KALUZA KLEIN COSMOLOGICAL MODELS WITH PERFECT FLUID IN SAEZ AND BALLESTER THEORY

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Abstract: An attempt has been made to solve the field equations with perfect fluid in a five dimensional space-time in the scalar tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A 113, 467, 1986). Some physical and geometrical properties of the models are discussed.

Keywords: Kaluza-Klein, perfect fluid, Saez and Ballester theory

1. Introduction: The study of higher dimensional cosmological models attracted the attention of many researchers due to its significant role in the description of the early Universe. Chodos and Detweiler [1] obtained cosmological solutions which yield the contraction of the extra dimensions as a consequence of cosmological evolutions. Guth [2] and Alvarez and Gavela [3] found that during contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problems. In addition many authors [4-8] claimed through the solutions of the field equations that there is an expansion of the four dimensional space-time while extra dimension contracts or remain constant.

Several alternative theories of gravitation are proposed to reveal the nature of the Universe at the early stage of the evolution. Saez and Ballester [9] formulated scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field. Singh and Agrawal [10], Shri Ram and Singh [11], Mohanty and Sahu [12-14], Reddy [15, 16] are some of the authors who have studied various aspects of this theory. Reddy and Naidu [17] constructed various string cosmological models in Saez and Ballester theory. Mohanty et al. [18] shown that five dimensional LRS Bianchi type-I string cosmological models do not survive for geometric and Takabayasi string whereas barotropic string survives where the sum of rest energy density and tension density for cloud of string vanish. In this paper Kaluza Klein cosmological models are obtained for perfect fluid in Saez and Ballester theory of gravitation.

2. Field Equations

Here we consider the five dimensional metric in the form

$$ds^2 = dt^2 - R^2\left(dx^2 + dy^2 + dz^2\right) - A^2 dm^2 \quad (1)$$

where *R* and *A* are functions of cosmic time *t* only.

The field equations given by Saez and Ballester [9] for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega V^n \left(V_{,i}V_{,j} - g_{ij}V_{,a}V^{,a}\right) = -T_{ij} \quad (2)$$

and the scalar field satisfies the equation

$$2V^n V_{;i}^{,i} + nV^{n-1}V_{,a}V^{,a} = 0 \quad (3)$$

where ω and n are arbitrary dimensionless constants. T_{ij} is the stress tensor of the matter and comma and semicolon denote partial and covariant differentiations respectively.

The energy momentum tensor for a perfect fluid is given by

$$T_{ij} = \left(P + \rho\right)u_i u_j - P g_{ij} \quad (4)$$

where *P* is the isotropic pressure, ρ is the energy density and u^i represent the four velocity vector of the fluid distribution. Further we have

$$T_{;j}^{ij} = 0 \quad (5)$$

as a consequence of field equations (2) and (3).

Using (3)-(6) the explicit form of field equations (2) for the line element (1) are obtained as

$$\frac{3R''}{R} + 3\left(\frac{R'}{R}\right)^2 - \frac{1}{2}\omega V^n V'^2 = -P \quad (6)$$

$$\frac{2R''}{R} + \left(\frac{R'}{R}\right)^2 + \frac{2R'A'}{RA} + \frac{A''}{A} - \frac{1}{2}\omega V^n V'^2 = -P \quad (7)$$

$$3\left(\frac{R'}{R}\right)^2 + \frac{3R'A'}{RA} + \frac{1}{2}\omega V^n V'^2 = \rho \quad (8)$$

$$V'' + V'\left(\frac{3R'}{R} + \frac{A'}{A}\right) + \frac{n}{2}\frac{V'^2}{V} = 0 \quad (9)$$

$$\frac{3R'}{R} + \frac{A'}{A} = \frac{-\rho'}{P + \rho} \quad (10)$$

Here afterwards the dash over the field variable represents ordinary differentiation with respect to cosmic time t only.

3. Consequences of the field equations and models

From equations (7) and (8) we obtain

$$\frac{R''}{R} + 2\left(\frac{R'}{R}\right)^2 = \frac{A''}{A} + \frac{2R'A'}{RA} \quad (11)$$

In view of this equation we consider

$$R = F(A(t)) \quad (12)$$

Now (11) with help of (12) becomes

$$\left(\frac{\dot F}{F}-\frac{1}{A}\right)A''+\left(\frac{\ddot F}{F}+2\frac{\dot F^2}{F^2}-2\frac{\dot F}{FA}\right)A'^2=0 \tag{13}$$

Here overhead dot denotes the differentiation w.r.t. the argument. Equation (14) is satisfied for the following cases:

Case (i):

$$\frac{\dot F}{F}-\frac{1}{A}=0 \text{ and } \frac{\ddot F}{F}+2\frac{\dot F^2}{F^2}-2\frac{\dot F}{FA}=0 \tag{14a, b}$$

Case (ii):

$$A''=0 \text{ and } \frac{\ddot F}{F}+2\frac{\dot F^2}{F^2}-2\frac{\dot F}{FA}=0 \tag{15a, b}$$

Case (iii): $A'=0$ (16a, b)

Case (iv): $A'=0$ and $\frac{\dot F}{F}-\frac{1}{A}=0$ (17a, b)

As the field equations are highly nonlinear in the subsequent sections we construct cosmological models corresponding to each of the above cases.

3.1 Case (i)

In this case we consider the stiff fluid case $(P=\rho)$ and obtained the explicit form of physical parameters as follows

$$A=(k_1t+k_2)^{1/4} \tag{18}$$

$$R=k_3(k_1t+k_2)^{1/4} \tag{19}$$

$$V^{\frac{n+2}{2}}=k\log\{k_4(k_1t+k_2)\} \tag{20}$$

and

$$P=\rho=\frac{k_5}{(k_1t+k_2)^2} \tag{21}$$

where $k_1\neq 0$, $k_i, i=2,3,4$ are constants of integration and

$$k=\frac{n+2}{k_3^3k_8},\quad k_5=\frac{\{3(n+2)^2k_1^2+16\omega k_1^2\}}{128(n+2)^2}.$$

Therefore the metric corresponding to the above solution through a proper choice of coordinates becomes

$$ds^2=dt^2-(k_1t+k_2)^{1/2}\left[k_3{}^2(dX^2+dY^2+dZ^2)+dm^2\right] \tag{22}$$

which represents the stiff fluid model in Saez and Ballester theory of gravitation.

3.2 Case (ii)

In this case we get $A=(at+b)$ (23)

$$R=\left[a_1(at+b)^3+a_2\right]^{1/3} \tag{24}$$

$$V^{\frac{n+2}{2}}=a_4\ln\frac{a_3(at+b)}{a_1(at+b)^3+a_2} \tag{25}$$

where $a\neq 0, a_i, i=1,2,3,4$ are constants of integration. For simplicity we take $a_1=a_3=a_4=1$ and $a_2=0$. Thus we have

$$P=\frac{a^2\left(8\omega-3(n+2)^2\right)}{(n+2)^2(at+b)^2} \tag{26}$$

and

$$\rho=\frac{a^2\left(8\omega+6(n+2)^2\right)}{(n+2)^2(at+b)^2} \tag{27}$$

In this case the metric (1) becomes

$$ds^2=dt^2-(at+b)^2\left(dx^2+dy^2+dz^2-dm^2\right) \tag{28}$$

which represents the perfect fluid Universe in the Saez and Ballester theory.

3.3 Case (iii):

In this case we have

$$A=c \tag{29}$$

$$R=(3c_1t+3c_2)^{1/3} \tag{30}$$

$$V^{\frac{n+2}{2}}=\frac{c_0(n+2)}{6c_1}\ln(3c_1t+3c_2) \tag{31}$$

$$P(=\rho)=\frac{3c_1{}^2}{2(3c_1t+3c_2)^{1/3}}+\frac{\omega c_0{}^2}{2(3c_1t+3c_2)^2} \tag{32}$$

where $c, c_0, c_1\neq 0, c_2$ are constants of integration. In this case metric (1) becomes

$$ds^2=dt^2-(3c_1t+3c_2)^{2/3}\left(dx^2+dy^2+dz^2\right)-c^2dm^2 \tag{33}$$

which represents the stiff fluid Universe in Saez and Ballester theory.

3.4 Case (iv)

In this case the scale factors are constants and the metric reduces to a flat metric. The pressure and density in the model are constants. Further we have

$$V^{\frac{n+2}{2}}=\frac{n+2}{2}(c_3t+c_4) \tag{34}$$

4. Some physical and geometrical properties

In this section we study some physical and geometrical properties of the models obtained in the preceding section.

In case (i) we obtained a stiff fluid model of the Universe. The volume of the Universe increases with the increase in cosmic time t. The pressure and density in the model decrease as t increases. The scalar of expansion (θ) and shear scalar (σ^2) for the model are obtained as

$$\theta = \frac{k_1}{k_1 t + k_2} \quad (35)$$

$$\sigma^2 = \frac{k_1^{\,2}}{(k_1 t + k_2)^2} \quad (36)$$

The scalar of expansion is infinitely large at $t = \frac{-k_2}{k_1}$ and tends to zero as *t* tends to infinite. There is a big-bang in the model at $t = \frac{-k_2}{k_1}$. The tensor of rotation in the model is zero. Hence the model represents a shearing and non-rotating Universe with the big-bang.

In case (ii) we obtained a perfect fluid model of the Universe which is expanding in nature. The pressure and density in this model decrease with increase in cosmic time *t*. There is a big-bang in the model at $t = \frac{-b}{a}$. The scalar of expansion and shear scalar for this model are

$$\theta = \frac{4a}{at + b} \quad (37)$$

$$\sigma^2 = \frac{32a^2}{9(at + b)^2} \quad (38)$$

In case (iii) we obtained the stiff fluid model of the Universe. In this model the extra dimension becomes constant. The scalar of expansion and shear scalar for this model are

$$\theta = \frac{3c_1}{3c_1 t + c_2} \quad (39)$$

$$\sigma^2 = \frac{2c_1^{\,2}}{(3c_1 t + c_2)^2} \quad (40)$$

In all the above three models we observed that $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$. Hence the models are anisotropic throughout the evolution.

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