SEMI NEAR-RINGS AND THEIR IDEALS

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Abstract: *Near-rings are algebraic systems with binary operations of addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. Semi rings are algebraic systems which are closed and associative under two operations, usual addition, multiplication, satisfying both the distributive laws. In this paper we consider an algebraic system semi near-ring, which is a generalization of both a semi ring and a near-ring. We have defined s-k-product of two fuzzy s-k-ideals and proved that the s-k-product of two fuzzy s-k-ideals is contained in their intersection.*

Keywords: semi near-ring, s-ideal, s-k-ideal

1. INTRODUCTION

A semi near-ring S is an algebraic system with two binary operations: usual addition and usual multiplication such that S forms a semi group with respect to both the operations and satisfies the right distributive law. A natural example of a semi near-ring is obtained by considering the operations usual addition and composition of mappings on a set of all mappings of an additive semi group S into itself.

Definition 1.1. A subset I of a Semi near-ring S is a right (respectively, left) s-ideal if
(i) $x + y \in I$ (ii) $xr \in I$ (right s-ideal), ($rx \in I$ (left s-ideal)) for all $x, y \in I$ and $r \in S$.

Definition 1.2. A non-empty fuzzy subset μ (that is, $\mu(x) \neq 0$ for some $x \in S$) of semi near-ring S is called a fuzzy s-ideal if it satisfies

(i) $\mu(x + y) \geq \min \{(\mu(x), \mu(y)\}$
(ii) $\mu(xy) \geq \max \{\mu(x), \mu(y)\}$.

Definition 1.3. A left (or right) s-ideal I of a semi near-ring S is called a left (or right) s-k-ideal of S if $y \in I$ and $x \in S$, $x + y \in I$ implies that $x \in I$.

Definition 1.4. A fuzzy s-ideal μ of a semi near-ring S is called a fuzzy s-k-ideal of S for all $x, y, z \in S$,
$x + y = z$ implies that $\mu(x) \geq \min \{\mu(y), \mu(z)\}$.

2. PRODUCT OF S-K-IDEALS

Definition 2.1. Let μ and σ be two fuzzy subsets of a semi near-ring S. Then the s-k-product of μ and σ denoted by $\mu \lozenge \sigma$ is defined as,

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$$(\mu \lozenge \sigma)(x) = \begin{cases} \text{Sup } \{ \min \{\mu(a_i), \sigma(b_i)\} \\ x + a_1 b_1 = a_2 b_2, \quad i = 1, 2 \\ 0, \text{ elsewhere} \end{cases}$$

Proposition 2.2. Let μ and σ be two fuzzy s-k-ideals of a semi near-ring S. Then $\mu \cap \sigma$ is also a fuzzy s-k- ideal of S. Also $\mu \lozenge \sigma \subseteq \mu \cap \sigma$.

Proof : First, we prove that $\mu \cap \sigma$ is a fuzzy s-k-ideal of S.

$(\mu \cap \sigma)(x + y) = \min \{\mu(x + y), \sigma(x + y)\}$
(by the definition of intersection)
$\geq \min \{\min\{\mu(x), \mu(y)\}, \min \{\sigma(x), \sigma(y)\}\}$
(since μ and σ are fuzzy ideals)
$= \min \{\min\{\mu(x), \sigma(x)\}, \min \{\mu(y), \sigma(y)\}\}$
$= \min \{(\mu \cap \sigma)(x), (\mu \cap \sigma)(y)\}$ (by the definition of intersection)

$(\mu \cap \sigma)(xy) = \min \{\{\mu(xy), \sigma(xy)\}$
(by the definition of intersection)
$\geq \min \{\max \{\mu(x), \mu(y)\}, \max \{\sigma(x), \sigma(y)\}\}$
(since μ and σ are fuzzy ideals)
$= \max \{\min\{\{\mu(x), \sigma(x), \min \{\mu(y), \sigma(y)\}\}$
(by the definition of intersection)
$= \max \{(\mu \cap \sigma)(x), (\mu \cap \sigma)(y)\}$

Therefore , $\mu \cap \sigma$ is a fuzzy s-ideal of S.

Next, we show that $\mu \cap \sigma$ is a fuzzy s-k-ideal of S.

Suppose $(\mu \cap \sigma)(x + y) = (\mu \cap \sigma)(0)$ and
$(\mu \cap \sigma)(y) = (\mu \cap \sigma)(0)$

To show $(\mu \cap \sigma)(x) = (\mu \cap \sigma)(0)$

Now $(\mu \cap \sigma)(x) = \min \{\mu(x), \sigma(x)\}$
$\geq \min \{\min\{\mu(y), \mu(x + y)\}, \min\{\sigma(y), \sigma(x + y)\}$
(since μ and σ are fuzzy s-k-ideals)
$= \min \{\mu(y), \mu(x + y), \sigma(y), \sigma(x + y)\}$
$= \min \{\min \{\mu(y), \sigma(y)\}, \min \{\mu(x + y), \sigma(x + y)\}$
$= \min \{(\mu \cap \sigma)(0), (\mu \cap \sigma)(0)\}$
(by supposition)
$= (\mu \cap \sigma)(0)$

Therefore , $\mu \cap \sigma$ is a fuzzy s-k-ideal of S.

Now we will show that $\mu \lozenge \sigma \subseteq \mu \cap \sigma$.

If $(\mu \lozenge \sigma)(x) = 0$ for all $x \in S$.

Then clearly, $\mu \lozenge \sigma \subseteq \mu \cap \sigma$.

On the other hand, take $a_1, b_1, a_2, b_2 \in S$ such that

$$x + a_1 b_1 = a_2 b_2$$

Now $\mu(x) \geq \min\{\mu(a_1b_1), \mu(a_2b_2)\}$

(since μ is a fuzzy s-k-ideal)

$\geq \min\{\mu(a_1), \mu(a_2)\}$

(since μ is a fuzzy s-ideal)

Similarly, $\sigma(x) \geq \min\{\sigma(b_1), \sigma(b_2)\}$.

Now

$$(\mu \lozenge \sigma)(x) = \sup_{x + a_1 b_1 = a_2 b_2} \{\min\{\mu(a_1), \mu(a_2), \sigma(b_1), \sigma(b_2)\}$$

$$\leq \min\{\mu(x), \sigma(x)\}$$

$$= (\mu \cap \sigma)(x).$$

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