THERMAL CONVECTION OF COMPRESSIBLE NON – NEWTONIAN WALTERS' B' VISCOELASTIC FLUID

[1]R. VASANTHAKUMARI, [2]K. THIRUMURUGAN

Abstract : The thermal instability of compressible Non-Newtonian Walters' B' viscoelastic fluid in the presence of uniform magnetic field is considered. It has been found that the magnetic field has both stabilizing and destabilizing effects. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. For stationary convection, Walters' B' viscoelastic fluid behaves like a Newtonian fluid. Tables are drawn to depict the stability characteristics.

Keywords: Non- Newtonian fluid, Walters B' fluid, viscoelasticity, compressibility, thermal convection.

Introduction: A Non-Newtonian fluid is a fluid in which viscosity changes with applied shear force. Therefore, Non-Newtonian fluids may not have a well-defined viscosity. Non - Newtonian fluids are applied in Geological, Biological, Pharmaceutical, Mechanical, Medical and Industrial areas. The theoretical and experimental results of thermal instability in a fluid layer under conditions of varying hydrodynamic and hydromagnetic stability have been treated in detail by Chandrasekhar [3]. Chandra [2] has studied the instability of fluid heated from below. Bhatia and Steiner [1] have studied the problem of thermal instability in a visco – elastic fluid layer in Hydromagnetics. Rayleigh [7] has analyzed the thermal instability of a fluid layer heated below with maintained adverse temperature gradient. Sharma [8] studied the thermal instability in a viscoelastic fluid in hydromagnetics. Oldroyd [5] has remarked Non – Newtonian effect in steady motion of some idealized elastic-viscous liquid. Sharma [9] has studied the thermal instability in compressible fluids in the presence of rotation and a magnetic fluid. Sharma et al. [11] have considered the thermosolutal instability of Walters B' rotating in porous medium. Spiegel [12] analyzed the convective instability in a compressible atmosphere. Sharma and Rana [10] have studied thermal instability of a Walters' B' elastico – viscous fluid in the present of a variable gravity field and rotation on a porous medium. Kumar et al. [4] have investigated the thermal convection in a Walters' (model B') elastic viscous dusty fluid in hydromagnetic with the effect of compressibility and rotation. Rana and Knago [6] have studied the thermal instability of compressible walters' (model B') elastic-viscous rotating fluid permitted with suspended dust particles in porous medium. The role of Non – Newtonian fluid dynamics is important as it relates to modern technology and industries is ever increasing and the investigation on such fluids are desirable, one such class of Non – Newtonian fluids is Walters' B' fluid. Walters' [13] has analyzed Non-Newtonian effects in some elastico-viscous liquids whose behavior at small rate of shear is characterized by a general linear equation of state. Walters' (model B') viscoelastic fluid forms the basis for the manufacture of many such important and useful products. The purpose of the present work is to study the effect of thermal convection in Non –Newtonian fluids, particularly compressible Walters' B' viscoelastic fluid.

Mathematical Formulation: Consider an infinite, horizontal, compressible electrically conducting Walters' B' viscoelastic fluid layer of thickness d, heated from below so that the temperatures and densities at the bottom surface $z = 0$ are T_0 and ρ_0 and at the upper surface $z = d$ are T_d and ρ_d respectively, and that a uniform temperature gradient β is replaced by its excess over the adiabatic (β – g/Cp), where Cp being specific heat of the fluid at constant pressure, the gravity field $\vec{g}(0,0,-g)$, and a uniform vertical magnetic field $\vec{H}(0,0,H)$ act on the system.

The governing equations are

$$\frac{\partial \vec{v}}{\partial t}+(\vec{v}.\nabla)\vec{v}=-\nabla\left(\frac{p}{\rho_0}\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\nabla^2\vec{v}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H} \quad (1)$$

$$\nabla . \vec{v} = 0, \quad (2)$$

$$\frac{\partial T}{\partial t}+(\vec{v}.\nabla)T=\chi\nabla^2 T \quad (3)$$

$$\nabla . \vec{H} = 0, \quad (4)$$

$$\frac{\partial \vec{H}}{\partial t}=(\vec{H}.\nabla)\vec{v}+\eta\nabla^2\vec{H}, \quad (5)$$

where $\vec{v}(u,v,w)$, p, ρ, T, υ, υ' denote the velocity, pressure, density, temperature, kinematic viscosity, and kinematic viscoelasticity respectively, and $\vec{r}(x,y,z)$.The density equation of state for the fluid is

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right], \quad (6)$$

where ρ_o, T_0 are, respectively, the density and and α

temperature of the fluid at the reference level $z = 0$ is the coefficient of thermal expansion. The basic state is

$\vec{v} = (o,o,o)$, $\rho = \rho(z)$, $p = p(z)$, $T = T(z)$.

The Perturbed equations are considered.

3. Dispersion Relations: In the normal mode analysis,

$$[w,\theta,h_z,\zeta,\xi] = [W(z),\Theta(z),K(z),Z(z),X(z)] \exp(ik_x x + ik_y y + nt) \quad (7)$$

where k_x, k_y are the wave numbers along x and y directions, respectively, $k^2 = (k_x^2 + k_y^2)$ is the resultant wave number, and n is the growth rate which is , in general, a complex constant. The non-dimensional transformed equations are

$$\sigma(D^2-a^2)W + \left(\frac{g\alpha d^2}{\nu}\right)a^2\Theta - \frac{\mu_e Hd}{4\pi\rho_0\nu}(D^2-a^2)DK = [1+F\sigma](D^2-a^2)^2 W$$

(8)

$$[\{1-F\sigma\}(D^2-a^2)-\sigma]Z = -\frac{\mu_e Hd}{4\pi\rho_0\nu}DX$$

(9)

$$[D^2-a^2-p_1\sigma]\Theta = -\left(\frac{(\beta - \text{g/Cp})\ d^2}{\chi}\right)W$$

(10)

$$[D^2-a^2-p_2\sigma]K = -\left(\frac{Hd}{\eta}\right)DW$$

(11)

$$[D^2-a^2-p_2\sigma]X = -\left(\frac{Hd}{\eta}\right)DZ$$

(12)

The boundary conditions are

$W = D^2W = 0$, $DZ = 0$, $\Theta = 0$

at $z=0$, $z=1$, $DX=0$, $K=0$ (13)

on a perfectly conducting boundary. Using the above boundary conditions, it can be shown that all the even-order derivatives of W must vanish for $z=0$ and $z=1$, and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin\pi z \quad (14)$$

where W_0 is a constant.

Simplifying we get,

$$R_1 = \left(\frac{1+x}{x}\right)\frac{[\{(1+iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\}\{1+x+i\sigma_1 p_2\}+Q_1][1+x+i\sigma_1 p_1]}{\{1+x+i\sigma_1 p_2\}}\left(\frac{\text{G}}{\text{G-1}}\right) \quad (15)$$

where $R = g\alpha\beta d^4/\nu\chi$, $Q = \mu_e H^2 d^2/4\pi\rho_0\nu\eta$ stand for the Rayleigh – number, the Chandrasekhar number, respectively, and G = (Cp/g)β and the non-dimensionalized parameters are

$$x = \frac{a^2}{\pi^2},\ R_1 = \frac{R}{\pi^4},\ i\sigma_1 = \frac{\sigma}{\pi^2},\ F_1 = \pi^2 F,\ Q_1 = \frac{Q}{\pi^2},\ i = (-1)^{\frac{1}{2}}$$

(16)

4. Stability Analysis: Stationary Convection

Taking $\sigma = 0$, $R_1 = \left(\frac{G}{G-1}\right)\left(\frac{1+x}{x}\right)[(1+x)^2 + Q_1]$

(17)

$$\frac{dR_1}{dQ_1} = \frac{1+x}{x}\left(\frac{\text{G}}{\text{G-1}}\right) \quad (18)$$

5. Result and Discussion: Stability analysis is being carried out and value of R_1-the Rayleigh Number is calculated for allowable range of parameters as listed below and tables are formed accordingly.

Table – 1: R_1 Vs Q_1 for different values of x, G = 10

x	$Q_1=50$	$Q_1=100$	$Q_1=150$	$Q_1=200$
	R_1	R_1	R_1	R_1
3	98	172	246	320
6	128	193	258	323
9	185	247	309	370

Table-1 is formed with values of x ranging from 3 to 9,G taking the value 10 and Q_1 taking values 50,100,150.

(Here $x = \frac{a^2}{\pi^2}$, G = (Cp/g)β., $Q_1 = \frac{Q}{\pi^2}$,and $R_1 = \frac{R}{\pi^4}$).

From the table it is clear that for G=10, as Q_1 increases, R_1 increases indicating that the system gets stabilised with respect to Q_1. Also as x increases , R_1 also increases resulting in stabilisation of the system.

Table – 2: R_1 Vs Q_1 for different values of x, G = 30

x	$Q_1=50$	$Q_1=100$	$Q_1=150$	$Q_1=200$
	R_1	R_1	R_1	R_1
3	91	160	229	298
6	118	178	237	297
9	172	230	287	345

Table-2 is formed with values of x ranging from 3 to 9,G taking the value 30 and Q_1 taking values 50,100,150. From the table it is clear that for G=30, as

Q_1 increases, R_1 increases indicating that the system gets stabilised with respect to Q_1. Also as x increases , R_1 also increases resulting in stabilization of the system.

Table – 3: R_1 Vs Q_1 for different values of x , G = 50

x	$Q_1 = 50$	$Q_1 = 100$	$Q_1 = 150$	$Q_1 = 200$
	R_1	R_1	R_1	R_1
3	90	158	226	294
6	118	177	237	296
9	170	227	283	340

Table-3 is formed with values of x ranging from 3 to 9,G taking the value 50 and Q_1 taking values 50,100,150. From the table it is clear that for G=50, as Q_1 increases, R_1 increases indicating that the system gets stabilised with respect to Q_1. Also as x increases , R_1 also increases resulting in stabilization of the system. Collectively from the tables 1,2 and 3 as G increases, R_1 increases indicating that with respect to G also the system gets stabilised.

6. Conclusion: We observe that the combined effect of magnetic field parameter and compressibility parameter has been investigated on thermal instability of a Walters' B' fluid.The principal conclusions from the analysis are as follows:

(i) For stationary convection, Walters' B' fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter.

(ii) It is clear that the effect of compressibility is to postpone the onset of instability.

(iii) As critical Rayleigh number R_1 increases with the increase in Q_1,thus indicates that the magnetic field stabilizes the system

(iv) For a stationary convection, dR_1 / dQ_1 may be positive as well as negative, thus the magnetic field has both stabilizing and destabilizing effects on the system.

(v) As $\sigma_i = 0$, oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for the Non-Newtonian fluid which is heated from above.

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[1]Principal, Kasthurba College for Women, Villianur,Puducherry – 605 110, India, vasunthara1@yahoo.com
[2]Research – Scholar , Department of Mathematics, Bharathiar University, Coimbatore, India
thirumurugan.kirithish@gmail.com