SOME FIXED POINT RESULTS OF PRESIC TYPE IN B-METRIC SPACES

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Abstract : A generalized common fixed point theorem of Presic type for two mappings and in a b-metric space and also fixed point theorems for Presic type contractions and quasi contractions in tvs cone b-metric space are proved. Our results extends and generalizes many well known results.

Keywords : Fixed points, rectangular metric space, rectangular b-metric space.

Introduction : Since the introduction of Banach contraction principle in 1922, because of its wide applications, the study of existence and uniqueness of fixed points and common fixed points has become a subject of great interest. Many authors proved the Banach contraction Principle in various generalized metric spaces to include among others partial metric space, cone metric space, tvs cone metric space, cone b-metric space, tvs cone b-metric space etc. For some work on these spaces see [12,13,14,15,16,17,18,19,20,23,24,25] and the references therein. Generalizing the concept of metric space Bakhtin[3] introduced the concept of b-metric space which is not necessarily Hausdorff and proved the Banach contraction principle in the setting of a b-metric space. Since then, several papers have dealt with fixed point theory or the variational principle for single-valued and multi-valued operators in b-metric spaces (see [4,5,6,8,9,10,11] and the references therein).

Note that spaces with non Hausdorff topology plays an importnat role in Tarskian approach to programming language semantics used in computer science (For some details see [20]).

In this paper we have proved a common fixed point theorem of Presic type for two mappings in a b-metric space. We have also proved the existence of fixed points of Presic type contractions and quasi contraction in the setting of a tvs cone b-metric space. Our results extends and generalises may well known results.

Preliminaries :

Definition 2.1 [3] : Let X be a nonempty set and the mapping $d : X \times X \to [0,\infty)$ satisfies:

(bM-1) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;

(bM-2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(bM-3) there exist a real number $s \geq 1$ such that $d(x, y) \leq s[d(x,z) + d(z, y)]$ for all $x, y, z \in X$.

Then d is called a b-metric on X and (X,d) is called a b-metric space (in short bMS) with coefficient s .

Note that every metric space is a b-metric space. However the converse is not necessarily true.

For any $x \in X$, the open ball with centre x and radius $r > 0$ is given by $B_r(x) = \{y \in X : d(x, y) < r\}$. The open balls in bMS are not necessarily open.

Let U be the collection of all subsets A of X satisfying the condition that for each $x \in A$ there exist $r > 0$ such that $B_r(x) \subseteq A$. Then U defines a topology for the bMS (X,d) , which is not necessarily Hausdorff.

Definition 2.2 : Let (X,d) be a b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then

(a) The sequence $\{x_n\}$ is said to be convergent in (X,d) and converges to x , if for every $\varepsilon > 0$ there exists $n_0 \in \Box$ such that $d(x_n, x) < \varepsilon$ for all $n > n_0$ and this fact is represented by $\lim_{n\to\infty} x_n = x$ or $x_n \to x\$as\$n \to \infty$.

(b) The sequence $\{x_n\}$ is said to be Cauchy sequence in (X,d) if for every $\varepsilon > 0$ there exists $n_0 \in \Box$ such that $d(x_n, x_{n+p}) < \varepsilon$ for all $n > n_0, p > 0$ or equivalently, if $\lim_{n\to\infty} d(x_n, x_{n+p}) = 0$ for all $p > 0$.

(c) (X,d) is said to be a complete b-metric space if every Cauchy sequence in X converges to some $x \in X$.

Definition 2.3 : Let (X,d) be a b-metric space. The mapping $T : X \to X$ is called a quasi contraction if there exists $0 \leq q < 1$ such that

$$d(Tx,Ty) \leq q.Max\{d(x, y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$$

The following result is from [2].

Theorem 2.4 : Let (X,d) be a complete b-metric space with coefficient $s \geq 1$ and the mapping $T : X \to X$ be a quasi contraction with $q \leq \frac{1}{s^2 + s}$.

Then T has a unique fixed point.

Let E be a topological vector space and P a solid cone in E. We define a partial ordering $\prec$ with repect to P by $x \prec y$ if and only if $y - x \in P$. We shall write $x < y$ if $x \prec y$ and $x \neq y$, and $x \ll y$ if $y - x \in intP$, where $intP$ denote the interior of P.

The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$\theta \prec x \prec y$ implies $\| x \| \leq K \| y \|$.

Definition 2.5 : Let X be a non empty set. Suppose that the mapping $d : X \times X \to E$ satisfies :

(tvs-CbM1) $\theta \prec d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$.

(tvs-CbM2) $d(x,y)=d(y,x)$ for all $x,y\in X$

(tvs-CbM3) $d(x, y) \prec s[d(x,z) + d(z, y)]$ for all $x, y, z \in X$ and some $s \geq 1$.

Then d is called a $tvs-cone\text{-}b-metric$ on X and (X, d) is called a $tvs-cone\text{-}b-metric$ space (in short $tvs-CbMS$) with coefficient s.

Remark 2.6 : The concept of a tvs cone metric space is more general than that of a cone metric space because every cone metric space is a tvs cone metric space with a normal cone, however tvs cone metric spaces with non normal cones are not necessarily a cone metric space.

Remark 2.7 : Note that any tvs-cone metric space is $tvs-CbMS$ with coefficient s=1 but the converse is not true in general.

Recall that (see [26]) if V is an absolutely convex and absorbing subset of a topological vector space E, then its Minkowski functional is defined by $E \ni x \mapsto q_V(x) = inf\{\lambda > 0 : x \in \lambda V\}$. It is a seminorm on E (i.e $q_V(x+y) \leq q_V x + q_V y$ for all $x, y \in E$ and $q_V(\lambda x) = |\lambda| q_V(x)$ for $x \in E$, (λ scalar) and $V \subset W$ implies that $q_W(x) \leq q_V(x)$ for $x \in E$. If V is an absolutely convex neighbourhood of θ in E then q_V is continuous and

$\{x \in E : q_V(S) < 1\} = intV \subset V \subset \overline{V}$

$= \{x \in E : q_V(S) \leq 1\}$. If (E, P) is an ordered topological vector space and $e \in intP$ then $[-e, e] = (P - e) \bigcap (e - P) = \{z \in E : -e \prec z \prec e\}$ is an absolutely convex neighbourhood of θ and its Minkowski functional is denoted by q_e (For some details see [20]). Moreover q_e is an increasing function over P for if $\theta \prec x_1 \prec x_2$ then $\{\lambda : x_1 \in \lambda[-e, e]\} \supset \{\lambda : x_2 \in \lambda[-e, e]\}$ and so $q_e(x_1) \leq q_e(x_2)$.

The following theorems appear in [24].

Theorem 2.8 : Let (X, d) be a complete tvs cone b-metric space with coefficient $s \geq 1$ and $e \in intP$. Let q_e be the Minkowski functional of $[-e, e]$. Then $d_b = q_e \circ d$ is a (real valued) b-metric on X.

Moreover, for arbitrary $x, y, x_1, y_1 \in X$, $d(x, y) \prec d(x_1, y_1)$ implies $d_b(x, y) \leq d_b(x_1, y_1)$.

Theorem 2.9 : Let $(X, d), e, q_e$ and d_b be as in the above theorem, let $x \in X$ and $\{x_n\}$ be a sequence in X. Then we have the following :

1. x_n d-converges to x if and only if x_n d_b-converges to x.
2. x_n is a d-Cauchy sequence if and only if it is a d_b-Cauchy sequence.
3. (X, d) is complete if and only if (X, d_b) is d_b-complete.

Definition 2.10 : Let (X, d) be a metric space, k a positive integer, $T : X^k \to X$ and $f : X \to X$ be mappings.

(a) An element $x \in X$ is said to be a coincidence point of f and T if and only if $f(x) = T(x, x, ..., x)$. If $x = f(x) = T(x, x, ..., x)$ then we say that x is a common fixed point of f and T. If $w = f(x) = T(x, x, ..., x)$ then w is called a point of coincidence of f and T.

(b) Mappings f and T are said to be commuting if and only if $f(T(x, x, ...x)) = T(fx, fx, ... fx)$ for all $x \in X$.

(c) Mappings f and T are said to be weakly compatible if and only if they commute at their coincidence points.

Remark 2.11 : For $k = 1$ the above definitions reduces to the usual definition of commuting and weakly compatible mappings in a metric space.

The set of coincidence points of f and T is denoted by $C(f, T)$.

Main Results :

Now we present our main results as follows :

Theorem 3.1 : Let (X, d) be a b-metric space with coefficient $s \geq 1$. For any positive integer k, let $T : X^k \to X$ and $f : X \to X$ be mappings satisfying the following conditions:

(3.1) $T(X^k) \subseteq f(X)$

(3.2) $d(T(x_1, x_2, \ldots x_k), T(x_2, x_3, \ldots x_{k+1}))$
$\leq \lambda Max\{d(fx_1, fx_2), d(fx_2, fx_3), \ldots (fx_k, fx_{k+1})\}$

where $x_1, x_2, \ldots x_{k+1}$ are arbitrary elements in X and $\lambda \in (0, \frac{1}{s^{k+1}})$ and

(3.3) f(X) is complete

Then f and T has a coincidence point, i.e. $C(f,T) \neq \varnothing$. Further if f and T are weakly compatible, then f and T has a unique common fixed point. Moreover if $x_1, x_2, \ldots, x_k$ are arbitrary points in X and for $n \in N$,

$y_{n+k} = f(x_{n+k}) = T(x_n, x_{n+1}, \ldots x_{n+k-1})$, $n = 1, 2, \ldots$,

then the sequence $< y_n >$ is convergent and

$lim\ y_n = f(lim\ y_n) = T(lim\ y_n, lim\ y_n, \ldots, lim\ y_n)$

Proof : By (3.1)and (3.4) we define sequence $< y_n >$ in $f(X)$ as

$y_n = fx_n$ for $n = 1, 2, \ldots k$ and

$y_{n+k} = f(x_{n+k}) = T(x_n, x_{n+1}, \ldots x_{n+k-1})$, $n = 1, 2, \ldots$

.Let $\alpha_n = d(y_n, y_{n+1})$. By the method of mathematical induction we will now prove that

$\alpha_n \leq R.\theta^n$ ... (3.4)

for all n, where $R = \text{Max}\{\frac{\alpha}{\theta}, \frac{\alpha^2}{\theta^2}, \cdots \frac{\alpha^k}{\theta^k}\}$, $\theta = \lambda^{\frac{1}{k}}$.

Clearly by the definition of R, (3.4) is true for $n = 1, 2, \ldots k$. Let the k inequalities

$\alpha_n \leq R\theta^n, \alpha_{n+1} \leq R\theta^{n+1}, \ldots\ldots, \alpha_{n+k-1} \leq R\theta^{n+k-1}$ be the induction hypothesis.

Then we have

$\alpha_{n+k} = d(y_{n+k}, y_{n+k+1})$
$= d(T(x_n, x_{n+1}, \ldots x_{n+k-1}), T(x_{n+1}, x_{n+2}, \ldots x_{n+k}))$
$\leq \lambda Max\{d(fx_n, fx_{n+1}), d(fx_{n+1}, fx_{n+2}), \ldots (fx_{n+k-1}, fx_n$
$= \lambda Max\{\alpha_n, \alpha_{n+1}, \ldots \alpha_{n+k-1}\}$
$\leq \lambda Max\{R\theta^n, R\theta^{n+1}, \ldots\ldots\ldots, R.\theta^{n+k-1}\}$
$\leq \lambda Max\{R\theta^n, R\theta^n, \ldots\ldots\ldots, R\theta^n)\}$
$\leq \lambda R\theta^n = R\theta^{n+k}$.

Thus inductive proof of (3.5) is complete. Now for $n, p \in N$, we have

$d(y_n, y_{n+p}) \leq$
$sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + \cdots s^p d(y_{n+p-1}, y_{n+p})$
$\leq sR\theta^n + s^2 R\theta^{n+1} + \cdots s^p R\theta^{n+p-1}$
$\leq sR\theta^n (1 + s\theta + s^2\theta^2 + \cdots)$
$= \frac{sR\theta^n}{1 - s\theta} \to 0$ as $n \to \infty$ and all $p \geq 1$.

Hence sequence $< y_n >$ is a Cauchy sequence in $f(X)$ and since $f(X)$ is complete, there exists $v, u \in X$ such that $lim_{n\to\infty} y_n = v = f(u)$. Now we have

$d(fu, T(u, u, \ldots u))$
$\leq d(fu, y_{n+k}) + d(y_{n+k}, T(u, u, \ldots u))$
$= d(fu, y_{n+k}) +$
$d(T(x_n, x_{n+1}, \ldots x_{n+k-1}), T(u, u, \ldots u))$
$\leq d(fu, y_{n+k}) + sd(T(u, \cdots u), T(u, \cdots x_n))$
$+ s^2 d(T(u, \cdots x_n), T(u, \cdots x_n, x_{n+1}))$
$+ \cdots s^k d(T(u, x_n, \ldots x_{n+k-2}), T(x_n, x_{n+1} \ldots x_{n+k-1})$
$\leq d(fu, y_{n+k}) + s\lambda Max\{d(fu, fu), \cdots d(fu, fx_n)\}$
$+ s^2 \lambda Max\{d(fu, fu), \cdots d(fu, fx_n), d(fx_n, fx_{n+1})\}$
$+ \cdots s^k \lambda Max\{d(fu, fx_n), d(fx_n, fx_{n+1}),$
$\ldots d(fx_{n+k-2}, fx_{n+k-1})\}$
$= d(fu, y_{n+k}) + s\lambda Max\{0, \cdots d(fu, fx_n)\}$
$+ s^2 \lambda Max\{0, \cdots d(fu, fx_n), d(fx_n, fx_{n+1})\}$
$+ \cdots s^k \lambda Max\{d(fu, fx_n), d(fx_n, fx_{n+1}),$
$\ldots d(fx_{n+k-2}, fx_{n+k-1})\} \to 0$ as $n \to \infty$.

Thus $fu = T(u, u, u, \ldots. u)$, i.e. $C(f,T) \neq \varnothing$. If f and T are weakly compatible then

$f(T(u, u, \ldots u) = T(fu, fu, fu \ldots fu)$.

By (3.2) we have

$d(ffu, fu) = d(fT(u, u, \ldots u), T(u, u, \ldots u))$
$= d(T(fu, fu, fu, \ldots. fu), T(u, u, ..u))$
$\leq sd(T(fu, \cdots fu), T(fu, \cdots fu, u))$
$+ s^2 d(T(fu, \cdots fu, u), T(fu, fu, \ldots, u, u))$
$+ \ldots.. + s^k d(T(fu, u, \cdots u), T(u, \cdots u))$
$\leq s\lambda Max(d(ffu, ffu), \cdots d(ffu, ffu), d(ffu, fu))$
$+ s^2 \lambda Max(d(ffu, ffu), \ldots d(ffu, fu), d(fu, fu))$
$+ \ldots s^k \lambda Max(d(ffu, fu), \ldots. d(fu, fu), d(fu, fu))$
$= s\lambda \frac{s^k - 1}{s - 1} d(ffu, fu)$.

Therefore $d(ffu, fu) = 0$ and so $ffu = fu$. Hence we have,

$fu = ffu = f(T(u,u,...u)) = T(fu, fu, fu....fu)$,

i.e. fu is a common fixed point of f and T, and

$lim\ y_n = f(lim\ y_n) = T(lim\ y_n, lim\ y_n, ...lim\ y_n)$.

Now suppose x, y be two fixed points of f and T. Then

$$\begin{aligned} d(x,y) &= d(T(x,\cdots x),T(y,\cdots y)) \\ &\le sd(T(x,...x),T(x,\cdots x,y)) \\ &+ s^2 d(T(x,\cdots x,y),T(x,\cdots x,y,y)) \\ &+ ... + d(T(x,y,\cdots y),T(y,\cdots y)) \\ &\le s\lambda Max\{d(fx,fx),d(fx,fx),.....,d(fx,fy)\} \\ &+ s^2\lambda\phi\{d(fx,fx),d(fx,fx),.....d(fx,fy),d(fy,fy)\} \\ &+ ... + \lambda\phi\{d(fx,fy),d(fy,fy),...d(fy,fy)\} \\ &= s\lambda\frac{s^k-1}{s-1}d(x,y). \end{aligned}$$

Thus $d(x,y) \le 0$ and so the common fixed point is unique.

Example 3.2 : Let $X = [0,2]$ and $d : X \times X \to R$ such that $d(x,y) = |x-y|^2$. Then, d is a b-metric on X. Indeed, $d(x,y) \le 4[d(x,z) + d(z,y)]$. However $d(0,2) = 4 > 2 = d(0,1) + d(1,2)$ and so d is not a metric on X.

Let $T : X^2 \to X$ and $f : X \to X$ be defined as follows:

$$Tx = \begin{cases} \frac{x^2+y^2}{32} + \frac{15}{16} & if \quad (x,y) \in [0,1]\times[0,1] \\ \frac{x+y}{32} + \frac{15}{16} & if \quad (x,y) \in [1,2]\times[1,2] \\ \frac{x^2+y}{32} + \frac{15}{16} & if \quad (x,y) \in [0,1]\times[1,2] \\ \frac{x+y^2}{32} + \frac{15}{16} & if \quad (x,y) \in [1,2]\times[0,1] \end{cases}$$

$$fx = \begin{cases} x^2 & if \quad x \in [0,1] \\ x & if \quad x \in [1,2] \end{cases}$$

We will prove that T and f satisfies condition (3.2). \\

Case 1. $x, y, z \in [0,1]$

$$\begin{aligned} d(T(x,y),T(y,z)) &= |T(x,y) - T(y,z)|^2 \\ &= |\frac{x^2 - z^2}{32}|^2 = |\frac{x^2 - y^2 + y^2 - z^2}{32}|^2 \\ &\le 4(|\frac{x^2-y^2}{32}|^2 + |\frac{y^2-z^2}{32}|^2) \\ &= \frac{1}{128}\frac{1}{2}[|x^2-y^2|^2 + |y^2-z^2|^2] \\ &\le \frac{1}{128}Max\{d(fx,fy),d(fy,fz)\}. \end{aligned}$$

Case 2. $x, y \in [0,1]$ and $z \in [1,2]$

$$\begin{aligned} d(T(x,y),T(y,z)) &= |\frac{x^2+y^2}{32} - \frac{y^2+z}{32}|^2 \\ &\le 4(|\frac{x^2-y^2}{32}|^2 + |\frac{y^2-z}{32}|^2 \\ &\le \frac{1}{128}Max\{d(fx,fy),d(fy,fz)\}. \end{aligned}$$

Case 3. $x \in [0,1]$ and $y, z \in [1,2]$

$$\begin{aligned} d(T(x,y),T(y,z)) &= |\frac{x^2+y}{32} - \frac{y+z}{32}|^2 \\ &= |\frac{x^2-z}{32}|^2 \\ &\le 4(|\frac{x^2-y}{32}|^2 + |\frac{y-z}{32}|^2) \\ &\le \frac{1}{128}Max\{d(fx,fy),d(fy,fz)\} \end{aligned}$$

Case 4. $x, y, z \in [1,2]$

$$\begin{aligned} d(T(x,y),T(y,z)) &= |\frac{x+y}{32} - \frac{y+z}{32}|^2 \\ &\le (4(|\frac{x-y}{32}|^2 + |\frac{y-z}{32}|^2) \\ &\le \frac{1}{128}Max\{(fx,fy),d(fy,fz)\}. \end{aligned}$$

Similarly in all other cases

$$d(T(x,y),T(y,z)) \le \frac{1}{128}Max\{(fx,fy),d(fy,fz)\}$$

.Thus, f and T satisfy condition (3.2)

With $\lambda = \frac{1}{128}$. Clearly $C(f, T) = \{1\}$, f and T commute at 1. Finally, 1 is the unique common fixed point of f and T.

Theorem 3.3 : Let (X,d) be a tvs cone b-metric space with coefficient $s \ge 1$ and solid cone P contained in a topological vector space E. For any positive integer k, let $T : X^k \to X$ and $f : X \to X$ be mappings satisfying the following.

$T(X^k) \subseteq f(X)$

$d(T(x_1,x_2,...x_k),T(x_2,x_3,...x_{k+1})) \prec \lambda w(x)$

where $x_1,x_2,...x_{k+1}$ are arbitrary elements in X,

$\lambda \in (0,\frac{1}{s^{k+1}})$ and

$w(x) \in \{d(fx_1,fx_2),d(fx_2,fx_3),...d(fx_k,fx_{k+1})\}$

$f(X)$ *is complete*

Then f and T has a coincidence point, i.e. $C(f,T) \neq \varnothing$. Further if f and T are weakly compatible, then f and T has a unique common fixed point. Moreover if $x_1,x_2,...,x_k$ are arbitrary points in X and for $n \in N$,

$y_{n+k} = f(x_{n+k}) = T(x_n,x_{n+1},...x_{n+k-1})$, $n = 1,2,...$,

then then the sequence $< y_n >$ is convergent and

$\lim y_n = f(\lim y_n) = T(\lim y_n, \lim y_n,...,\lim y_n)$.

Proof : The condition that

$d(T(x_1,x_2,...x_k),T(x_2,x_3,...x_{k+1})) \prec \lambda w(x)$ for some

$w(x) \in \{d(fx_1,fx_2),d(fx_2,fx_3),...(fx_k,fx_{k+1})\}$

implies by Theorem 2.2 that

$d_b(T(x_1,x_2,...x_k),T(x_2,x_3,...x_{k+1}))$

$\leq Max\{d_b(fx_1,fx_2),d_b(fx_2,fx_3),...d_b(fx_k,fx_{k+1})\}$.

Theorem 2.3 implies that $(F(X),d_b)$ is complete.

Thus all conditions of Theorem 3.1 is satisfied and it follows that T and f has a unique common fixed point.

Using the same technique as in the last proof and using Theorems 2.1, 2.1 and 2.3, we have the following :

Theorem 3.4 : Let (X,d) be a complete tvs cone b-metric space with coefficient $s \geq 1$ and solid cone P contained in a topological vector space E. Let $T: X \to X$ be such that

$d(Tx,Ty) \prec q.u(x,y), \quad q \in [0,\frac{1}{s^2+s})$

where $u(x,y) \in \{d(x,y),d(x,Tx),d(y,Ty),$ $d(x,Ty),d(y,Tx)\}$. Then T has a unique fixed point.

Remark 3.5 : Theorems 3.1 and 3.3 are proper extension and generalisation of the corresponding results of [7,21,22,23,24].

Remark 3.6 : Theorem 3.4 is a proper generalisation of the corresponding results of [15,16,25].

Open Problems : In Theorem 3.1 can we extent the range of λ to the case $\frac{1}{s^{k+1}} < \lambda < 1$$.

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