A STUDY ON THE DOMINATION PARAMETERS OF P-REGULAR BI-PARTITE GRAPHS

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Abstract: In this paper, we study the domination parameters of p-regular bipartite graphs and obtain some relation between them.

Introduction: I.E. Zverovich and V.E. Zverovich [1] have discussed about the domination parameters namely the irredundance number, the domination number, the independent domination number, the independent number, the upper domination number and the upper irredundance number and found certain results for cubic graphs.
N. Murugesan and D.S. Nair [2] discussed only for the cubic bipartite graphs and found the results that the cardinality of independent set is equal to n/2 and dominating set is less than n/3. In this paper we present the relation between the domination parameters of p-regular bipartite graphs.

Definitions:
A graph G is a p-regular bipartite graph, it is satisfies both
the regular and the bipartite conditions and deg (v) = p for all v ∈ V (G).
The open neighborhood of v ∈ V (G) is the set of vertices adjacent to v, N(v) = {u | u v ∈ E(G)} and the closed neighborhood of v is the set N[v] = N(v) ∪ {v}.
For any subset D, D ⊆ V(G), $N(D) = \cup_{v\in D} N(v)$ and $N[D] = \cup_{v\in D} N[v]$
A subset D ⊆ V(G) is an independent set, if no two vertices in D are adjacent. The minimum and maximum cardinalities of the maximal independent sets of G are the independent domination number, i(G) and the independent number, β(G).
A subset D ⊆ V(G) is a dominating set, if every vertex in V-D is adjacent to at least one vertex in D. The minimum and maximum cardinalities of the minimal dominating sets of G are the domination number, γ(G) and the upper domination number, Γ(G).
For any vertex v ∈ D ⊆ V(G), if the set N[v] – N[D-{v}] = ϕ, then v is said to be redundant vertex in D. A set D without a redundant vertex is called an irredundant set. The minimum and maximum cardinalities of the maximal irredundant sets of G are the irredundance number, ir(G) and the upper irredundance number, IR(G).
The parameters γ(G), i(G), ir(G), β(G), Γ(G) and IR(G) are refered as the domination parameters of G.
A subset M ⊆ E(G) is called a matching in G, if no two edges in M are adjacent. The matching number, α(G) is the cardinality of the maximum matching.

Theorem:
In a p-regular bipartite graph G of order n > 2p (p ≥ 3), the following conditions are satisfied:
(i) β(G) = Γ(G) = IR(G) = n/2.
(ii) For 2r +1 ≤ p ≤ 2r+2 (r ≥ 1), γ (G) = ir(G) ≤ n/(r+2).
(iii) For k ≥ 1,

$$\beta(G) - i(G) \le \begin{cases} kp - k, & if\ n = 2kp + 2k, 2kp + (2k+2), \\ & \quad 2kp + (2k+4); \\ kp - k + 1, & if\ n = 2kp + (2k+6); \\ & \quad . \\ & \quad . \\ & \quad . \\ (k+1)p - (k+2), & if\ n = (2k+2)p + 2k . \end{cases}$$

(iv) α(G)+ β(G) = n.
Proof: (i). Let X and Y be two independent vertex sets of G. Since G is a p-regular bipartite graph, each set has n/2 vertices. The maximal independent set and the minimal dominating set can also have at most n/2 vertices. This leads to β(G) = n/2 and Γ(G) = n/2.
If D is a minimal dominating set, then we prove that D is an irredundant set. Suppose x ∈ D is a redundant vertex, then N[x] ⊆ N[D-{x}]. Any vertex y ∈ N[D] implies y ∈ N[D-{x}]. Hence D - {x} is a dominating set. This contradicts the assumption that D is a minimal dominating set. We next assert that D is a maximal irredundant set. Suppose D ∪ {z} is an irredundant set, for some z ∈ V(G). By the definition of irredundant set, N[z] ⊈ N[D]. Hence D is not a dominating set. This is a contradiction. Therefore D is a maximal irredundant set and the upper irredundance number, IR(G) = n/2. So β(G) = Γ(G) = IR(G) = n/2 .
(ii). From result (i) we have the condition that every minimal dominating set is a maximal irredundant set. If G is regular, the minimum cardinality of the minimal dominating set and maximal irredundant set are equal. That is γ(G) = ir(G).
When p=3 or 4, then γ(G) ≤ n/3. When p=5 or 6, then γ(G) ≤ n/4. When p=7 or 8, then γ(G) ≤ n/5 and so on. In general, for every r ≥ 1, 2r+1 ≤ p ≤ 2r+2, γ (G) ≤ n/(r+2).
(iii). Let D be a maximal independent set of G. Then

D must also be an independent dominating set. Suppose D - {x} is an independent dominating set, then by definition N[D-{x}] = V(G). This contradicts our assumption that D is a maximal independent set. Therefore D is a minimal independent dominating set and hence the independent domination number, i(G) < n/2. But β(G) = n/2. Therefore the minimum values of β(G) - i(G) are as follows:

For 3-regular bipartite graph, we have
β(G)-i(G) ≤ 2 for n = 8, 10, 12
≤ 3 for n = 14
≤ 4 for n = 16, 18, 20
≤ 5 for n = 22, and the pattern is similar for every two terms of n.
For 4-regular bipartite graph, we have
β(G)-i(G) ≤ 3 for n = 10, 12, 14
≤ 4 for n = 16
≤ 5 for n = 18
≤ 6 for n = 20, 22, 24
≤ 7 for n = 26
≤ 8 for n = 28, and the pattern is similar for every three terms of n.
For 5-regular bipartite graph, we have
β(G)-i(G) ≤ 4 for n = 12, 14, 16
≤ 5 for n = 18
≤ 6 for n = 20
≤ 7 for n = 22
≤ 8 for n = 24, 26, 28
≤ 9 for n = 30
≤ 10 for n = 32
≤ 11 for n = 34 and the pattern is similar for every four terms of n.
The general case is as follows:

$$\beta(G) - i(G) \leq \begin{cases} kp - k, & if\ n = 2kp + 2k, 2kp + (2k+2), 2kp + (2k+4); \\ kp - k + 1, & if\ n = 2kp + (2k+6); \\ \vdots & \\ (k+1)p - (k+2), & if\ n = (2k+2)p + 2k. \end{cases}$$

and the pattern is similar for every (n-1) terms.
(iv). The maximum edge independent set of any graph G with vertices contains n/2 edges. The matching number, α(G) = n/2. But from the result (i), β(G) = n/2. Hence α(G)+ β(G) = n.
Illustration 1: Consider a 4-regular bipartite graph with n =12 as shown in figure-1.
The minimum dominating set D1 has elements v1, v2 and v7, i.e., D1 = {v1, v2, v7}. It is clear that D1 satisfies the definition of minimum irredundant set and minimum independent dominating set. Therefore γ(G) = ir(G) = i(G) =3.

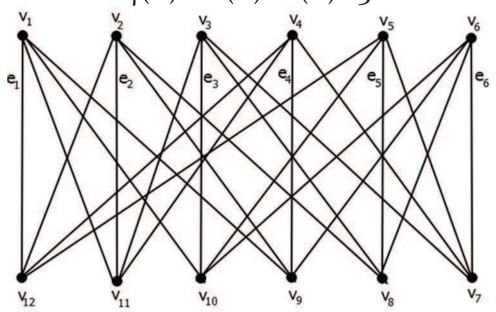

Figure. 1.

The maximum dominating set D_2 has elements v_1, v_2, v_3, v_4, v_5 and v_6, i.e., $D_2 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$. It is clear that D_2 also satisfies the definition of maximum irredundant set and maximum independent dominating set. Therefore β(G) = Γ(G) = IR(G) = 6. The maximum edge independent set is given by $M = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Therefore α(G) = 6.
Illustration 2: Consider a 5-regular bipartite graph with n=14 as shown in figure-2.

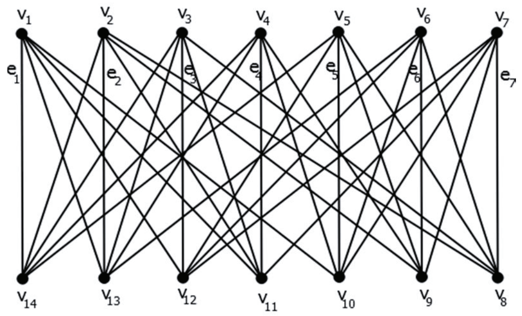

Figure. 2.

Consider the set $D_1 = \{v_1, v_4, v_8\}$ which is the minimum dominating set. D_1 is as well as the minimum irredundant set and minimum independent dominating set. Therefore γ(G) = ir (G) = i(G) = 3. Consider the set $D_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ which is the maximum dominating set. D_2 is as well as the maximum irredundant set and maximum independent dominating set. Therefore β(G) = Γ(G) = IR(G) = 7. The maximum edge independent set is given by $M = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. Therefore α(G) = 7.

Conclusion:
We developed some relation between the domination parameters of p-regular bipartite graphs.

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