CONSTRUCTION AND SELECTION OF GENERALIZED TWO PLAN SYSTEM WITH REPETITIVE DEFERRED SAMPLING PLAN AS REFERENCE PLAN

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Abstract: This paper presents a new procedure and tables for the construction and selection of a Generalized Two Plan system of type (n,cN,cT) with Repetitive Deferred Sampling Plan as reference plan indexed through Acceptable Quality Level (AQL), Limiting Quality Level (LQL), and Indifference Quality Level (IQL). Tables are constructed by considering various combinations of acceptable and limiting quality levels, and illustrations are also provided for ready- made selection of plan parameters.

Keywords: Acceptable Quality Level(AQL), Indifference Quality level(IQL), Limiting Quality Level(LQL), Repetitive Deferred Sampling Plan.

Introduction: Inspection of raw materials, semi finished products, or a finished product is an important part of quality assurance. When inspection is for the purpose of acceptance or rejection of a product, based on the adherence to a standard, the type of inspection procedure employed is usually called acceptance sampling. The primary objective of sampling inspection is to reduce the cost of inspection while at the same time assuring the customer of an adequate level of quality in the items being inspected.

Dodge (1969) proposed a sampling inspection involving normal and tightened inspection plans which are usually referred as a generalized two-plan system. The tightened inspection can be used when the quality of a product deteriorated and normal inspection is used when the quality is found to be good. This system is largely incorporated in MIL-STD-105E (1989) which forms an integrated sampling inspection system guaranteeing the consumer that the outgoing quality will be better than the specified AQL and at the same time assuring the producer that the risk of rejection will be smaller for products of AQL quality or better.

Kuralmani (1992) has designed two-plan switching system involving acceptable and limiting quality levels. The procedure with a pair of plans gives an overall OC curve that generally lies in between the OC curve of the normal and tightened plans in a Two-Plan switching system. Balamurali and Chi-Hyuck Jun (2009) have made contributions to designing of a variables two-plan system by minimizing the average sample number (ASN).

The Repetitive Deferred Sampling plan has been developed by Shankar and Mohapatra(1991) and this plan is essentially an extension of the Multiple Deferred Sampling plan MDS-(c_1, c_2) due to RambertVaerst(1981). In this plan the acceptance or rejection of a lot in deferred state is dependent on the inspection results of the preceding or succeeding lots under Repetitive Group Sampling (RGS) inspection. So, RGS is a particular case of RDS plan. Wortham and Baker(1976) have developed Multiple Deferred State Sampling (MDS) plans and also provided tables for construction of plans. Suresh(1993) has proposed procedures to select Multiple Deferred State Plan of type MDS and MDS-1 indexed through producer and consumer quality levels considering filter and incentive effects.

Lilly Christina (1995) has given the procedure for the selection of RDS plan with given acceptable quality levels and also compared RDS plan with RGS plan with respect to operating ratio(OR) and ASN curve. Suresh and Saminathan (2010) have studied the selection of Repetitive Deferred Sampling Plan through acceptable and limiting quality levels. The operating ratio was first proposed by Peach (1947) for measuring quantitatively the relative discrimination power of sampling plans. Hamaker (1950) has studied the selection of Single Sampling Plan assuming that the quality characteristics follow Poisson model such that the OC curve passes through indifference quality level.

This paper provides a new procedure for selection of Generalized Two-Plan System with RDS plan indexed through certain basic quality levels, which are tailor made for industrial shop floor applications towards quality of a product.

Selection Of Sampling Plan: Conditions for application of a Generalized Two-Plan System

1. The production is steady so that results on current and preceding lots are broadly indicative of the continuing process and submitted lots are expected to be of essentially the same quality
2. Lots are submitted substantially in the order of production.
3. The product comes from a source in which the consumer has confidence.

Operating Procedure for Generalized Two-Plan System Switching rules for generalized Two-plan Systems are:

Normal to Tightened: When normal inspection is in effect, tightened inspection shall be instituted when'(s)' out of 'm' consecutive lots or batches have been rejected on original inspection($s \leq m$).

Tightened to Normal: When tightened inspection is in effect, normal inspection shall be instituted when'd' consecutive lots or batches have been considered acceptable on original inspection.

A number of important measures of performance are to be determined and used in the evaluation of OC function which will be discussed.

P_N = the proportion of lots expected to be accepted under normal inspection.

P_T = the proportion of lots expected to be accepted under tightened inspection.

I_N = the expected proportion of lots inspected on normal inspection.

I_T = the expected proportion of lots inspected on tightened inspection.

Dodge (1959) has provided a performance measure with a composite of function for the probability of acceptance, $Pa(p) = I_N P_N + I_T P_T$ ...(2.1)

The method for deriving various measures of performance for the Generalized Two- Plan System are also studied.

All probabilities can now be evaluated using the condition that the sum of all probabilities equals to one,ie, $I_N + I_T = 1$ ...(2.2)

one can get, $I_N = \dfrac{\mu}{\mu+\tau}$ ...(2.3)

$I_T = \dfrac{\tau}{\mu+\tau}$ ...(2.4)

Where, $$\mu = \frac{1+(1-a)^{s-2}(2a-a^{m-s+2}-1)}{a(1-a^{m-s+2})(1-a)^{s-1}} \quad ...(2.5)$$

= the average number of lots inspected using the normal plan before going to tightened inspection.

and $$\tau = \frac{1-b^d}{(1-b)b^d} \quad ...(2.6)$$

= the average number of lots inspected using the normal plan before going to tightened inspection.

Here, a as P_N and b as P_T, the composite OC and ASN functions are, respectively, obtained as

$$P_a(p) = \frac{\mu P_N + \tau P_T}{\mu+\tau} \quad ...(2.7)$$

Where, P_N = Probability of acceptance under the normal inspection. $P_N = p(d \leq c_N / n,p)$

P_T= Probability of acceptance under the tightened inspection. $P_T = p(d \leq c_T / n,p)$

Note that where μ and τ are the average number of lots inspected using normal inspection before going to tightened inspection and average number of lots inspected using tightened inspection before going to normal inspection respectively.

Conditions for the application of RDS plan

1. Production is steady so that result of past, current and future lots are broadly indicative of a continuing process.
2. Lots are submitted substantially in the order of their production.
3. A fixed sample size, n from each lot is assumed.
4. Inspection is by attributes with quality defined as fraction non-conforming.

Operating Procedure for RDS Plan

1. Draw a random sample of size n from the lot and determine the number of defectives (d) found therein.
2. Accept the lot if $d \leq c_1$.Reject the lot if $d > c_2$.
3. If $c_1 < d < c_2$, accept the lot provided 'i' proceeding or succeeding lots are accepted under RGS inspection plan, otherwise reject the lot.

Here c_1 and c_2 are acceptance number such that $c_1 < c_2$, when i=1 this plan reduces to RGS plan.The operating characteristic function $P_a(p)$ for Repetitive Deferred Sampling Plan is derived by Shankar and Mohapatra (1991) using the Poisson Model as,

$$P_a(p) = \frac{p_a(1-p_c)^i + p_c p_a{}^i}{(1-p_c)^i} \quad ...(2.8)$$

where $$p_a = p[d \leq c_1] = \sum_{r=0}^{c_1} \frac{e^{-x}(x)^r}{r!} \quad ...(2.9)$$

$$p_c = p[c_1 < d < c_2] = \sum_{r=0}^{c_2} \frac{e^{-x}(x)^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x}(x)^r}{r!} \quad ...(2.10)$$

also x=np

Selection Procedure: Designing Of AQL, LQL, α And B

Table 2 is used to design Generalized Two-Plan system of type (n, c_N, c_T) with RDS as reference plan for give p_o, p_1, p_2, α and β. To design a TPRDSS for the given two points on the OC curve (p_1, 1-α) and (p_2, β), first calculate the operating ratio OR = p_2/p_1. Find the value in the Table 2 under the column for the appropriate α and β, which is closed to the desired ratio. The $u_1, u_2, v_1, v_2, c_N, c_T$,s,m,d and i values corresponding to the p_2/p_1 value found in Table 2 can be used in Table 1 to obtain the value of np equal to np_1 where $P_a(p) = 1-α$. The sample size is obtained by dividing np_1 by p_1.

For example, let $p_1 = 0.0194$, α=0.05,p_2=0.0633 and β=0.10. For given p_1 and p_2, OR = p_2/p_1=0.0633/0.0194 = 3.26288. From table 2, values of OR (α=0.05, β=0.10) which is nearest to the desired ratio 3.2617.

Corresponding to this one obtains $i=1, u_1=0, u_2=4, v_1=1, v_2=3, c_N=3, c_T=0, s=1, m=2, d=2$ and $np_1=1.261$. The sample size obtained as $n=np_1/p_1=1.261/0.0194=65$.Thus the desired system is a TPRDSS(65,0,4,1,3,3,0,1,2,2).

4. Construction of Tables: The expression for probability of acceptance under the assumption of Poisson model, for the quality characteristics of the composite OC function of TPRDSS is given by

$$P_a(p) = \frac{\mu P_N + \tau P_T}{\mu + \tau} \quad \ldots(4.1)$$

$$P_N = \frac{p_a(1-p_c)^i + p_c p_a{}^i}{(1-p_c)^i} \quad \ldots(4.2)$$

where $p_a = p[d \le u_1] = \sum_{r=0}^{u_1} \frac{e^{-x}(x)^r}{r!}$

$$p_c = p[u_1 < d < u_2] = \sum_{r=0}^{u_2} \frac{e^{-x}(x)^r}{r!} - \sum_{r=0}^{u_1} \frac{e^{-x}(x)^r}{r!}$$

$$P_T = \frac{p_a(1-p_c)^i + p_c p_a{}^i}{(1-p_c)^i} \quad \ldots(4.3)$$

$p_a = p[d \le v_1] = \sum_{r=0}^{v_1} \frac{e^{-x}(x)^r}{r!}$ $\quad p_c = p[v_1 < d < v_2] = \sum_{r=0}^{v_2} \frac{e^{-x}(x)^r}{r!} - \sum_{r=0}^{v_1} \frac{e^{-x}(x)^r}{r!}$ where $x=np$

For various values given in Tables the different quality levels, one can solve the equation (4.2) and (4.3), by substituting in equation (4.1), using iterative technique for selected combinations of u_1 , u_2, v_1, v_2, s, m, d, c_N, c_T and i, accordingly the operating ratio values can be calculated. Using iterative solution for x=np are obtained and provided respectively.

5. Conclusion:Generalized Two-Plan System and Repetitive Deferred Sampling Plan have wide potential applicability in industries to ensure higher standard of quality attainment and increased customer satisfaction. In this paper, the concept of Generalized Two-Plan System with RDS plan as a reference plan is studied to obtain a new plan designated as Two-Plan Repetitive Deferred Sampling System (TPRDSS), which is a disposal of a lot on the basis of normal and tightened plan. Poisson unity values have been tabulated for a wider range of plan parameters. The present development would be a valuable addition to the literature and useful device to the quality practitioners. This concept of this article is useful for assistance to quality control engineers and plan designers in the development of further plans towards the quality improvement of a product.

Table 1 Values of np for Generalized TPS (n,c_N,c_T) plan with RDS plan when i=1

u_1	u_2	v_1	v_2	c_N	c_T	S	m	d	Probability of Acceptance						
									0.99	0.95	0.90	0.50	0.10	0.05	0.01
0	0	0	0	1	0	1	1	5	0.010	0.051	0.105	0.693	2.302	2.995	4.530
2	1	0	1	1	0	1	1	4	0.130	0.275	0.463	0.987	2.306	2.996	4.605
0	2	0	2	1	0	1	1	3	0.372	0.611	0.756	1.213	2.350	3.007	4.651
0	4	0	3	1	0	1	1	2	0.669	0.972	1.141	1.603	2.469	3.044	4.607
0	5	0	4	1	0	2	2	5	0.995	1.343	1.528	1.973	2.680	3.139	4.611
0	6	0	5	2	0	2	2	4	1.337	1.719	1.914	2.335	2.962	3.545	4.624
0	7	0	6	1	0	2	2	2	1.688	2.096	2.301	2.715	3.284	3.578	4.661
1	2	0	1	1	0	1	1	5	0.138	0.307	0.437	1.055	2.527	3.141	4.650
1	3	0	2	3	0	1	2	4	0.413	0.706	0.939	1.363	2.582	3.159	4.698
1	4	0	3	1	0	1	1	3	0.761	1.103	1.207	1.661	2.706	3.208	4.653
1	5	0	4	1	0	1	1	2	1.143	1.564	1.565	1.999	2.912	3.319	4.660
3	6	2	3	2	0	1	1	2	0.815	1.990	1.690	2.835	5.185	6.345	7.979
4	5	3	4	1	0	1	2	1	1.544	2.412	2.389	3.925	6.621	4.763	9.727
2	3	1	2	1	0	2	2	5	0.426	0.769	1.006	1.955	4.079	4.859	6.671
0	4	1	3	3	0	1	2	2	0.802	1.261	1.538	2.358	4.113	4.869	6.705
2	5	1	4	1	0	1	1	4	1.226	1.759	1.937	2.650	4.191	4.894	6.672
2	6	1	5	1	0	1	1	3	1.671	2.249	2.385	2.993	4.335	4.951	6.675
2	7	1	6	1	0	1	2	2	2.123	2.726	2.823	3.363	4.551	5.062	6.681
3	4	2	3	1	0	2	2	4	0.813	1.317	1.648	3.159	5.495	6.398	8.433
3	5	2	4	2	0	2	5	3	1.258	1.867	2.230	3.352	5.523	6.406	8.434
3	6	2	5	1	0	1	2	5	1.736	2.413	2.687	3.633	5.581	6.423	8.520
3	7	2	6	1	0	1	1	4	2.227	2.944	3.172	3.971	5.690	6.460	8.436
3	8	2	7	1	0	2	2	4	2.721	3.460	3.647	4.342	5.863	6.534	8.440
4	5	3	4	1	0	2	2	5	1.269	1.921	2.335	4.161	6.844	7.848	10.070
1	6	3	5	3	0	2	2	1	1.765	2.511	2.871	4.346	6.868	7.855	10.095
4	7	3	6	2	0	2	2	3	2.284	3.092	3.446	4.617	6.916	7.869	10.071
4	8	3	7	1	0	1	1	3	2.811	3.655	3.969	4.948	7.004	7.896	10.072
4	9	3	8	1	0	2	2	2	3.339	4.201	4.466	5.317	7.147	7.950	10.349
5	6	4	5	3	0	2	2	5	1.775	2.564	3.054	5.162	8.150	9.243	11.627

Table 2 Operating Ratio Values for TPRDSS (n, c_N, c_T) for i=1

u_1	u_2	v_1	v_2	c_N	c_T	s	m	d	p_2/p_1 for α = 0.05			p_2/p_1 for α = 0.01		
									α = 0.05 β = 0.10	α = 0.05 β = 0.05	α = 0.05 β = 0.01	α = 0.01 β = 0.10	α = 0.01 β = 0.05	α = 0.01 β = 0.01
0	0	0	0	1	0	1	1	5	45.1373	13.5882	88.8235	230.2000	69.3000	453.0000
2	1	0	1	1	0	1	1	4	8.3855	3.5891	16.7455	17.7385	7.5923	35.4231
0	2	0	2	1	0	1	1	3	3.8462	1.9853	7.6121	6.3172	3.2608	12.5027
0	4	0	3	1	0	1	1	2	2.5401	1.6492	4.7397	3.6906	2.3961	6.8864
0	5	0	4	1	0	2	2	5	1.9955	1.4691	3.4334	2.6935	1.9829	4.6342
0	6	0	5	2	0	2	2	4	1.7231	1.3583	2.6899	2.2154	1.7464	3.4585
0	7	0	6	1	0	2	2	2	1.5668	1.2953	2.2238	1.9455	1.6084	2.7613
1	2	0	1	1	0	1	1	5	8.2313	3.4365	15.1466	18.3116	7.6449	33.6957
1	3	0	2	3	0	1	2	4	3.6572	1.9306	6.6544	6.2518	3.3002	11.3753
1	4	0	3	1	0	1	1	3	2.4533	1.5059	4.2185	3.5558	2.1827	6.1143
1	5	0	4	1	0	1	1	2	1.8619	1.2781	2.9795	2.5477	1.7489	4.0770
3	6	2	3	2	0	1	1	2	2.6055	1.4246	4.0095	6.3620	3.4785	9.7902
4	5	3	4	1	0	1	2	1	2.7450	1.6273	4.0328	4.2882	2.5421	6.2999
2	3	1	2	1	0	2	2	5	5.3043	2.5423	8.6749	9.5751	4.5892	15.6596
0	4	1	3	3	0	1	2	2	3.2617	1.8699	5.3172	5.1284	2.9401	8.3603
2	5	1	4	1	0	1	1	4	2.3826	1.5065	3.7931	3.4184	2.1615	5.4421
2	6	1	5	1	0	1	1	3	1.9275	1.3308	2.9680	2.5943	1.7911	3.9946
2	7	1	6	1	0	1	2	2	1.6695	1.2337	2.4508	2.1437	1.5841	3.1470
3	4	2	3	1	0	2	2	4	4.1724	2.3986	6.4032	6.7589	3.8856	10.3727
3	5	2	4	2	0	2	5	3	2.9582	1.7954	4.5174	4.3903	2.6645	6.7043
3	6	2	5	1	0	1	2	5	2.3129	1.5056	3.5309	3.2149	2.0927	4.9078
3	7	2	6	1	0	1	1	4	1.9327	1.3488	2.8655	2.5550	1.7831	3.7881
3	8	2	7	1	0	2	2	4	1.6945	1.2549	2.4393	2.1547	1.5957	3.1018
4	5	3	4	1	0	2	2	5	3.5627	2.1661	5.2421	5.3932	3.2790	7.9354
1	6	3	5	3	0	2	2	1	2.7352	1.7308	4.0203	3.8912	2.4623	5.7195
4	7	3	6	2	0	2	2	3	2.2367	1.4932	3.2571	3.0280	2.0215	4.4094
4	8	3	7	1	0	1	1	3	1.9163	1.3538	2.7557	2.4916	1.7602	3.5831
4	9	3	8	1	0	2	2	2	1.7013	1.2657	2.4635	2.1405	1.5924	3.0994

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