, 0.01; r=1(1)10, k=2(1)10, in Tables 2.1,2.2 for exponentiated exponential distribution these tables can be as another reliability test plans.

Table 2.1
EXPONENTIATED EXPONENTIAL DISTRIBUTION RELIABILITY TEST PLAN GIVEN CONSUMER'S RISK 1-α=0.95

c \ t/σ_0	2	1.5	1	0.4	0.2	0.15	0.1
0	3	4	6	27	90	153	330
1	5	6	10	42	143	243	522
2	6	8	14	56	190	323	693
3	8	11	17	69	234	398	854
4	10	13	21	82	276	470	1008
5	11	15	24	94	318	540	1158
6	13	17	27	106	358	608	1305
7	14	19	30	118	398	675	1449
8	16	21	33	130	437	742	1591
9	17	23	36	142	475	807	1731
10	19	24	39	153	513	871	1869
11	20	26	42	164	551	935	2007
12	22	28	45	176	588	999	2143
13	23	30	48	187	626	1062	2278
14	25	32	51	198	663	1125	2412
15	26	34	54	209	699	1187	2546

Table 2.2
EXPONENTIATED EXPONENTIAL DISTRIBUTION RELIABILITY TEST PLAN GIVEN CONSUMER'S RISK 1-α=0.99

C \ t/σ_0	2	1.5	1	0.4	0.2	0.15	0.1
0	4	5	10	41	138	236	506
1	6	8	14	59	200	340	730
2	8	11	18	75	254	431	925
3	10	13	22	89	303	515	1106
4	11	15	25	103	350	595	1278
5	13	17	29	116	395	672	1443
6	15	20	32	127	440	747	1604
7	16	22	36	143	483	821	1762
8	18	24	39	156	525	893	1917
9	19	26	42	169	567	964	2069
10	21	28	45	181	608	1034	2219
11	23	30	49	193	649	1103	2367
12	24	32	52	205	690	1171	2514
13	26	34	55	217	729	1239	2659
14	27	36	58	229	769	1306	2803
15	29	38	61	241	808	1373	2946

Table 2.3
EXPONENTIATED EXPONENTIAL DISTRIBUTION
RELIABILITY TEST PLAN GIVEN PRODUCER'S RISK
$1-\alpha=0.95$

r\n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.17409	0.14019	0.11997	0.10710	0.09747	0.08949	0.08379	0.07888	0.07474
2	0.37554	0.28935	0.24274	0.21292	0.19244	0.17657	0.16344	0.15373	0.14494
3	0.49750	0.37554	0.31211	0.27264	0.24452	0.22420	0.20775	0.19413	0.18322
4	0.57963	0.43246	0.35728	0.31115	0.27910	0.25440	0.23563	0.21984	0.20689
5	0.63798	0.47233	0.38988	0.33836	0.30256	0.27633	0.25531	0.23829	0.22420
6	0.68445	0.50332	0.41418	0.35928	0.32078	0.29217	0.26988	0.25169	0.23741
7	0.72150	0.52811	0.43355	0.37554	0.33541	0.30541	0.28189	0.26256	0.24721
8	0.75098	0.54864	0.44998	0.38894	0.34728	0.31595	0.29123	0.27172	0.25531
9	0.77674	0.56588	0.46333	0.40043	0.35627	0.32466	0.29971	0.27910	0.26256
10	0.79846	0.57963	0.47459	0.40993	0.36535	0.33149	0.30637	0.28561	0.26805

Table 2.4
EXPONENTIATED EXPONENTIAL DISTRIBUTION
RELIABILITY TEST PLAN GIVEN PRODUCER'S RISK
$1-\alpha=0.99$

r\n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.07405	0.05958	0.05171	0.04616	0.04214	0.03862	0.03637	0.03404	0.03223
2	0.22946	0.17906	0.15132	0.13390	0.12074	0.11085	0.10338	0.09674	0.09165
3	0.34429	0.26347	0.22072	0.19329	0.17409	0.15938	0.14812	0.13861	0.13078
4	0.42813	0.32272	0.26896	0.23475	0.21033	0.19244	0.17906	0.16752	0.15776
5	0.49173	0.36738	0.30446	0.26530	0.23829	0.21724	0.20092	0.18824	0.17740
6	0.54135	0.40253	0.33247	0.28935	0.25893	0.23652	0.21897	0.20433	0.19244
7	0.58215	0.43138	0.35527	0.30923	0.27633	0.25169	0.23298	0.21724	0.20518
8	0.61684	0.45441	0.37452	0.32466	0.29029	0.26438	0.24452	0.22858	0.21464
9	0.64602	0.47459	0.38998	0.33836	0.30256	0.27540	0.25440	0.23741	0.22333
10	0.67193	0.49173	0.40464	0.35027	0.31211	0.28468	0.26256	0.24542	0.23122

Table 2.5
EED Common Entries of scaled termination time (t/σ_0) $\alpha = 0.05$

n c	r	2r	3r	4r	5r	6r
0	1		2 0.1402	1.5 0.11997		1 0.0975
1	2		1.5 0.2893		1 0.2129	
2	3	2 0.4975				
4	5	2 0.6379				
5	6			1 0.41418		

Table 2.6

EED Common Entries of scaled termination time (t/σ_0) $\alpha = 0.01$

c \ n r		2r	3r	4r	5r	6r	7r	10r
0	1			2 0.05171	1.5 0.04616			1 0.03223
1	2		2 0.17906	1.5 0.15132			1 0.11085	
2	3					1 0.17409		
4	5		1.5 0.36738		1 0.21653			
7	8	2 0.61684						
8	9	2 0.64602						

Table 3.1

O.C. VALUES OF SELECTED RELIABILITY TEST PLANS FOR A GIVEN CONSUMER'S RISK

σ_o/σ	EED		
	$T/\sigma_o=2$		
	n= 6, r= 2	n=6, r=3	n= 8, r=4
	1-α=0.99	1-α=0.95	1-α=0.95
0.1	0.98517	0.99934	0.99993
0.2	0.86823	0.98007	0.99319
0.3	0.64659	0.89668	0.94038
0.4	0.41320	0.73828	0.79997
0.5	0.23395	0.54521	0.59508
0.6	0.12070	0.36588	0.38909
0.7	0.05800	0.22692	0.22777
0.8	0.02639	0.13209	0.12179
0.9	0.01150	0.07309	0.06055
1.0	0.00485	0.03886	0.02840
1.1	0.00199	0.01999	0.01271
1.2	0.00080	0.01003	0.00548
1.3	0.00032	0.00493	0.00229
1.4	0.00012	0.00238	0.00094
1.5	0.00005	0.00113	0.00037
1.6	0.00002	0.00053	0.00015
1.7	0.00001	0.00025	0.00006
1.8	0.00000	0.00012	0.00002
1.9	0.00000	0.00005	0.00001

able 3.2

EXPONETIATED EXPONENTIAL DISTRIBUTION

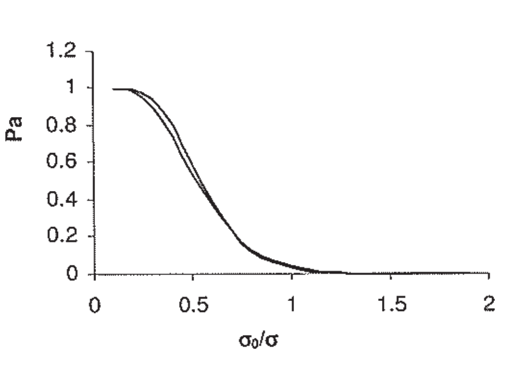
O.C. VALUES OF SELECTED RELIABILITY TEST PLANS FOR A GIVEN PRODUCER'S RISK

	n=2, r=1		n=10, r=2		n=6, r=2		n=12, r=3		n=8, r=4	
	T/σ_0=0.17409, .07405		T/σ_0=0.21292, 0.13390		T/σ_0=0.28935, 0.17906		T/σ_0=0.31211, 0.22072		T/σ_0=0.57963, 0.42813	
σ \ σ_0	P_a		P_a		P_a		P_a		P_a	
	1-α=0.95	1-α=0.99	1-α=0.95	1-α=0.99	1-α=0.95	1-α=0.99	1-α=0.95	1-α=0.99	1-α=0.95	1-α=0.99
0.1	0.99941	0.99989	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
0.2	0.99766	0.99957	0.99986	0.99998	0.99985	0.99998	0.99998	0.99999	0.99999	0.99999
0.3	0.99483	0.99903	0.99935	0.99989	0.99929	0.99989	0.99989	0.99998	0.99999	0.99999
0.4	0.99097	0.99829	0.99807	0.99967	0.99793	0.99966	0.99947	0.99992	0.99979	0.99997
0.5	0.98615	0.99736	0.99557	0.99923	0.99529	0.99921	0.99826	0.99973	0.99908	0.99988
0.6	0.98042	0.99623	0.99138	0.99845	0.99096	0.99843	0.99552	0.99928	0.99708	0.99958
0.7	0.97385	0.99490	0.98507	0.99724	0.98451	0.99722	0.99032	0.99835	0.99261	0.99886
0.8	0.96648	0.99339	0.97625	0.99546	0.97562	0.99546	0.98164	0.99669	0.98416	0.99735
0.9	0.95838	0.99017	0.96462	0.99302	0.96403	0.99306	0.96847	0.99396	0.97025	0.99456
1.0	0.94959	0.98984	0.94998	0.98979	0.94959	0.98992	0.94998	0.98983	0.94949	0.98989
1.1	0.94018	0.98780	0.93223	0.98567	0.93226	0.98593	0.92565	0.98391	0.92095	0.98274
1.2	0.93017	0.98559	0.91135	0.98058	0.91205	0.98104	0.89523	0.97585	0.88452	0.97252
1.3	0.91964	0.98323	0.88744	0.97442	0.88909	0.97515	0.85886	0.96533	0.84026	0.95861
1.4	0.90862	0.98069	0.86068	0.96713	0.86357	0.96824	0.81703	0.95208	0.78944	0.94064
1.5	0.89715	0.97802	0.83132	0.95866	0.83573	0.96024	0.77048	0.93591	0.73318	0.91845
1.6	0.88529	0.97519	0.79966	0.94897	0.80584	0.95116	0.72019	0.91672	0.67304	0.89178
1.7	0.87305	0.97222	0.76605	0.93805	0.774236	0.94094	0.66724	0.89449	0.61098	0.86087
1.8	0.86050	0.96909	0.73084	0.92588	0.741238	0.92962	0.61279	0.86927	0.54851	0.82590
1.9	0.84766	0.96585	0.69446	0.91249	0.707202	0.91721	0.55795	0.84123	0.48724	0.78755

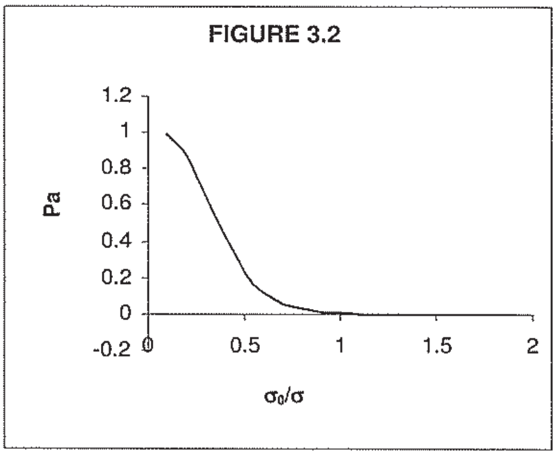
EXPONENTIATED EXPONENTIAL DISTRIBUTION

O.C. CURVES OF Table 3.1(a) (1-α=0.95)

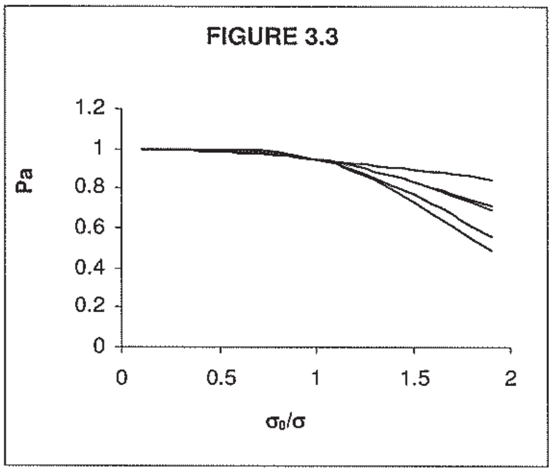
FIGURE 3.1



EXPONENTIATED EXPONENTIAL DISTRIBUTION
O.C. CURVES OF Table 3.1 (b)(1-α=0.99)

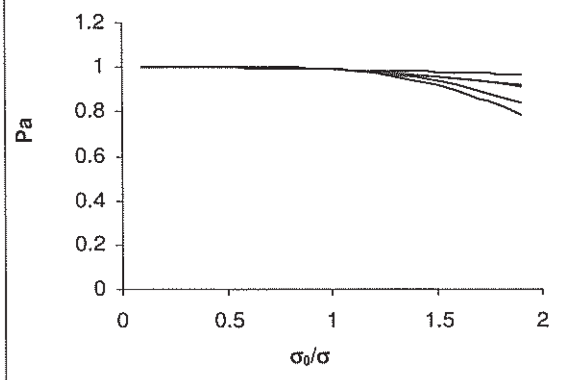


EXPONENTIATED EXPONENTIAL DISTRIBUTION
O.C. CURVESOF Table 3.2 (1-α=0.95)



EXPONENTIATED EXPONENTIAL DISTRIBUTION
O.C. CURVESOF Table 3.2 (1-α=0.99)

FIGURE 3.4



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