OPTIMAL DESIGNING OF THREE STAGE CHAIN SAMPLING PLAN WITH REPETITIVE GROUP SAMPLING PLAN AS REFERENCE PLAN

K.K.SURESH , ANAMIYA BABY

Abstract : This paper deals with the optimal designing of a three stage chain sampling plan of type ChSP(0,1,2) by considering the repetitive group sampling plan as the reference plan indexed through Acceptable Quality Level (AQL), Limiting Quality Level (LQL), Indifference Quality Level (IQL) and its operating ratio. Poisson unity values have been tabulated to facilitate the operation and construction of the plan. The tables are constructed by considering the various combinations of acceptable and limiting quality levels, and an example is given for illustration purpose.

Keywords : Acceptable Quality Level, Indifference Quality Level, Limiting Quality Level and Three Stage Chain Sampling Plan.

Introduction : Acceptance sampling is a statistical tool used to make decisions concerning whether or not a lot of products should be released for consumer use. An acceptance sampling plan is a statement regarding the required sample size for product inspection and the associated acceptance or rejection criteria for sentencing individual lots. The criteria used for measuring the performance of an acceptance sampling plan, is usually based on the operating characteristic (OC) curve which quantifies the risks for producers and consumers. The OC curve plots the probability of accepting the lot versus the lot fraction nonconforming, which displays the discriminatory power of the sampling plan. The basic acceptance sampling plan called the single-sampling plan is widely used in industry to inspect items due to its easiness of implementation. A single sampling attribute inspection plan calls for acceptance of a lot under consideration .If the number of non-conforming units found in a random sample of size n is less than or equal to the acceptance number Ac.

Whenever a sampling plan for costly or destructive testing is required, it is common to force the OC curve to pass through a point, say,(LQL,β). Unfortunately the Ac=0 plan has the following disadvantages.

- The OC curve of the Ac=0 plan has no point of inflection and hence it starts to drop rapidly even for the smallest increases in the fraction nonconforming p.
- The producer dislikes an AC=0 plan since a single occasional nonconformity will call for the rejection of the lot.

The chain sampling plan Chsp-1 by Dodge is an answer to the question of whether anything can be done to improve the pathological shape of the OC curve of a zero-acceptance –number plan.

Review Of Literature : Clark (1960) has provided a discussion on the OC curves for chain sampling plans .Soundarajan(1984) constructed tables for the selection of chain sampling plans for given acceptable quality level (AQL,denoted as p_1),producer risk α),LQL and β. Govindaraju(1990) has discussed the design ChSP -1 plan for minimum average total inspection. Dodge and Stephens (1966) viewed the chain sampling approach as a cumulative results criterion applied in to two stages and extended it to include larger acceptance numbers. Stephen and Dodge (1976) presented a further generalization of the family of two-stage chain sampling inspection plans by using different sample sizes in the two stages. Chain sampling is extended to two or more stages of cumulation of inspection results with appropriate acceptance criteria for each stage. Frishman (1960) extended the ChSP -1 plan and developed ChSP-4 and ChSP-4a plans

Raju(1991) has extended the two-stage chain sampling to three stages .The three stage cumulation procedure becomes complex, and will pay a limited role for costly or destructive inspection. The three stage plan will however be useful for general type B lot by lot inspection. Soundararajan(1984) and Raju (1984) gave the structure and operating procedure for generalized three-stage chain sampling plan and expression for OC curve of certain three-stage plans are also given.Suresh and Sripriya (2007) has developed a method for designing plans from the desired operating ratio where procedures and tables for the construction of three stage chain sampling plan of type ChSP (0, 1, 2) and for selection of plans by specified parameters are given . Suresh and Anamiya(2012) has developed three –stage Chain Sampling Plans indexed through Minimum angel method with producer quality level and consumer quality level.

The concept of Repetitive Group Sampling plan (RGS) plan was introduced by Sherman (1965) in which acceptance and rejection of the lot is based on the repeated sample results of the same lot. The detailed procedure and tables for the construction and selection of RGS plans have been given by Soundarajan and Ramaswamy (1984) and Singh et al.(1989).The purpose of present investigation is two-fold. Firstly following Stephens and Dodge (1976) proposed plan which uses different sample size in the normal and tightened phases of inspection.

Selection Of Sampling Plan :

3.1Repetitive Group Sampling plan (RGS)

Conditions for the Repetitive Group Sampling Plan

- The size of the lot is taken to be sufficiently large
- Under normal conditions, the lots are expected to be of eventually same quality
- The producer comes from a sources in which the consumer has confidence

Operating Procedure

Draw a random sample of size n_1 from the lot of normal inspection and determine the number of defectives (d) found therein

- Accept the lot id $d \le c_1$
- Reject the lot if $d < c_2$

If $c_1 < d \le c_2$ repeat the steps (1),(2) and (3).it is also noted that $c_1 < c_2$ .Thus the RGS plan is determined by the parameters n,c_1 and c_2

The probability of acceptance in a particular group sample is

$$P_1(p) = \sum_{k_1=0}^{c_1} \frac{e^{-np}(np)^{k_1}}{k_1!} \qquad (3.1.1)$$

The probability of rejection in a particular group sample is

$$P_1^1(p) = \sum_{k_2=c_2}^{\infty} \frac{e^{-np}(np)^{k_1}}{k_2!} \qquad (3.1.2)$$

The probability of eventually accepting the lot is given as

$$P_1(n, c_1, c_2 / p) = \frac{p_1}{p_1 + p_1^1}. \qquad (3.1.3)$$

Then from (1) and (2)

$$P_a = \frac{\sum_{k_1=0}^{c_1} \frac{e^{-np}(np)^{k_1}}{k_1!}}{1 - \sum_{k_2=c_2}^{\infty} \frac{e^{-np}(np)^{k_1}}{k_2!}} \qquad (3.1.4)$$

3.2 Three Stage Chain Sampling Plan

As mentioned already, three stage ChSP (0,1,2) plan is An extension of two stage ChSP plan. Before presenting the procedure for the three stage chain sampling plan of type ChSP (0,1,2) with a repetitive group as the reference plan, we recall the operation of three stage chain sampling plan. The operating procedure of such plan provided as follows.

Operating procedure for Three stage Chain sampling plan

Step 1: At the outset, select a random sample of n units from the lot and from each succeeding lot.

Step2: Record the number of defectives d, in each sample and sum the number of defectives, D, in all samples from the first up to and including in the current sample.

Step 3: Accept the lot associated with each new sample during the cumulaion as long as $D_i \le c_1$; $1 \le i \le k_1$.

Step4: When k_1 consecutive samples have all resulted in acceptance continue to sum the defectives in the k_1 samples plus additional samples upto not more than k_2 samples.

Step 5: Accept the lot associated with each new sample during cumulation as long as $D_i \le c_2$; $k_1 \le i \le k_2$.

Step 6: When k_2 consecutive samples have all resulted in acceptance continue to sum the defectives in the k_2 samples plus additional samples upto not more than k_3 samples.

Step 7: Accept the lot associated with each new sample during cumulation as long as $D_i \le c_3$; $k_2 \le i \le k_3$.

Step 8: When the third stage of the restart period has been successfully completed (i.e.,k_3 consecutive samples have been resulted in acceptance),start cumulation of defectives as moving total over k_3 samples by adding the current sample result while dropping from the sum, the sample result of the k_3th preceding sample. Continue this procedure as long as $D_i \le c_3$ and in each instance accept the lot.

Step 9: If for any sample at any stage of the above procedure, D_i is greater than the corresponding c, reject the lot.

Step 10: When a lot is rejected return to step-1 and fresh restart of the cumulation procedure.

When k_1=1, k_2=2 and k_3=3 and $c_2 = c_1 + 1$, $c_3 = c_2 + 1$, the three stage chain sampling plan becomes a multiple sampling plan

Selection Procedure :

4.1Selection of ChSPRGS (0,1,2)

For construction and evaluation of the Three Stage Chain Repetitive Group Sampling plan, the np values presented in tables were derived under the procedure stated by Duncan [1965]. Tables are used to derive individual plan to meet specified values of fraction defectives and probability of acceptance. It requires the specifications of AQL (p_1), LTPD (p_2), Producers risk (α), Consumers risk (β) and acceptance criteria i. The steps to be followed are,

1. Specify p_1 - Acceptable Quality Level (AQL), p_2 - Lot Tolerance Proportion Defective (LTPD), producer risk (α) and consumer risk (β).
2. The operating ratio is OR = p_2 / p_1 and m = np.
3. Choose the plan parameters having k_1,k_2,k_3,c_1,c_2 and i associated with an operating ratio which is nearest in the corresponding table.
4. Determine the sample size n = np_2 / p_1.
5. The OC Curve may be drawn by dividing the values of np shown for the plan by sample size n to obtain p associated with 0.95 for Pa(p).
6. Thus, the plan consists of six parameters namely: $n,,k_1,k_2,k_3,c_1,c_2,c_3$ and i may chosen from the given tables.

4.2 Designing of three stage ChSPRGS (0,1,2) plan

In general, any sampling plan or any sampling system can be designed for specified two points on the operating characteristic (OC) curve namely, acceptable quality level (AQL) and limiting quality level (LQL), along with the corresponding producer's risk (α) and the consumer's risks (β). AQL is usually defined as the worst-case quality level, in percentage or ratio, which is still considered acceptable.As an AQL is an acceptable level, the probability of acceptance of a lot at the AQL should be high. LQL is used as an index for consumer protection for designing an acceptance sampling plan. AQL is denoted by p_1 and the LQL is denoted by p_2.

4.3 Construction and Evaluation of the Plan:

Based on the principle of two points on the OC curve, the designing methodology of the ChSPRGS plan is explained below.According to Raju, the OC function of an ChSP (0,1,2) plan is given by,

$$P_a(P)=\frac{\begin{array}{l}p_0+p_1p_0^{k_2-1}+(k_3-k_2-1)p_1^2p_0^{k_3-2}+p_0^{k_3-1}+p_1p_0^{k1}\left[\frac{1-p_0^{k_2-k_1-1}}{1-p_0}\right]+(k_2-k_1)p_1^2p_0^{k_2-1}+\\ p_1^2p_0^{k_2}\left[\frac{1-(k_3-k_2-1)p_0^{k_3-k-2_2}}{1-p_0}+\frac{p_{0(}(1-p_0^{k_3-k-2_2})}{(1-p_0)^2}\right]+\frac{p_a(1-p_c)^i+p_cp_a^i}{(1-p_c)^i}\,p_0^{k_2}\left[\frac{1-p_0^{k-k_2-1_3}}{1-p_0}\right]\end{array}}{\begin{array}{l}1+p_1p_0^{k_1}\left[\frac{1-p_0^{k_2-k_1-1}}{1-p_0}\right]+(k_2-k_1)p_1^2p_0^{k_2-1`}+\\ p_1^2p_0^{k_2}\left[\frac{1-(k_3-k_2-1)p_0^{k_3-k-2_2}}{1-p_0}+\frac{p_{0(}(1-p_0^{k_3-k-2_2})}{(1-p_0)^2}\right]+\frac{p_a(1-p_c)^i+p_cp_a^i}{(1-p_c)^i}\,p_0^{k_2}\left[\frac{1-p_0^{k-k_2-1_3}}{1-p_0}\right]\end{array}} \quad (4.3.1)$$

It is well known that for a series of lots from a process, the binomial model for the OC curve will be exact in the case of fraction non-conforming. One can be satisfactorily approximated with the Poisson model where p is small, n is large, and np<5 when the quality is measured in terms of non-conformities, the Poisson model is the appropriate one.

The expression for probability of acceptance under the assumption with Poisson model, the composite OC function of ChSPRGS (0,1,2) is given through equation (5) with equation (4).

$$P_a(P)=\frac{\begin{array}{l}p_0+p_1p_0^{k_2-1}+(k_3-k_2-1)p_1^2p_0^{k_3-2}+\frac{\sum_{k_1=0}^{c_1}\frac{e^{-np}(np)^{k_1}}{k_1!}}{1-\sum_{k_2=c_2}^{\infty}\frac{e^{-np}(np)^{k_1}}{k_2!}}\,p_0^{k_3-1}+p_1p_0^{k1}\left[\frac{1-p_0^{k_2-k_1-1}}{1-p_0}\right]+(k_2-k_1)p_1^2p_0^{k_2-1}+\\ p_1^2p_0^{k_2}\left[\frac{1-(k_3-k_2-1)p_0^{k_3-k-2_2}}{1-p_0}+\frac{p_{0(}(1-p_0^{k_3-k-2_2})}{(1-p_0)^2}\right]+\frac{\sum_{k_1=0}^{c_1}\frac{e^{-np}(np)^{k_1}}{k_1!}}{1-\sum_{k_2=c_2}^{\infty}\frac{e^{-np}(np)^{k_1}}{k_2!}}\,p_0^{k_2}\left[\frac{1-p_0^{k-k_2-1_3}}{1-p_0}\right]\end{array}}{\begin{array}{l}1+p_1p_0^{k_1}\left[\frac{1-p_0^{k_2-k_1-1}}{1-p_0}\right]+(k_2-k_1)p_1^2p_0^{k_2-1`}+\\ p_1^2p_0^{k_2}\left[\frac{1-(k_3-k_2-1)p_0^{k_3-k-2_2}}{1-p_0}+\frac{p_{0(}(1-p_0^{k_3-k-2_2})}{(1-p_0)^2}\right]+\frac{\sum_{k_1=0}^{c_1}\frac{e^{-np}(np)^{k_1}}{k_1!}}{1-\sum_{k_2=c_2}^{\infty}\frac{e^{-np}(np)^{k_1}}{k_2!}}\,p_0^{k_2}\left[\frac{1-p_0^{k-k_2-1_3}}{1-p_0}\right]\end{array}} \quad (4.3.2)$$

Where

$P_o=e^{-np}$,$P_1=npe^{-np}$, $P_2=((np)^2/2)e^{-np}$

Table 1.1:np values for given probability of acceptance by three stage chain sampling plan ChSP (0, 1, 2) with Repetitive Group Sampling plan i=1

k1	k2	k3	99	95	90	75	50	0.25	0.1	0.05	0.01
1	2	3	0.5591	0.5941	0.6414	0.8063	1.1813	1.7862	2.565	3.152	4.629
1	2	5	0.4442	0.4854	0.5406	0.7324	1.1453	1.7757	2.561	3.151	4.629
2	2	5	0.442	0.474	0.5164	0.6611	0.9939	1.5801	2.394	3.043	4.575
2	3	4	0.4127	0.4398	0.4766	0.6079	0.928	1.5062	2.334	3.005	4.562
2	4	5	0.3366	0.3643	0.4023	0.5419	0.878	1.4748	2.321	3.002	4.561
2	5	4	0.3364	0.3549	0.3974	0.5101	0.8354	1.4567	2.319	3.001	4.561
3	4	5	0.3357	0.3594	0.3917	0.51	0.8167	1.4201	2.301	2.995	4.559
4	5	6	0.2869	0.3085	0.3384	0.4515	0.759	1.3946	2.301	2.994	4.559
5	6	7	0.2521	0.2729	0.3012	0.4117	0.729	1.3872	2.302	2.994	4.558
6	7	8	0.2265	0.2462	0.2735	0.3831	0.712	1.3871	2.302	2.994	4.558
7	8	9	0.2067	0.2254	0.252	0.3619	0.7029	1.3869	2.301	2.994	2.301
8	9	10	0.1902	0.2086	0.2347	0.3456	0.698	1.3868	2.301	2.994	2.3
9	10	11	0.1771	0.1947	0.2204	0.3329	0.6954	1.3862	2.299	2.994	2.3
9	10	11	0.1771	0.1947	0.2204	0.3329	0.6954	1.3862	2.3	2.994	4.559
10	11	12	0.1653	0.183	0.2084	0.3227	0.694	1.3861	2.298	2.994	4.559
11	12	13	0.1558	0.1729	0.1981	0.3157	0.6932	1.3861	2.297	2.994	4.559
11	12	13	0.1558	0.1729	0.1981	0.3157	0.6932	1.3861	2.297	2.994	4.559
11	12	14	0.1501	0.1676	0.1935	0.3141	0.6931	1.3861	2.296	2.994	4.559
12	13	14	0.1472	0.1642	0.1892	0.3096	0.6928	1.3861	2.295	2.994	4.559

K1	k2	k3	99	95	90	75	50	0.25	0.1	0.05	0.01
13	14	15	0.1397	0.1565	0.1821	0.3047	0.6925	1.3861	2.293	2.994	4.558
13	14	16	0.1353	0.1522	0.1784	0.3036	0.6925	1.3861	2.302	2.994	4.56
13	14	16	0.1353	0.1522	0.1784	0.3036	0.6925	1.3861	2.302	2.994	4.56
13	15	16	0.1334	0.1502	0.1763	0.3022	0.6924	1.3861	2.302	2.994	4.56
14	15	16	0.1305	0.1499	0.1753	0.3007	0.6924	1.3861	2.302	2.994	4.56
14	15	17	0.1289	0.1458	0.172	0.2999	0.6923	1.3861	2.302	2.994	4.56
14	17	18	0.1223	0.14	0.1656	0.2974	0.6919	1.386	2.302	2.994	4.56
14	17	18	0.1223	0.14	0.1656	0.2974	0.6919	1.386	2.302	2.994	4.56
15	16	18	0.1236	0.14	0.1653	0.297	0.6919	1.386	2.302	2.994	4.56
15	14	19	0.123	0.14	0.1651	0.297	0.6919	1.386	2.302	2.994	4.56
15	16	19	0.12	0.1367	0.1636	0.2964	0.6916	1.386	2.302	2.994	4.56
15	18	19	0.1174	0.1338	0.1606	0.295	0.6913	1.386	2.302	2.994	4.56
16	18	19	0.1174	0.1335	0.1598	0.2941	0.6913	1.386	2.302	2.994	4.56
16	18	19	0.1174	0.1335	0.1598	0.2941	0.6913	1.386	2.302	2.994	4.56
16	17	20	0.1154	0.1335	0.1587	0.2941	0.6913	1.386	2.302	2.994	4.56
16	16	20	0.1153	0.1335	0.1587	0.294	0.6913	1.386	2.302	2.994	4.56
17	17	20	0.1153	0.1314	0.158	0.2936	0.6912	1.386	2.302	2.994	4.56
18	19	20	0.1129	0.1286	0.1547	0.292	0.6912	1.386	2.301	2.994	4.56
19	20	21	0.109	0.1245	0.1508	0.2908	0.6912	1.386	2.301	2.994	4.56
19	20	22	0.1061	0.1223	0.1487	0.2906	0.6912	1.386	2.301	2.994	4.56
20	21	22	0.1051	0.1207	0.1472	0.2899	0.6911	1.386	2.301	2.994	4.56

Conclusion : The present development would be a valuable addition to the literature and a useful device to the quality practitioners. The concept of this article may be used for assistance to quality control engineers and plan designers in the development of further plans, which were useful and tailor made for industrial shop-floor situations.

.

References :

1. B.Anamiya." Selection of Three Stage Chain Sampling Plan of type ChSP(0,1,2) through Minimum Angle Method", submitted for publication" JQT,2012
2. S.B. Bachi, "An extension of chain sampling plan",IAPQR Trans.Vol.1,1976,pp19-22.
3. H.F.Dodge ,and K.S.Stephens,"Some new chain sampling inspection plans",Ind,Qual.Contr,Vol.23,1966,pp61-67.
4. C.R.Clark,"O-C Curves for ChSP-1 Chain sampling plans",Ind.Qual.Contr,Vol.17,1960, pp10-12.
5. F .Frishman ."An extended chain sampling plan",Ind.Qual.Contr,Vol.17, 1960,pp 10-12.
6. K .Govindaraju ."Selction of Chsp -1 chain sampling plans for given acceptable quality levels and limiting quality level",Commu.Sat.Theor.Methods Vol.19, 1990,pp 2179-2190.
7. C.Raju.'Three stage chain sampling plans",Commun.Stat.Theor.Methods,Vol 20, 1991,1777-1801.
8. E.S.Robert."Design and Evaluvation of repetitive group sampling plan",Technometrics,Vol 7.No.1, 1965,pp 11-21.
9. K.K Suresh and K .Sripriya. "Selection of three Chain Sampling Plan of type ChSP (0,1,2) indexed with AQL and AOQL ",Acta Ciencia Indica,Vol.xxxiv M,No.2, 2008,pp 501-511.
10. K.S Stephen, and H.F .Dodge ."Two stage chain sampling inspection plans with different sample sizes in two stages", Journal of Quality Technology Vol.8, 1976,pp209-224.
11. V .Soundarajan." Procedures, tables for construction, selection of chain sampling plans(ChSP-1)", Journal of Quality Technology Vol.16, 1984,pp 219-231.

***

Department of Statistics,
Bharathiar University,Coimbatore-641 046, Tamilnadu.
Email: sureshkk1@rediffmail.com, Email: anamiyababy@gmail.com