AN APPLICATION OF FUZZY GRAPH IN TRAFFIC LIGHT CONTROL

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Abstract : Let G= (V_F,E_F) be a simple connected undirected fuzzy graph where V_F is a fuzzy set of vertices where each vertices has membership value σ and E_F is a fuzzy set of edges where each edge has a membership value μ. Vertex coloring is a function which assigns colors to the vertices so that adjacent vertices will receive different colors. As the number of vehicles increases rapidly, traffic congestion has become a serious problem in a city. The traffic light setting problem is to investigate how to set the given traffic lights such that the total waiting time of vehicles on the road is minimized. In this paper, we use a fuzzy graph model to represent the traffic network. We apply the vertex coloring function of a fuzzy graph (crisp mode) to the traffic light problem. The function is based on α cut of graph Gα=(Vα,Eα),the α cuts of fuzzy graph G. The traffic light problem is analyzed following this approach.

Keywords : Fuzzy set, Fuzzy graph, α cut, Traffic light problem

Introduction : Graph coloring is one of the most important concepts in graph theory and is used in many real time applications like Job scheduling [8] , Aircraft scheduling [8], computer network security[9], Map coloring and GSM mobile phone networks[10] Automatic channel allocation for small wireless local area networks[11].The proper coloring of a graph is the coloring of the vertices with minimal number of colors such that no two adjacent vertices should have the same color. The minimum number of colors required for proper coloring is called as the chromatic number of the graph and the graph is called properly colored.

Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. Precision assumes that parameters of a model represent exactly either our perception of the phenomenon modeled or the features of the real system that has been modeled. Now, as the complexity of a system increases our ability to make precise and yet significant statements about its behavior diminishes

Until a threshold is reached beyond which precision and significance becomes almost mutually exclusive characteristics. Moreover in constructing a model, we always attempt to maximize its usefulness. This aim is closely connected with the relationship among three key characteristics of every system model: complexity, credibility and uncertainty. Uncertainty has a pivotal role in any efforts to maximize the usefulness of system models. All traditional logic habitually assumes that precise symbols are being employed. One of the meanings attributed to the term 'uncertainty' is "vagueness".

A mathematical frame work to describe this phenomenon was suggested by Lotfi.A.Zadeh in his seminal paper entitled "Fuzzy Sets" [12]. We know that graphs are simply model of relation. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In many real world problems, we get partial information about that problem. So there is vagueness in the description of the objects or in its relationships or in both. To describe this type of relation, we need to design fuzzy graph model. Fuzzy graph coloring is one of the most important problems of fuzzy graph theory. It can be used to solve many real world problems. Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph.

The first definition of a fuzzy graph was by Kaufmann in 1973, based on Zadeh's fuzzy relations. But it was Azriel Rosenfeld [2] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. During the same time, R.T.Yehand S.Y. Bang has also introduced various connectedness concepts in fuzzy graphs.

Preliminaries : In this section we define some basic concept of fuzzy set and fuzzy graph used in this paper.

Definition 2.1:α cut set of fuzzy set A is defined as A_α is made up of members whose membership is not less than α .A_α ={xϵX | $\mu_A(x) \geq \alpha$.}. α cut set of fuzzy set is crisp set.

Definition 2.2: Blue et al. have given five types of graph fuzziness .Fuzzy graph is a graph G_F satisfying one of the following types of fuzziness (G_F of the ith type) or any of its combination:

(i) G_{F_1} = { G_1 , G_2 , G_3 , . . . , G_F } where fuzziness is on each graph G_i.

(ii) G_{F_2} = { V, E_F } where the edge set is fuzzy.

iii) G_{F_3} = { V, E(t_F , h_F) } where both the vertex and edge sets are crisp, but the edges have fuzzy heads h(e_i) and fuzzy tails t(e_i.)

iv) G_{F_4} = { V_F , E } where the vertex set is fuzzy.

(v) G_{F_5} = {V, E(w_F) } where both the vertex and edge sets are crisp but the edges have fuzzy weights.

In this paper, we use a fuzzy graph G which is a combination of G_{F_2} and G_{F_4} . So fuzzy graph G= $G_{F_2} \cup G_{F_4}$. We can define this fuzzy graph using their membership value of vertices and edges. Let V be a finite nonempty set. The triple G= (V, σ, μ) is called a fuzzy graph on V where μ and σ are fuzzy sets on V and E (V × V), respectively, such that $\mu(\{u, v\}) \le \min\{\sigma(u), \sigma(v)\}$ for all u, v∈ V.

Note that a fuzzy graph is a generalization of crisp graph in which

$\mu(v) = 1$ for all $v \in V$ and $\rho(i, j) = 1$ if $(i, j) \in E$

$=0$ otherwise

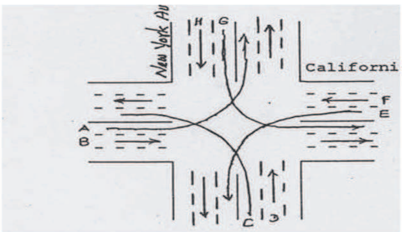
so all the crisp graph are fuzzy graph but all fuzzy graph are not crisp graph.

Definition 2.3:The α cut of fuzzy graph defined as $G_\alpha = (V_\alpha, E_\alpha)$ where $V_\alpha = \{v \in V | \sigma \ge \alpha\}$ and $E_\alpha = \{e \in E | \mu \ge \alpha\}$.

The Traffic Lights Problem As A Crisp Graph Coloring Problem:

Problem:

Here we consider a very common traffic flow problem shown in Figure.1. We assumed that intersection of two divided zones where all left and right turns are permitted. The arrows indicate the traffic flow along to each avenue. Also it is assumed that all the direction are equally heavy.



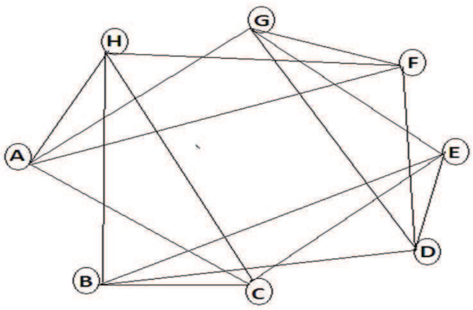


Figure 1

Figure 2

This traffic flow can be modeled as a graph where each traffic flow is represented as vertex. Two vertices are adjacent if the corresponding traffic flows cross each other. For the problem of Figure.1, eight vertices A,B,C,D,E,F,G,H are assigned for all direction of traffic flow. Since the four right turns do not interfere with the other traffic flows, they can safely be dropped from our discussion. The vertices C and H are adjacent since the corresponding directions C and H interest. The graphical representation of this traffic flow problem is depicted in Figure.2

From the problem of traffic light, it is well known that if(two flow cross) two vertices are adjacent, then they should have different light signals(colors). That is, the traffic light problem is equivalent to the graph coloring problem. Now, color its vertices, since the graph contains C3 as sub graph, its chromatic number ≥ 3. Suppose we color A with color 1, C with color 4 and H with color 3. Then B cannot be color with color 3 or color 4 but can be color with color 1. Then E must be color with color 1. Now H and D are not adjacent. So we can color the vertex D with color 3. Similarly C and G can be given same color. But B,F cannot be color with color 1, color 3 or color 4, so it must be assigned a new color, say, color2. Thus, the total number of colors needed for the graph is 4.

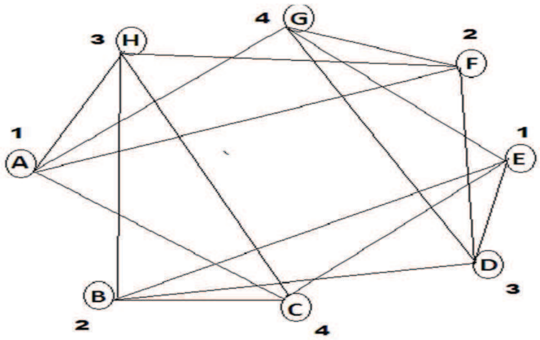


Figure 3

Figure 3 shows a coloring of graph with exactly four colors, which depicts an efficient way of designing the traffic signal pattern. It consists of four phases:

Vertices B and F are 2, so traffic flows B and F proceed, while other are waiting. Vertices D and H are 3; that is, only traffic directions D and H precede simultaneously. Vertices A and E are 1; that is, traffic flows A and E continue at the same time, while others are Stopped. Vertices C and G are 4; that is, only traffic directions D and H precede simultaneously.

In this problem, we assume that all paths are equally heavy in each direction. That means number of vehicles in all path are same. But in real world suction, that is not possible. For instance number of vehicles in B and F are greater than D and H. In this problem all traffic lights have the same cycle time T. If D and H need T time to pass all the vehicle then B and F will need more time than T. So total waiting time of vehicles on the roads will be increase and there may be a suction of traffic jam or accident. In next section this paper, we introduce a method to solve those problems using the coloring of fuzzy graph.

The Traffic Lights Problem As Coloring Of Fuzzy Graph: The control policy of the traffic light depends mainly on the number of vehicles in the intersection line. If the traffic flow in the intersection line is high then there is a possibility of accident. When the number of vehicles in the intersection line is low then there may be less possibility of accident. The concept of accident and number of vehicles in each line could be fuzzy and it could be graduated. This graduation, which does not need to be numerical, is associated to the desired security level for the traffic. Here we represent each traffic flow with a fuzzy edge whose membership value depends on the number of the vehicles in that path. Two fuzzy vertices are adjacent if the corresponding traffic flows cross each other then there is a possibility of accident. Possibility of accident value will depend on vertex membership value. The maximum security level is attained when all lanes are considered to be in intersection with each other and the number of vehicles in each line is also high. So graph will be a complete graph. In this case, the chromatic number is the number of lanes and the control policy of the lights assure that only one movement is allowed in any slot of the cycle. On the other hand, the minimum security level is attained when the intersection edge set is empty; in this case, the chromatic number is 1 and all movements are allowed at any instant.

Since the four right turns do not interfere with the other traffic flows, they can safely be dropped from our discussion. The remaining traffic directions are in Figure 1 and are labeled A through H and their membership value are in table 1.If the number of vehicles in any path is greater than 10000 per hour than we consider the membership value of that path is high. If the number of vehicles in any path is greater than or equal to 5000 per hour than we consider the membership value of that path is medium. If the number of vehicles in any path is less than 5000 per hour than we consider the membership value of that path is low. Membership values are represented by symbolic name H for high, M for medium for low respectively.

Vertex	A	B	C	D	E	F	G	H
σ	M	H	M	L	M	H	M	L

Table 1

We need to develop a traffic pattern so that vehicles can pass through the intersection without interfering with other traffic flows. In this problem, we represent each traffic flow with a vertex and their membership value. Two vertices are adjacent if the corresponding traffic flows cross each other. For instance, direction C and H intersect, so vertices C and H are adjacent. If two vertices are adjacent then there is a possibility of accident. The possibility of accident depends on the adjacent vertices membership value. If membership value of the adjacent two vertices is high (H) then there is more possibility of accident. So we consider the membership value of that arc is high (H).If membership value of one adjacent vertex is high (H) and another is low (L) then we consider the membership value of that arc is medium (M).If membership value of the adjacent two vertices is medium (M) then we consider membership value of that arc is medium (M).

. In this paper, we represent each possibility of accident with a edge and their membership value. Membership values of edges are given below in Table2.

Table 2

Edge	AH	AG	AF	AC	BC	BD	BH	BE	CE	CH	DE	DF	DG	GE	FG	FH
μ	L	M	H	M	H	M	M	H	M	L	L	M	L	M	H	H

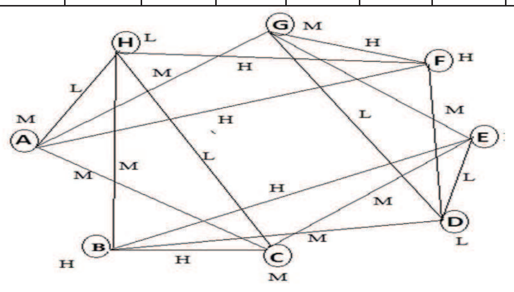


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Figure 4

Coloring function of fuzzy graph (crisp mode)

3.1Definition: Given a fuzzy graph G= (V,σ,μ),its chromatic number is fuzzy number $\chi(G)=\{(x_\alpha, \alpha)\}$ where x_α is the chromatic number of G_α and α values are the different membership value of vertex and edge of graph G. In this paper, we use α values are all different membership value of vertex and edge of fuzzy graph G. We find the all graph G_α which is a crisp graph for all α. Then we find minimum number of color needed to color the graph G_α . In such way, we find the fuzzy chromatic number which is a fuzzy number is calculated by its α cut.

For $\alpha=L \rightarrow G_L=(V_L, E_L)$

$V_L=\{A, B, C, D, E, F, G, H \mid \sigma(v) \geq L\}$

$E_L=\{AC, AF, AG, AH, BC, BD, BE, BH, CE, CH, DE, DF, DG, EG, FG, FH \mid \mu \geq L\}$ $\chi_L = \chi(G_L)=4$

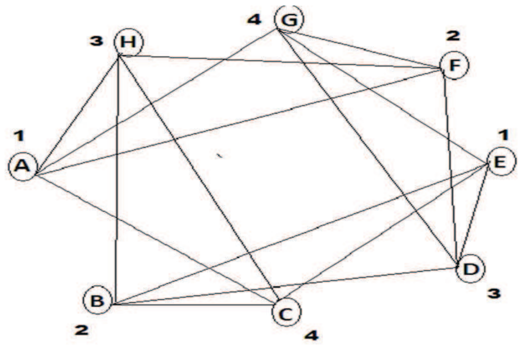

Figure 5

Using same approach, we can calculate when α=M then $\chi_M = \chi(G_M)=3$ and when α=H then $\chi_H = \chi(G_H)=1$ The chromatic number of a fuzzy graph is a normalized fuzzy number whose modal value is associated with the empty edge-set graph. Its depends on the sense of index α, and it can be interpreted in the following way: for lower values of α there are many number of node and high number of intersection links between nodes and, consequently, more colors are needed in order to consider these intersection; on the other hand, for higher values of α there are fewer number of nodes and low value of intersection links between nodes and less colors are needed. The chromatic number sums up all this information in order to manage the fuzzy problem. The fuzzy coloring problem consists of determining the chromatic number of a fuzzy graph and an associated coloring function. In this approach, for any level α, the minimum number of colors needed to color the crisp graph $G\alpha$ will be computed. In this way, the fuzzy chromatic number will be defined as a fuzzy number through its α-cuts.

Conclusion: The chromatic number of G is $\chi\ (G)$ = {(4, L), (3,M)(1,H)}. The interpretation of $\chi(G)$ is the following: lower values of α are associated to lower driver aptitude levels and, consequently, the traffic lights must be controlled conservatively and the chromatic number is high; on the other hand, for higher values of α, the driver aptitude levels increase and the chromatic number is lower, allowing a less conservative control of the traffic lights and a more fluid traffic flow. The duration time of the traffic light depends on the membership value of the node. If the node membership value is high then it needs more time to flow the entire vehicle. So the duration time of this node is greater than other lower membership value node. In this problem duration of the light 2 will be maximum and light 3 will be minimum. Using this concept, total waiting time of the vehicles will be minimizing. In our next paper, we will introduce a method of robust coloring approach to solve the traffic light problem.

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