CONSTRUCTION AND SELECTION OF ONE PLAN SUSPENSION SYSTEM WITH BAYESIAN CHAIN SAMPLING PLAN AS REFERENCE PLAN

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Abstract : Acceptance sampling plans are practical tools for quality assurance applications involving productquality control. Acceptance sampling systems are advocated when small sample size are necessary or desirabletowards costlier testing for product quality. When single plan is used with suspension rule, the system is calledOne-Plan (OP) Suspension System. In OP suspension system, a lot-by-lot sampling plan is used to decide whether individual lots are accepted or rejected.
This paper provides a new procedure for Designing and Selection of One Plan Suspension System with Bayesian Chain Sampling Plan as Reference Plan. Tables and procedures are also provided for the selection of the parameter for the plan. Numerical illustrations are also provided for the shop floor applications of these procedures.

Keywords: One Plan Suspension System, Bayesian Chain Sampling Plan, Gamma-Poisson Distributions, Stopping rule, Average probability of Acceptance, Operating Ratio.

Introduction : To improve the quality of any product and services, it is customary to modernize the quality practices and simultaneously reduce the cost for quality improvement. An efficient quality improvement Program can be instrumental in increasing productivity at reduced cost. As a result of increasing customer quality requirements and development of new product technology, many existing quality assurance practices and techniques need to be modified. The need for statistical and analytical techniques in quality assurance is rapidly increasing owing to stiff competition in industry towards product quality.

Acceptance sampling is a tool for the consumer to reject bad lots as well as the producer to quicken the process control. In a progressive atmosphere of production with increased chances of occurrence of non-conforming material, statistical process control optimizes the process capability and acceptance sampling acts logically to prevent passing out non-conforming units.

This paper is concerned with acceptance sampling plans when small samples are necessary or desirable, for example, when production quantities are small or when inspection is either costly or destructive. Under these conditions, an attribute plan with small sample size is not very effective, since the discrimination between good and bad quality is not sufficient. Nor does lot-by-lot inspection provide an incentive for the producer to turn out consistently good quality.

Review Of Literature : Calvin (1984) has provided procedures and tables for implementing Bayesian Sampling Plans. A set of tables presented by Oliver and Springer (1972) which are based on assumption of Beta prior distribution with specific posterior risk to achieve minimum sample size, which avoids the problem of estimating cost parameters. It is generally true that Bayesian Plan requires a smaller sample size than a conventional sampling plan with the same producer and consumer risks. Schafer (1967) discusses single sampling by attributes using three prior distributions for lot quality. Hald (1960) has given an extensive account of sampling plans based on discrete prior distributions of product quality. Case and Keats (1982) have provided a table for the classification of attributes sampling plan design methodologies.

Lilly Christina (1995) have studied the Selection of One Plan Suspension System with Repetitive Group Sampling (RGS) plan as reference plan indexed with Operating ratio and Average run length and also provided the One Plan Suspension System with Chain Sampling Plan as reference plan indexed with Probability of lot acceptance and also studied Modified Suspension System.

Deepa (2002) has studied the formulation of a Bayesian Sampling Plan using acceptance probability with Gamma prior distribution for product quality using producer and consumer quality levels with selection procedure for Bayesian Special type of Double Sampling Plan through MAAOQ and also the sum of weighted risks and also designing sampling plans of the one plan suspension system and Quick switching system.

Latha (2002) has studied average probability of acceptance function for single sampling plan with Gamma prior distribution. Formula of inflection point and tangent at the inflection point are also derived. A selection procedure for Bayesian Single sampling attributes plan (with Gamma prior distribution) based on AQL and LQL, point of control and relative slope at that point, MAPD and K, measure for sharpness are also explained, and lot acceptance procedures are developed for Bayesian Single sampling attributes plans when the acceptance number is fixed and when the sample size is fixed.

Sreeja Unnikrishnan (2007) has studied with One Plan Suspension System having Repetitive Group Sampling Plan and Special Type of Double Sampling Plan as reference plans and designing of Suspension System through Average Outgoing Quality Limit and Maximum Allowable Average Outgoing Quality.

Pradeepaveerakumari (2009) has studied with designing of Bayesian Chain sampling plan indexed with relative slopes and also provided the designing of One plan Suspension System with Bayesian Single sampling plan as reference plan indexed with Reference Quality Level and Overall Average Outgoing Quality Limit (OAOQL). Sangeetha (2010) have studied the Selection of One Plan Suspension System with Single Sampling Plan as reference plan using Minimum Angle Method, Minimum Sum of Risks, Weighted Risks and Quality Regions.

Selection Of Sampling Plan :

3.1 Bayesian Chain Sampling Plan (BChSP-1)

According to Dodge (1955) the operating characteristic function of ChSP-1 plan is

$$P_a(p) = P_0 + P_1 (P_0)^i, \quad (3.1.1)$$

The Chain Sampling Plan (ChSP-1) is characterized with two parameters n and i, where n is the sample size and i is the number of preceding samples with zero defective, using the OC curve, Dodge (1955) has studied the properties of the Chain Sampling Plan. Clark (1960) has presented additional OC curves, which cover most of the situations. Soundararajan (1978 a, b) has described procedures and tables for construction and selection of Chain sampling plans (ChSP-1) indexed through specified parameters.

The probability of acceptance of ChSP-1 based on Poisson model is given as

$$P(n,i/p) = e^{-np} + e^{-np(+i)}np \quad (3.1.2)$$

Using the past history of inspection, it is observed that p follows Gamma distribution with density function,

$$w(p) = e^{-pt} p^{s-i} t^s / \Gamma(s), \quad s,t > 0, \quad p > 0 \quad (3.1.3)$$

The average probability of acceptance is given as

$$\overline{P} = \int_0^\infty P(n,i/p)w(p)dp \quad (3.1.4)$$

$$= s^S/(s+n\mu)^S + ns^{S+1}\mu/(s+n\mu+ni\mu)^{S+1} \quad (3.1.5)$$

Where $\mu = s/t$, is the mean value of the product quality p.

3.2 One Plan Suspension system

Cone and Dodge (1962) have first shown that the effectiveness of a small sample lot-by-lot sampling system can be greatly improved by using cumulative results as a basis for suspending inspection. Suspending inspection required the producer to correct what is wrong and submit satisfactory written evidence of action taken before inspection is resumed. The small sample is due to small quantity of production or costly or destructive nature of sample. Usually small sample size is not very effective since the discrimination between good and bad quality is not sufficient. Hence Cone and Dodge used the cumulative results principle to suspend inspection.

Troxell (1972) has applied the suspension principle to acceptance sampling system. A suspension rule is a procedure used to decide when to suspend inspection of a production process, where product is submitted for inspection in lots. The decision to suspend is based on the observed sequence of lot acceptance and rejections.

A suspension rule, which is designated (j,k), $2 \le j \le k$ is a rule of suspending inspection based on finding j lot rejections in k or less lots. Given j and k at least j lots must be inspected before a decision is possible upon the beginning of a new process or from the time of the last suspension.

A suspension system is a combination of suspension rule and a single lot-by-lot sampling plan or pair plans. When a single plan is used with a suspension rule it is called One Plan (OP) Suspension System and when two plans, Tightened and Normal are used, it is called Two Plan (TP) Suspension System.

Conditions for application

Production is reasonably steady. So that results on current and proceeding lots are broadly indicative of a continuous process.

Samples are taken from lot substantially in the order of production so that observed variations in quality of product reflect process performance.

Inspection is performed close to the production source so that inspection information can be made available promptly.

Inspection is by attributes, with quality measured in terms of fraction defective p.

A single sample of size n or double or multiple samples of equal size n is taken from each sampled lot.

Operating Procedure

For the product under consideration establish a reference quality level (RQL).This RQL termed as np represents the desired quality at delivery considering the needs of service and cost of production.

Consider the established RQL, select a suspension system.

Apply the suspension rule to the first, second.........k lot, then to each successive group of k lots.

If any lot is rejected, declare the lot nonconforming and dispose it in accordance with standard procedures.

If for any lot, the suspension rule occurs, declare the current lot nonconforming and also declare the process nonconforming.

When the process is judged nonconforming:

Notify the submitting agency that no additional lots may be submitted for inspection until that agency has furnished evidence, satisfactory to the inspection agency that action has been taken to assure the submission of satisfactory material.

Dispose the current nonconforming lot in accordance with standard procedures.

When satisfactory evidence of corrective action is furnished, start inspection again with the next succeeding lot and with this lot begin accumulation.

If it becomes necessary to refuse lot submissions a second time, so advice an appropriate higher authority and notify the submitting agency that further submissions will be refused until evidence satisfactory to the higher authority has been approved.

Average Run Length

According to Troxell (1972) the expected time for suspension or average run length of a rule is important in the evaluation of the suspension system. The average run length of the suspension rule (j,k) designated as ARL (j,k) can be calculated in the following way.

First, the expected number of lot rejections until suspension is calculated. Since lot rejections are interspaced with lot acceptances, the second step is to find the total expected number of lots inspected, including the rejected lot, between successive lot rejections, the ARL equals the sum of the total number of lots inspected until suspension.

It is shown that, in fact the total number of inspected lots between consecutive rejections are independently and identically distributed for all rejections so that:

ARL (j,k) = Total number of inspected lots between two Rejections) x (expected number of rejection until suspension.) (3.2.1)

Using this fact, for j=2, the expression is given by a single term and for j=3, the result is best expressed in the form of a continued fraction, which is found by solving for the stationary distribution of a particular Markov chain. For higher rules, a discussion is given indicating the method of solving for the expected number of rejections until suspension. Troxell (1972) has derived the following results:

i) ARL for the rule (j, j), j≥ 2 is

$$\text{ARL (j, j)} = \frac{1-(1-P_a)^j}{P_a(1-P_a)^j} \quad (3.2.2)$$

ii) ARL for the rule (j, ∞) is

$$\text{ARL (j, ∞)} = \frac{j}{1-P_a} \quad (3.2.3)$$

This is the waiting time for the j occurrence of a lot rejection, or the mean of the negative distribution with parameter j

iii) ARL for the rule (2, k) is

$$\text{ARL (2, k)} = \frac{\left(2-P_a^{\,k-1}\right)}{(1-P_a)\left(1-P_a^{\,k-1}\right)}, \ k\geq 2 \quad (3.2.4)$$

For any k such that j < k < ∞ and 0< P_a < 1

ARL (j, j) > ARL (j, k) > ARL (j, ∞) (3.2.5)

So that the rules (j, j) and (j, ∞) respectively are upper and lower bounds for all rules in the class (j, k).

Operating Characteristic Curve

A different type of OC curve which has features not common to type B OC curve (1959) has been used here to study the suspension system. Since ARL for the rule (j,k) is some function of incoming quality p, this correspondence allows an operating characteristic to be plotted in the following way.

In a large number of lots N, the number of lots for which the process is judged conforming, that is the number of lots for which suspension does not occur, is given approximately by N(1-1/ARL). Therefore 1-1/ARL is interpreted as the average fraction of lots for which the process is acceptable, or the probability of accepting the process. This value is denoted as P_A,

$P_A(j,k) = 1 - 1/ARL(j,k)$ and hence (3.2.5)

$$P_A(2,k) = \frac{1+P_a-P_a^{\,k}}{2-P_a^{\,k-1}} \quad (3.2.6)$$

$$P_A(2,\infty) = \frac{1+P_a}{2} \quad (3.2.7)$$

The OC Curve is a graph of P_A as a function of fraction defective.

Selection Procedure :

Designing of One Plan Suspension System with Bayesian Chain sampling Plan as reference plan

In this section, a new procedure has been proposed by Bayesian Chain Sampling plan as reference plan in One-Plan Suspension System. Hence the plan is named One Plan Suspension System with Bayesian Chain sampling plan as reference plan (or) Bayesian One Plan Suspension

System with Chain Sampling plan as reference plan. The plan is designed with Average Probability of Acceptance and Operating Ratio the comparison is made with the conventional sampling plan. The construction and evaluation of the plan is studied.

4.1 Construction and Evaluation of the Plan:

The probability of Acceptance for One Plan Suspension System plan as defined in the equation 3.2.6 is given below

$$P_A(2,k) = \frac{1 + P_a - P_a^{\ k}}{2 - P_a^{\ k-1}}$$

Here P_a is the probability of acceptance for Bayesian Chain Sampling Plan (BCHSP-(0,1)). The Probability of acceptance for Bayesian Chain Sampling Plan- (0,1) as defined in the equation 3.1.5 is given as

$$s^S/(s+n\mu)^S + ns^{S+1}\mu/(s+n\mu+ni\mu)^{S+1}$$

Now, the average probability of acceptance (APA) for One Plan Suspension System with Bayesian Chain Sampling Plan as reference plan is defined as,

$$P_A(2.k) = \frac{1 + s^s/(s+n\mu)^s + ns^{s+1}\mu/(s+n\mu+ni\mu)^{s+1} - (s^s/(s+n\mu)^s + ns^{s+1}\mu/(s+n\mu+ni\mu)^{s+1})^k}{2 - (s^s/(s+n\mu)^s + ns^{s+1}\mu/(s+n\mu+ni\mu)^{s+1})^{k-1}}$$

(4.1.1)

Where $\mu = s/t$, is the mean value of the product quality p.

Example : If the sample size is set at n_k =10, and it is deires to accept material with proportion defective p_k =0.025 (or defects per units) 95 percent of the time so that L(p_k) =0.95, scan the column headed L(p)=0.95, find out the value in Table 1 equal to (just less than) $n_k\ p_k$ =(10)*0.025=0.25. Using Table 1 this turns out to be 0.25328. This is Opposite the k=13, s=3 and i=2 for getting the parameters of the sampling Plan. Further it reveals that higher probability of acceptance with One Plan Suspension System is established at good incoming quality.

Construction Of Tables : For any Suspension rule (j, k) ARL is a function of P_a, the probability of lot acceptance. The expression for P_a is given in equations (4.1.1) respectively. Using the equation (4.1.1) different ARL values such as (2,1) it is equal to 2 by the method of successive approximation and it given a name in k, and also given values of s and i it is a parameters for Bayesian Chain sampling plan. Using these values the Table1 was obtained. Table 1 the 'np' values are tabulated which are found out using the equation (4.1.1) and also find out operating ratio in Table 2.

Conclusion : Bayesian Acceptance sampling is the technique, which deals with the procedures in which decision to accept or reject lots or process based on their examination of past history or knowledge of samples. The present work mainly relates to the Construction and Selection of performance measures of one Plan.

Table 1 Probability of acceptance for One Plan Suspension System with Bayesian Chain Sampling Plan as reference plan.

k	S	i	$P_a(p)$						
			0.99	0.98	0.95	0.90	0.80	0.75	0.50
2	1	0	0.13524	0.63998	1.00117	1.55329	2.96629	4.00488	0.5001
3	1	1	0.09296	0.30060	0.46621	0.71813	1.35414	1.81666	10.313
4	1	2	0.07318	0.21721	0.33722	0.52484	1.02021	1.38053	11.512
5	1	3	0.06341	0.17065	0.27483	0.43815	0.88301	1.22132	91.008
6	1	4	0.03611	0.06736	0.08135	0.09450	0.10968	0.11502	0.1322
7	1	5	0.05361	0.12876	0.21516	0.35701	0.76474	1.10350	66.623
8	1	6	0.04751	0.11602	0.19775	0.33498	0.73691	1.07313	68.593
9	1	7	0.04857	0.10612	0.18476	0.31821	0.72073	1.05449	65.841
10	1	8	0.04435	0.10175	0.17468	0.30618	0.70929	1.04231	65.482
11	1	9	0.04305	0.09600	0.16663	0.29690	0.70097	1.03252	64.732
12	3	0	0.28216	0.36402	0.55301	0.82877	1.46763	1.88413	37.104
13	3	1	0.1856	0.20912	0.32065	0.49173	0.88430	1.13768	24.028
14	3	2	0.03765	0.16275	0.25328	0.39277	0.73396	0.95910	22.979
15	3	3	0.03659	0.13958	0.21927	0.34602	0.66622	0.88348	22.296

16	3	4	0.03554	0.12109	0.19698	0.31846	0.62867	0.84492	22.111
17	3	5	0.03454	0.11118	0.18275	0.29821	0.60741	0.82279	21.973
18	3	6	0.03364	0.10258	0.17090	0.28520	0.59353	0.80939	21.940
19	3	7	0.03279	0.09576	0.16199	0.27564	0.58411	0.80081	21.923
20	3	8	0.03201	0.09020	0.15500	0.26818	0.57752	0.79112	21.910
21	3	9	0.03126	0.08559	0.14802	0.26229	0.57280	0.78917	21.901
22	5	0	0.25679	0.33702	0.52582	0.81379	1.43031	1.79227	20.482
23	5	1	0.14145	0.19504	0.30743	0.39680	0.85577	1.07958	13.815
24	5	2	0.02896	0.15253	0.24357	0.38621	0.70838	0.90919	14.068
25	5	3	0.02881	0.13188	0.21197	0.33975	0.64024	0.83458	14.046
26	5	4	0.08609	0.11526	0.19050	0.31254	0.60532	0.79867	12.626
27	5	5	0.02808	0.10557	0.17709	0.29262	0.58258	0.77791	14.046
28	5	6	0.02769	0.09780	0.16574	0.27960	0.56912	0.76582	14.0521
29	5	7	0.02729	0.09165	0.15727	0.26993	0.56014	0.75836	14.0546
30	5	8	0.02691	0.08665	0.15066	0.2986	0.5540	0.75035	14.0540
31	5	9	0.02654	0.08250	0.14533	0.25662	0.54972	0.74966	14.0543
32	7	0	0.24414	0.32547	0.52726	0.81546	1.41068	1.75678	16.3169
33	7	1	0.14440	0.18879	0.30364	0.48377	0.84584	1.05928	10.4303
34	7	2	0.104621	0.14786	0.23885	0.38650	0.69851	0.89369	10.3708
35	7	3	0.02700	0.12637	0.21001	0.33930	0.62957	0.81349	11.5409
36	7	4	0.02679	0.11373	0.18871	0.31098	0.5947	0.77866	11.5923
37	7	5	0.02659	0.10186	0.17538	0.29136	0.57201	0.75909	11.6149
38	7	6	0.02640	0.09538	0.16427	0.27795	0.55868	0.74763	11.6250
39	7	7	0.02621	0.08952	0.15592	0.26804	0.54989	0.74075	11.6296
40	7	8	0.02604	0.08475	0.14939	0.26047	0.54398	0.73439	11.6318
41	7	9	0.02587	0.08079	0.14410	0.25454	0.53994	0.73412	11.6329

Suspension System with Bayesian Chain Sampling Plan as Reference Plan. The present development would be valuable addition to the literature and a useful device for quality control practitioners. These tables are useful for both producer and consumer for obtaining good quality products with less inspection cost.

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