WAVE PROPAGATION IN MAGNETOPYROELECTRIC PLATES

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Abstract: In this paper, we considered the boundary value problem concerning the wave propagation of plane Harmonic waves in a thin, flat homogeneous orthotropic rectangular plate of finite width and infinite length in vacuum under uniform temperature. The frequency equations for harmonic wave propagation in this magneto pyroelectric plate in symmetric and anti-symmetric modes are derived when surfaces of the plate are traction free and completely coated with electrodes that are shorted and thermally and magnetically insulated.

Keywords: Wave propagation, magnetopyro-electric plate, frequency equations, $BaTio_3$ and $CoFe_2O_4$

Introduction: A deterministic modelling of structures can be done using the four physical fields namely elastic, electric, thermal and magnetic. All these fields are measurable using the material properties. Multifield problems are the one, using more than one of the above fields according to which the structures are modelled. The sun irradiation over the wings of aircraft is an example for multifield problem. In general, piezoelectric materials are mainly used in vibration control of distributed systems. Pyroelectric materials are used as detectors for laser fusion applications. Magneto electro-elastic materials are widely used as magneto-electric memory elements, smart sensors and transducers etc. **John.H.Lehamn**(19) built and evaluated a pyroelectric detector coated with Single Wall Nano Tubes (SWNTS). **J.Hasegawa et al(11)** presented a microcomputer aided pyroelectric thermal imaging system, developed for medical use. Most recent advances are the smart or intelligent materials where piezoelectric or piezo-magnetic material are involved and these materials have the ability to convert energy from one form to the other (22). Also composite materials made of piezoelectric/piezomagnetic material exhibit a magneto-electric effect that is not present in single phase piezoelectric or piezomagnetic effect (23).

The first rigorous analysis of wave propagation in an orthotropic plate was given by **LAMB (1)** for the two dimensional problem of a plane harmonic wave travelling in a direction parallel to the medial plane. **Iya Abubakar (2)** studied the propagation of plane harmonic waves in arbitrary direction on a flat homogeneous transversely isotropic plate. **Warren etal** (9) proved that piezoelectric can produce waves form 100 hertz region up to the billions of hertz and also it is proved that this is the most efficient means for producing the waves with higher amplitudes. **S.Syngelakis (13)** extended the linear vibration analysis to crystal plates of any symmetry and any orientation relative to their crystallographic axes. **H.S.Paul and V.K. Nelson (15) analysed** Harmonic Wave propagation in an infinite piezocomposite plate for three dimensional cases. **Adnan et al** (12) presented analytical solution for the piezoelectric plates which are assumed to be immersed in water and subjected to incident acoustic beams at arbitrary poles and azimuthal angles. **R.A.Kline(6)** investigated modes of plate wave propagation in fibre reinforced composite and in that paper, surfaces were assumed to be stress free and it was also discussed about the application to non-destructive characterization of laminated composites. **R.D.Mindlin (3)** derived constitutive equation which accommodate coupling with electric-elastic-thermal fields for crystal plates. **S.S.Rao et al (14)** developed a finite element formulation of distributed thermo-piezoelectric media and concluded that thermal effects are important in the precision control of intelligent structures. **H.S.Paul and Renganathan(10)** analysed free vibration of pyroelectric crystal class 6mm. **H.S.Paul and G.V.Raman (12)** discussed free vibrations of pyroelectric layer of hexagonal (6mm) class. **V.K.Nelson and S.Karthikeyan (25)** carried out the free vibration of crystal class 6mm of sandwich plates made of pyroelectric materials. **D.F. Nelson (5)** included magnetic effect and moving media electromagnetic effect in his formulation and accounted all the modes of mechanical motion of crystal. **Kolumbin Hutter (4)** derived a magneto elastic wave propagation for Para and soft Ferro magnetic material both analytically and numerically. **Pan (16)** first derived exact solutions for three dimensional anisotropic, linearly magneto-electro-elastic simply supported and multi-layered rectangular plates under static loadings using a propagator matrix method. **Bogdan et al (7 &8)** derived non-linear magneto-thermo-elastic equations in soft ferromagnetic and elastic bodies and investigated the symmetry of couplings in these equations and also derived analytical solutions. **W.Q.Chen et al (17 & 18)** discussed the magneto electric coupling between the two phases due to material synthesis through numerical investigation and found that when the plate is subjected to a magnetic force, the influence of this coupling becomes significant on the electric field and which is an important issue in practical design of magneto electro-elastic structures with heterogeneous material properties. **A.R.Anniegeriet al (20)** analysed the vibrational behaviour of magneto-electro-elastic cylindrical shells, using a semi analytical finite element approach. He used BaTio3 as piezoelectric material and $CoFe_2O_4$ as piezomagnetic material.**E.Carrera et al (21)** derived constitutive equations in terms of coupled mechanical-thermal-electrical-magnetic field variables, on the basis of thermodynamics approach and finite elements have been considered in the frame work of Unified formulation. P.F.**Hou et al (23)** derived a two dimensional general solution for the plane problem of electro-magneto-thermo-elastic materials. **Peng-Fei Hou et al (24)** derived a three dimensional general solution for transversely isotropic electro-magneto-thermo-elastic materials.

In this paper, we studied the simple harmonic wave propagation in a magneto pyroelectric, orthotropic infinite plate of finite thickness under uniform temperature, where the axis of isotropy being normal to the faces of the plate. We have obtained exact frequency equation for the free vibration of magneto pyroelectric plate, in both symmetric and anti-symmetric modes. Since the frequency equations are transcendental equations, we, therefore consider, the results for some limiting cases such that waves are short or long compared to the thickness of the plate. In general, waves are dispersive, numerical work has been carried out for composite of BaTio3 and $CoFe_2O_4$ and dispersions curves are drawn for both symmetric and anti-symmetric modes of vibrations using the numerical algorithm given in **R.A.Kline(6).** The results are tabulated and dispersion curves are drawn for both symmetric and anti-symmetric modes of vibrations using MATLAB software. In the first part, we discussed the constitutive equations and governing equations of the problem. In the second part, the frequency equations were derived for both symmetric and anti-symmetric modes of vibrations using the boundary conditions. In the third part, numerical work is carried out to solve the frequency equations; the material coefficients were taken for the material $BaTiO_3$ and $CoFe_2O_4$ also numerical algorithm was given.

Constitutive Equations:

Constitutive equations characterise the individual materials and its reaction to applied loads. For elastic materials, the constitutive behaviour is the function of the current state of deformation. The constitutive equations considered are assumed to be linear. Standard tensor notation is used and Einstein's summation convention is implied over the repeated indices. In the constitutive equations, the set of intensive variable $(\varepsilon, E, \theta, H)$ are assumed to be independent variables. According to thermo-dynamic principle, the Gibbs free energy / unit volume is given by

$$G = U_E - \eta\theta - T_{ij}\varepsilon_{ij} - E_i D_i - H_i B_i$$

.

The quadratic form of the above equation is given by

$$G = \frac{1}{2}\begin{bmatrix} \theta^2 \frac{\partial^2 G}{\partial \theta^2} + \varepsilon_{ij}\varepsilon_{lm}\frac{\partial^2 G}{\partial \varepsilon_{ij}\varepsilon_{lm}} + E_i E_l \frac{\partial^2 G}{\partial E_i E_l} + H_i H_l \frac{\partial^2 G}{\partial H_i H_l} + \theta\varepsilon_{lm}\frac{\partial^2 G}{\partial \theta \partial \varepsilon_{lm}} + \theta E_l \frac{\partial^2 G}{\partial \theta \partial E_l} + \theta H_l \frac{\partial^2 G}{\partial \theta \partial H_l} \\ + \varepsilon_{ij}\theta \frac{\partial^2 G}{\partial \varepsilon_{ij}\partial \theta} + \varepsilon_{ij} E_l \frac{\partial^2 G}{\partial \varepsilon_{ij}\partial E_l} + \varepsilon_{ij} H_l \frac{\partial^2 G}{\partial \varepsilon_{ij}\partial H_l} + E_i\theta \frac{\partial^2 G}{\partial E_i \partial \theta} + E_i \varepsilon_{lm}\frac{\partial^2 G}{\partial E_i \partial \varepsilon_{lm}} + E_i H_l \frac{\partial^2 G}{\partial E_i \partial H_l} \\ + H_i\theta \frac{\partial^2 G}{\partial H_i \partial \theta} + H_i \varepsilon_{lm}\frac{\partial^2 G}{\partial H_i \partial \varepsilon_{lm}} + H_i E_l \frac{\partial^2 G}{\partial H_i \partial E_l} \end{bmatrix}$$

The physical constants are introduced by second derivative of the Gibbs function. Therefore each coupling constant is a second derivative with respect to two different variables and the change of order of differentiation has different meaning. For example,

$\frac{\partial^2 G}{\partial \varepsilon_{ij}\partial E_l} = \frac{\partial D_l}{\partial \varepsilon_{ij}}$ which indicates the direct piezoelectric effect.

$\frac{\partial^2 G}{\partial E_i \partial \varepsilon_{lm}} = \frac{\partial T_{lm}}{\partial E_i}$ which indicates the inverse piezoelectric effect. But both have the equal effect.

The independent variables are given by

$$T_{ij} = -\left(\frac{\partial G}{\partial \varepsilon_{ij}}\right) \quad D_i = -\left(\frac{\partial G}{\partial E_i}\right) \quad \eta = -\left(\frac{\partial G}{\partial \theta}\right) \quad B_i = -\left(\frac{\partial G}{\partial H_i}\right)$$

Therefore the governing equation for magneto pyroelectric material is given by

$\widetilde{T}_{ij,j} = \rho u_{,tt}$ (By Hooke'sLaw)

$D_{i,i} = 0$ (Maxwell's equation)

$k_{ij}\theta_{,ij} = \theta_0 \eta_{,t}$ (Heat conduction equation)

$B_{i,i} = 0$ (Gauss'sequation of magnetism)

-----------------------(1)

where

$$\widetilde{T} = c\varepsilon - e^T E - \lambda\theta - q^T H$$
$$D = e\varepsilon + \xi E + p\theta + dH$$
$$\eta = \lambda^T \varepsilon + p^T E + \chi\theta + m^T H$$
$$B = q\varepsilon + dE + m\theta + \mu H$$

-----------------------(2)

and

U_E- internal energy / unit volume

$\widetilde{T}_{ij}$ – stress tensor

Di – electric displacement

kij – heat conduction coefficient or conductivity tensor (symmetric and Positive semi definite tensor)

ρ – material density

Bi – magnetic displacement

η – entropy / unit volume

θ_0 – initial temeperature

θ_{ij} – change in temperature with reference to initial

e- strain tensor

E- electric field and

$E = -\nabla\varphi$ where φ - electric potential

and H- Magnetic field

η – variation of entropy / unit volume

$\hbar$ – heat flux

c- elastic stiffness coefficient matrix

e- piezo electric coefficient

λ – thermo mechanical coupling coefficient

q- piezo-magnetic coefficient

ξ – dielectric or permeability coefficiient

p- pyroelectric coefficient

d-magneto electric coefficient

$\chi = \frac{\rho C_v}{\theta_0}$, C_v – specific heat/unit mass

m- pyromagnetic coefficient

μ – magneto permeability coefficient

the material constants are given by

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \text{ where}$$

$$c_{66} = \frac{c_{11} - c_{12}}{2}$$

$$\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & \lambda_6 \end{bmatrix}^T$$

$H = -\nabla\psi$ where ψ - magnetic potential

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$q = \begin{bmatrix} 0 & 0 & 0 & 0 & q_{15} & 0 \\ 0 & 0 & 0 & q_{24} & 0 & 0 \\ q_{31} & q_{32} & q_{33} & 0 & 0 & 0 \end{bmatrix} \xi = \begin{bmatrix} \xi_{11} & 0 & 0 \\ 0 & \xi_{22} & 0 \\ 0 & 0 & \xi_{33} \end{bmatrix} d = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \mu = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix}$$

$$k = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} p = \begin{bmatrix} 0 \\ 0 \\ p_3 \end{bmatrix} m = \begin{bmatrix} 0 \\ 0 \\ m_3 \end{bmatrix}$$

$$\varepsilon_{ij} = \left(U_{i,j} + U_{j,i}\right) \text{(ie) } \varepsilon = \begin{pmatrix} u_{,x} & v_{,y} & w_{,y} & w_{,y} + v_{,z} & u_{,z} + w_{,x} & v_{,x} + u_{,y} \end{pmatrix}$$

Now we consider a wave propagating along (l,m,o) direction, then the solutions of equations (1) can be taken in the form

u(x,y,z,t) = U(z) expi(klx+kmy-pt)

v(x,y,z,t) = V(z) expi(klx+kmy-pt)
w(x,y,z,t) = iW(z) expi(klx+kmy-pt)
ϕ (x,y,z,t) = $i(c_{44}/e_{33})\phi(z)$expi(klx+kmy-pt)
θ (x,y,z,t) = $i(c_{44}/\lambda_3)T(z)$expi(klx+kmy-pt)
ψ (x,y,z,t) = $i(c_{44}/q_{33})\psi(z)$expi(klx+kmy-pt)-- (3)

Where k-wave number , p- angular frequency and $l^2 + m^2 = 1$ and $i = \sqrt{-1}$.

Now we introduce the non-dimensional quantities r and δ such that r=z/h and δ =kh(h is the thickness of the plate).

Using this solution we can rewrite equation (2) as

$$\begin{bmatrix} \bar{c}_{55}\frac{d^2}{dr^2}+A_1 & A_2 & -A_3\frac{d}{dr} & -A_4\frac{d}{dr} & A_5 & -A_{14}\frac{d}{dr} \\ A_2 & \frac{d^2}{dr^2}+A_6 & -A_7\frac{d}{dr} & -A_8\frac{d}{dr} & A_9 & -A_{15}\frac{d}{dr} \\ A_3\frac{d}{dr} & A_7\frac{d}{dr} & \bar{c}_{33}\frac{d^2}{dr^2}+A_{10} & \frac{d^2}{dr^2}+A_{11} & -h\frac{d}{dr} & \frac{d^2}{dr^2}+A_{16} \\ A_4\frac{d}{dr} & A_8\frac{d}{dr} & \frac{d^2}{dr^2}+A_{11} & -\bar{\xi}_{33}\frac{d^2}{dr^2}+A_{12} & h\bar{p}\frac{d}{dr} & -\bar{d}_{33}\frac{d^2}{dr^2}+A_{17} \\ A_5 & A_9 & h\frac{d}{dr} & -h\bar{p}\frac{d}{dr} & i\bar{k}_{33}\frac{d^2}{dr^2}+A_{13} & -h\bar{m}\frac{d}{dr} \\ A_{14}\frac{d}{dr} & A_{15}\frac{d}{dr} & \frac{d^2}{dr^2}+A_{16} & -\bar{d}_{33}\frac{d^2}{dr^2}+A_{17} & \bar{m}\frac{d}{dr} & -\bar{\mu}_{33}\frac{d^2}{dr^2}+A_{18} \end{bmatrix} \begin{bmatrix} U(z) \\ V(z) \\ W(z) \\ \phi(z) \\ T(z) \\ \psi(z) \end{bmatrix} = 0$$

------------(4)

Where

$$A_1 = (ch)^2 - \delta^2(\bar{c}_{11}l^2 + \bar{c}_{66}m^2) \; A_2 = -\delta^2(\bar{c}_{12} + \bar{c}_{66})lm \; A_3 = \delta(\bar{c}_{13} + \bar{c}_{55})l \; A_4 = \delta(\bar{e}_{31} + \bar{e}_{15})l$$

$$A_5 = \delta h(l\bar{\lambda}_1 + m\bar{\lambda}_6) \quad A_6 = (ch)^2 - \delta^2(\bar{c}_{66}l^2 + \bar{c}_{22}m^2) \; A_7 = \delta(\bar{c}_{23} + 1)m \; A_8 = \delta(\bar{e}_{24} + \bar{e}_{32})m$$

$$A_9 = \delta h(l\bar{\lambda}_6 + m\bar{\lambda}_2) \; A_{10} = (ch)^2 - \delta^2(\bar{c}_{55}l^2 + m^2) \; A_{11} = -\delta^2(\bar{e}_{15}l^2 + \bar{e}_{24}m^2) \; A_{12} = -\delta^2(\bar{\xi}_{11}l^2 + \bar{\xi}_{22}m^2)$$

$$A_{13} = \bar{d}h - i\delta^2(\bar{k}_{11}l^2 + \bar{k}_{22}m^2 + 2\bar{k}_{12}lm) \; A_{14} = \delta(\bar{q}_{31} + \bar{q}_{15})l \; A_{15} = \delta(\bar{q}_{24} + \bar{q}_{32})m$$

$$A_{16} = -\delta^2(\bar{q}_{15}l^2 + \bar{q}_{24}m^2) \; A_{17} = \delta^2(\bar{d}_{11}l^2 + \bar{d}_{22}m^2) \; A_{18} = \delta^2(\bar{\mu}_{11}l^2 + \bar{\mu}_{22}m^2).$$

And $\bar{c}_{ij} = \frac{c_{ij}}{c_{44}} \; \bar{e}_{ij} = \frac{e_{ij}}{e_{33}} \; \bar{q}_{ij} = \frac{q_{ij}}{q_{33}} \; \bar{\lambda}_i = \frac{\lambda_i}{\lambda_3}$

$$\bar{d}_{ij} = \frac{d_{ij}c_{44}}{q_{33}e_{33}} \; \bar{\xi}_{ij} = \frac{\xi_{ij}c_{44}}{e_{33}^2} \; \bar{\mu}_{ij} = \frac{\mu_{ij}c_{44}}{q_{33}^2} \; \bar{k}_{ij} = \frac{k_{ij}\sqrt{\rho c_{44}}}{\lambda_3^2 h\theta_0 ch} \; \bar{p} = \frac{p_3c_{44}}{e_{33}\lambda_3} \; \bar{d} = \frac{\rho c_{44}C_v}{\theta_0\lambda_3^2} \; \bar{m} = \frac{m_3c_{44}}{q_{33}\lambda_3} \; c = \sqrt{\rho}\,p$$

The symmetric mode solution of equation(4) are taken as

$$U = \sum_{j=1}^{6} F_j \;, \quad V = \sum_{j=1}^{6} t_jF_j \,, \quad W = \sum_{j=1}^{6} q_jG_j \,, \quad \phi = \sum_{j=1}^{6} r_jG_j \,, \quad T = \sum_{j=1}^{6} s_jF_j \,, \quad \psi = \sum_{j=1}^{6} p_jG_j$$

Where $F_j = \sum_{j=1}^{6} C_j \cosh(\alpha_j hr) + D_j \sinh(\alpha_j hr)$

$$G_j = \sum_{j=1}^{6} D_j \cosh(\alpha_j hr) + C_j \sinh(\alpha_j hr)$$

and $(\alpha_j h)^2$ are the six roots of the equation

$$\begin{vmatrix} \bar{c}_{55}(\alpha h)^2 + A_1 & A_2 & -A_3(\alpha h) & -A_4(\alpha h) & A_5 & -A_{14}(\alpha h) \\ A_2 & (\alpha h)^2 + A_6 & -A_7(\alpha h) & -A_8(\alpha h) & A_9 & -A_{15}(\alpha h) \\ A_3(\alpha h) & A_7(\alpha h) & \bar{c}_{33}(\alpha h)^2 + A_{10} & (\alpha h)^2 + A_{11} & -h(\alpha h) & (\alpha h)^2 + A_{16} \\ A_4(\alpha h) & A_8(\alpha h) & (\alpha h)^2 + A_{11} & -\bar{\xi}_{33}(\alpha h)^2 + A_{12} & h\bar{p}(\alpha h) & -\bar{d}_{33}(\alpha h)^2 + A_{17} \\ A_5 & A_9 & h(\alpha h) & -h\bar{p}(\alpha h) & i\bar{k}_{33}(\alpha h)^2 + A_{13} & -h\bar{m}(\alpha h) \\ A_{14}(\alpha h) & A_{15}(\alpha h) & (\alpha h)^2 + A_{16} & -\bar{d}_{33}(\alpha h)^2 + A_{17} & \bar{m}(\alpha h) & -\bar{\mu}_{33}(\alpha h)^2 + A_{18} \end{vmatrix} = 0 \quad \text{-------(6)}$$

The constants t_j, q_j, r_j, s_j and p_j can be evaluated using the following relations

$$[\bar{c}_{55}(\alpha_j h)^2 + A_1] + A_2 t_j - A_3(\alpha_j h) q_j - A_4(\alpha_j h) r_j + A_5 s_j - A_{14}(\alpha_j h) p_j = 0$$

$$A_2 + [(\alpha_j h)^2 + A_6] t_j - A_7(\alpha_j h) q_j - A_8(\alpha_j h) r_j + A_9 s_j - A_{15}(\alpha_j h) p_j = 0$$

$$A_3(\alpha_j h) + A_7(\alpha_j h) t_j + [\bar{c}_{33}(\alpha_j h)^2 + A_{10}] q_j + [(\alpha_j h)^2 + A_{11}] r_j - h(\alpha_j h) s_j + [(\alpha_j h)^2 + A_{16}] p_j = 0$$

$$A_4(\alpha_j h) + A_8(\alpha_j h) t_j + [(\alpha_j h)^2 + A_{11}] q_j + [-\bar{\xi}_{33}(\alpha_j h)^2 + A_{12}] r_j + h\bar{p}(\alpha_j h) s_j + [-\bar{d}_{33}(\alpha_j h)^2 + A_{17}] p_j = 0$$

$$A_5 + A_9 t_j + h(\alpha_j h) q_j - h\bar{p}(\alpha_j h) r_j + [i\bar{k}_{33}(\alpha_j h)^2 + A_{13}] s_j - h\bar{m}(\alpha_j h) p_j = 0 \quad \text{-----------------(7)}$$

Frequency Equations:

The frequency equation has been derived for magneto pyroelectric plates of orthotrophic material using the boundary conditions

Boundary conditions:

Surfaces are traction free (ie) $T_{33}=T_{23}=T_{31}=0$ at $z=\pm h$

Completely coated with electrodes that are shorted and also assumed that the weight and stiffness of the electrodes were neglected (ie) $\phi = 0$ at $z = \pm h$

Thermally insulated (ie) $\theta_{,z} = 0$ at $z = \pm h$

Magnetically insulated (ie) $\psi = 0$ at $z = \pm h$

Eliminating the arbitrary constants C_j and D_j we get the frequency equations as

$|S_{ij}| = 0$ (for symmetric mode) and

$|\beta_{ij}| = 0$ (foranti-symmetric mode)------------(8)

Where

$$S_{1j} = (\bar{c}_{13} l\delta + \bar{c}_{23} m\delta q_j + \bar{c}_{33}(\alpha_j h) q_j + (\alpha_j h) r_j - s_j + i$$

$$S_{2j} = ((\alpha_j h) t_j - m\delta q_j - \bar{e}_{24} m\delta r_j - \bar{q}_{24} m\delta p_j)$$

$$S_{3j} = (\bar{c}_{55}(\alpha_j h) - \bar{c}_{55} l\delta q_j - \bar{e}_{15} l\delta r_j - \bar{q}_{15} l\delta p_j)$$

$$S_{4j} = r_j$$

$$S_{5j} = (\alpha_j h) s_j$$

$S_{6j} = p_j$ in which j = 1, 2, ... 6

The elements of β_{ij} can be obtained from S_{ij} by replacing $\coth(\alpha_j hr)$ by $\tanh(\alpha_j hr)$. The order of the frequency equations is six.

Numerical results and Dispersion curves:

Algorithm:

Step1: Fix the wave number k

Step:2 Initialize w=0. For each pair of values of k and w calculate the wave propagations parameters and associated polarization.

Step:3 Find the determinant of the coefficient matrix given equation 6.

Step:4 Check for possible root by comparing the sign of the determinant with that obtained for the previous value of w. If there was no sign change, increment w and repeat the process. Otherwise go to step 5.

Step: 5 Find a root of the determinant using any of the numerical methods.

Step:6 Increment w and repeat the process until desired number of roots are found for given k.

Step:7 Increment k and repeat procedure (from step 2) until desire maximum wavenumber is reached.

References:

1. H.Lamb, "On Waves in an Elastic Plate", *Proc. R. Soc. Lond. A* vol.**93(A)**, pp.114-128 March 1917.
2. Iya Abubakar, " Free vibrations of a Transversely isotropic plate", Quart. Journ.Mech and applied Math., Vol XV(PtI), PP129-136, July 1961.
3. R.D.Mindlin, " Equations of high frequency vibrations of thermo piezoelectric crystal plates", Int.J.Solids structures, vol.10, pp.625-637, 1974.
4. Kolumban Hutter, " Wave propagation and attenuation in paramagnetic and soft ferromagnetic materials", IJES, vol.13 pp.1067-1084, 1975.
5. D.F.Nelson, " Theory of nonlinear electroacoustics of dielectric, piezoelectric, and puroelectric crystals", JASA, vol.63(6) , pp.1738-1748, June 1978.

6. . R.A.Kline, M.M.Doroudian and C.P.Hsiao, " Plate wave propagation in transversely isotropic materials", J.of Composite materials, vol.23, pp.505-533, May 1980.
7. Bogdan Maruszewski, "Dynamical magnetothermoleastic problem in circular cylinders-I", IJES,vol.19(9), pp.1233-1240, 1981.
8. Bogdan Maruszewski, "Dynamical magnetothermoleastic problem in circular cylinders-II", IJES,vol.19(9), pp.1241-1253, 1981.
9. Warren.P.Mason, " Piezoelectricity, its history and applications", JASA, vol 70 (6) pp. 1561-1566, Dec 1981.
10. H.S.Paul and K.Renganathan, " Free vibrations of pyroelectric layer of hexagonal (6mm) class", JASA, vol.78(2), pp.395-397, Aug. 1985.
11. J.Hasegawa and E.Takaya, " Development of microcomputer-aided pyroelectric thermal imaging system and application to pain management", Med. & Biol.Engg. & Comput., vol.24, pp.275-280, May 1986.
12. H.S.Paul and G.V.Raman, " Vibrations of pyroelectric plates" , JASA, vol.90(4), PP. 1729-1732, Oct. 1991.
13. Adnan H. Nayfeh and Hual-Te Chien, " The influence of piezoelectricity on free and reflected waves from fluid-loaded anisotropic plates", JASA, vol.91(3), PP.1250-1261,March 1992.
14. S.Syngellakis and P.C.Y.Lee, " Piezoelectric wave dispersion curves for infinite anisotropic plates", J.Appl.Phys. vol.73(11) pp.7152-7160, June 1993.
15. S.S.Rao and M.Sunar, " Analysis of distributed thrmopiezoelectric sensors and actuators in advanced intelligent structures", AIAA Journal, vol.31(7), pp.1280-1285, July 1993.
16. H.S.Paul and V.K.Nelson, " Wave Propagation in piezocomposite plate " , Proc. Indian natn. Sci,Acad., Vol 61, (3-4) pp. 221-228, Apr 1995.
17. E.Pan and P.R.Heyliger, " Free vbrations of simply supported and multilayred magneto-electro-elastic plates", J. of Sound and vibration vol.252(3), pp.429-442, 2002.
18. W.Q.Chen and Kang Yong Lee," Alternative state space formulations for magnetoelectric themoeleasticity with transverse isotropy and the application to bending analysis of non-homogeneous plates", Int.J.Solids and structures , vol.40, pp.5689-5705, June 2003.
19. W.Q.Chen , K.Y.Lee and H.J.Ding, " General solution for transversely isotropic magneto-electro-thermo-elasticity and the potential theory method, int.J.eng.sci., vol42, pp.1361-1379, 2004.
20. John H Lehman, Chaiwat Engtrakul, Thomas Gennett, and Anne C.Dillon, "Single-wall carbon nanotube coating on a puroelectric detector", Applied Optics, Fol.44(4), pp.483-488, Feb 2005.
21. A.R.Annigeri, N.Ganesan,S.Swarnamani," Free vibrations of clamped-clamped magneo-electro-elastic cylindrical shells", J.of Sound and vibration, vol.292,pp.300-314, 2006.
22. E.Careera, S.Brichetto and P.Nalli, " Variational statements and computation models for multifield problems and multi-layered structures", Mechanics of Advanced Materials and Struycutures, vol.15, pp.182-198, Jan 2008.
23. W.J.Feng , E.Pan and X.Wang, " Stress analysis f a penny-shaped crack in a magneto-electro-themro-elastic layer under uniform heat flow and shear loads", J. of Themrla stresses, vol.31, pp.497-514, Jan 2008.
24. P.F.Hou, T.Yi, and L.Wang, " 2D general solution and fundamental solution for orthotropic electro-magneto-thermo-elastic mateials", J.of thermal stresses, Vol.31(9), pp.807-822, Aug.2008.
25. Peng-Fei Hou, Hao-Ran Chen and Sha He, " Three dimensional fundamental solution for transversely isotropic lector-magneto-thermo-elastic materials", J. of thermal stresses, vol.32, pp.887-904, Jan 2009.
26. 25. S.Karthikeyan and V.K.Nelson, " Vibration of pyroelectric sandwich plate", i-Manager's journal on Engineering and Tenchnology, vol. 6(1), pp. Oct. 2010.

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