RAYLEIGH-BENARD-MARANGONI INSTABILITY IN A MICROPOLAR DIELECTRIC LIQUID USING THE GALERKIN TECHNIQUE

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Abstract: The effects of a non-uniform temperature gradient on the onset of convection driven by combined surface tension and buoyancy force in a horizontal layer of micropolar dielectric fluid with upper boundary free and adiabatic and lower boundary rigid and isothermal is studied. The microrotation is assumed to vanish at the boundaries. A linear stability analysis is performed. The Galerkin technique is used to obtain the eigen values. The influence of various parameters on the onset of convection has been analysed. Six different non-uniform temperature profiles are considered and their comparative influence on onset is discussed. It is observed that the most destabilising temperature profile is step function and the most stabilising temperature profile is inverted parabolic. A micropolar dielectric liquid is more stable compared to a dielectric liquid. The electric Rayleigh number has a destabilising effect on the onset of Rayleigh-Benard-Marangoni convection. The problem has possible applications in the microgravity environment.

Keywords: Rayleigh-Benard-Marangoni, micropolar, dielectric liquid, galerkin technique.

Introduction : The mechanism of controlling convection in a fluid generated either by buoyancy force or by the variation in surface tension with temperature or by both is important in material processing in space because of its applications to the possibility of producing various new materials. The range of possibilities extends from producing large crystals of uniform properties to manufacturing materials with unique properties. The results of space exploration, particularly the mechanism of prevention of buoyancy driven convection are useful in understanding the physical processes involved in manufacturing these materials. Even though the microgravity environment in space is known to reduce the convection driven by buoyancy force, Marangoni convection will be generated due to the variation of surface tension with temperature. The present paper attempts to suggest additional mechanisms to suppress convection using a dielectric micropolar fluid description and non-uniform basic temperature gradients.

The problem of the onset of convective instability in a horizontal layer of fluid heated from below has its origin the experimental observations of Benard[1]. This was confirmed by Block[2] in his experiments. The convection induced by the variation of surface tension with temperature is generally referred to as Marangoni instability. Pearson[3] was the first person to make an analytical study of this effect. According to Pearson's theory, for a critical value of a Marangoni number, the layer displays a short wave pattern of stationary cellular convection. The combined effect of buoyancy and surface tension has been investigated by Neild[4] using linear perturbation technique who concluded that for the case of linear density variation, the coupling between the buoyancy and surface tension effects causing instability reinforce one another and are tightly coupled. Rudraiah et al[5] have studied the effect of non-uniform basic temperature gradients in Rayleigh-Benard-Marangoni convection.

The application of a strong electric field in a poorly conducting fluid can induce bulk motions. This phenomenon known as electroconvection or electrohydrodynamics is gaining importance due to the technological stimulus of designing more efficient heat exchangers as required for jet engines [6]. Boiling of dielectric fluids was proposed as a promising cooling mechanism for future microelectronic chips [7] and for dielectric fluid motor [8]. Convective heat transfer through polarised dielectric liquids was studied by P. J. Stiles [9].

In the study of fluids with suspended particles we use the "micropolar fluid model" proposed by Eringen [10] which takes into account the translational and rotational motion of the freely suspended particles relative to the fluid. The limitation in this micro-continuum approach is the assumption of minute rigid suspended particles without collision. The Rayleigh-Benard situation in Eringen's micropolar fluids has been thoroughly investigated by many authors [11].

The objective of this paper is to study the effect of non-uniform basic temperature gradient on the onset of convection driven by combined buoyancy and surface tension forces in a dielectric micropolar fluid.

Mathematical Formulation And Solution :

We consider an infinite horizontal layer of a micropolar dielectric fluid of thickness d.

The lower plane surface is at z = 0 and the upper one is at z = d. We assume a temperature drop ΔT across the boundaries. Provided ΔT is not too large we can invoke the Boussinesq approximation. The interface has a surface-tension σ which, following Pearson [3], can be assumed to vary linearly with temperature according to the formula $\sigma = \sigma_0 - \sigma_1 \Delta T$, (1)

where σ_0 is the unperturbed value of σ and $\sigma_1 = -\left(\frac{d\sigma}{dT}\right)_{T_0}$.

The Navier – Stokes equations describing flow in an incompressible fluid are

$$\nabla.\vec{q} = 0 \quad (2)$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g} + (2\zeta + \eta)\nabla^2\vec{q} + \zeta\nabla\times\omega + \vec{P}.\nabla\vec{E} \quad , (3)$$

$$\rho_0 I\left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = \zeta(\nabla\times\vec{q} - 2\vec{\omega}) + (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + \eta'\nabla^2\vec{\omega} \quad , (4)$$

$$\frac{\partial T}{\partial t} + \vec{q}.\nabla T = \frac{\delta}{\rho_0 c_v}(\nabla\times\vec{\omega}).\nabla T + K_c\nabla^2 T, \quad (5)$$

$$\rho = \rho_0[1 - \alpha(T - T_0)], \quad (6)$$

$$\nabla.\vec{D} = 0 \text{ where } \vec{D} = \varepsilon_0\vec{E} + \vec{P} \quad (7)$$

$$\nabla\times\vec{E} = 0, \quad (8)$$

In the above equations, $\vec{q}$ is the velocity , $\vec{\omega}$ is the spin, T is the temperature, p is the pressure, $\vec{\omega}$ is the acceleration due to gravity, η is the shear kinematic viscosity co-efficient, λ' and η' are the bulk and shear spin viscosity coefficients, ζ is the coupling viscosity co-efficient or vortex viscosity, δ is the micropolar heat conduction co-efficient, C_v is the specific heat, α is the co-efficient of thermal expansion, ρ is the density, ρ_0 is the density of the fluid at temperature $T=T_o$, I is the moment of inertia, K_c is thermal diffusivity. Equation (8) allows us to express the irrotational electric field $\vec{E}$ as

$$\vec{E} = E_0\left[1 + \beta z\left(\frac{\partial \ln \varepsilon_r}{\partial T}\right)\right]\hat{k} - \nabla\phi, \quad (9)$$

and the polarization field $\vec{P}$ can be expressed as

$$\vec{P} = \varepsilon_0 E_0\left[\varepsilon_r - 1 - \left(\frac{\partial \ln \varepsilon_r}{\partial T}\right)(\beta z - T)\right]\hat{k} - (\varepsilon_r - 1)\varepsilon_0\nabla\phi \quad , (10)$$

where ϕ is the perturbation to the electric scalar potential due to convection, ε_r and ε_0 are the relative permittivity and permittivity of free space respectively

2.1 Basic state:

The basic state is one in which

$$\vec{q}_b = (0,0,0,), \quad \vec{\omega}_b = (0,0,0), \quad p=p_b(z),$$

$$\rho = \rho_b(z), \quad \frac{\partial T_b}{\partial t} = K_c\frac{\partial^2 T_b}{\partial z^2}, \quad \vec{E} = (0,0,0), \quad (11)$$

The non – uniformity in T_b may find its origin in transient heating or cooling at the boundaries. As propounded by Nield [4] and Lebon and Cloot [12].

Denoting $-\frac{d}{\Delta T}\frac{dT_b}{dz}$ by f(z), a non–dimensional basic temperature gradient, satisfying the condition

$$\int_0^1 f(z)dz = 1, \quad (12)$$

We have considered various non–uniform temperature gradients in this paper. Let the basic state be slightly perturbed. The principle of exchange of stability can be assumed to be valid and hence we consider only stationary convection. The linearised equations (2) to (9) are non-dimensionalized using the following definitions: $(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right)$,

$$\vec{q}^* = \frac{\vec{q}'}{(K_c/d)}, \quad \omega^* = \frac{\omega}{(K_c/d^2)}, \quad T^* = \frac{T'}{\Delta T},$$

$$\phi^* = \frac{\phi'}{E_0\beta d^2\left(\frac{d\ln\varepsilon_0}{dT}\right)}. \quad (13)$$

Operating curl twice on the resulting momentum equation and operating curl once on the resulting angular momentum equation, we get

$$(1 + N_1)\nabla^4 w + N_1\nabla^2\Omega_z + (R + L)\nabla_1^2 T - L\frac{\partial(\nabla_1^2\phi)}{\partial z} = 0 \quad , (14)$$

$$N_3\nabla^2\Omega_z - N_1\nabla^2 w - 2N_1\Omega_z = 0, \quad (15)$$

$$\nabla^2 T + f(z)(w - N_5\Omega_z) = 0, \quad (16)$$

$$\nabla^2\phi - \frac{\partial T}{\partial z} = 0, \quad (17)$$

where the asterisks have been dropped for simplicity and

$N_1 = \frac{\zeta}{\eta + \zeta}$, $(0 \le N_1 \le 1)$ (coupling parameter)

$N_3 = \frac{\eta'}{(\eta + \zeta)d^2}$ $(0 \le N_3 \le m)$ (couple stress parameter)

$$N_5 = \frac{\delta}{\rho_0 C_v d^2}, \qquad \left(0 \le N_5 \le n\right)$$

(micropolar heat conduction parameter)

$$R = \frac{\rho_0 \alpha g \Delta T d^3}{(\eta + \zeta) K_c} \qquad \text{(Rayleigh number)},$$

$$L = \left(\frac{d\varepsilon}{dT}\right)^2 \frac{E^2 (\Delta T)^2 d^2}{\varepsilon K_c (\eta + \zeta)}$$

(Electric Rayleigh number).

The linearized equations of motion allow the solution for the perturbed field quantities in the form [13]

$$\left[w, \Omega_z, T, \phi\right] = \left[W(z), G(z), T(z), \phi(z)\right] \exp\left[i(lx + my)\right] \qquad (18)$$

Substituting equation (18) into equations (14) to (17),which are solved subject to the following boundary conditions:

$$W = DW = T = G = \phi = 0 \quad \text{at } z=0,$$

$$W = D^2W + a^2 MT = DT = G = \phi = 0 \quad \text{at } z=1, \qquad (23)$$

where $M = \dfrac{\sigma_T \Delta T d}{\rho_0 \nu K_c}$ is the Marangoni number

and $\sigma_T = \left(\dfrac{\partial \sigma}{\partial T}\right)_{T=T_0}$, σ being the surface tension.

We use the single term Galerkin expansion to obtain the condition for the onset of convection. Multiplying equation (19) by W, equation (20) by G, Equation (21) by T and equation (22) by ϕ and integrating the resulting equations by parts with respect to z from 0 to 1, using the boundary condition (23) and taking $W=AW_1$, $G=BG_1$, $T=CT_1$ and $\phi = D\phi_1$ in which A, B, C and D are constants and W_1, G_1, T_1 and ϕ_1 are trial functions, yields the following equation for the eigen value

$$M = \frac{\left\{\dfrac{-(1+N_1)C_2C_5C_8}{(C_3C_2 - C_1C_4)} + \dfrac{N_1C_1^2C_5}{(C_3C_2 - C_1C_4)} + (R+L)a^2C_9 + La^2\dfrac{C_{10}C_6}{C_7}\right\}}{(1+N_1)a^2C_{11}} \qquad (24)$$

where

$$C_1 = N_1\left\{\langle DG_1 DW_1\rangle + a^2\langle G_1 W_1\rangle\right\}$$

$$C_2 = N_3\left\{\langle (DG_1)^2\rangle + a^2\langle G_1^2\rangle\right\} + 2N_1\langle G_1^2\rangle$$

$$C_3 = \langle f(z) W_1 T_1\rangle$$

$$C_4 = N_5\langle f(z) T_1 G_1\rangle$$

$$C_5 = \langle (DT_1)^2\rangle + a^2\langle T_1^2\rangle$$

$$C_6 = \langle \phi_1 DT_1\rangle$$

$$C_7 = \langle (D\phi_1)^2\rangle + a^2\langle \phi_1^2\rangle$$

$$C_8 = \langle (D^2W_1)^2\rangle + 2a^2\langle (DW_1)^2\rangle + a^4\langle W_1^2\rangle$$

$$C_9 = \langle W_1 T_1\rangle$$

$$C_{10} = \langle W_1 D\phi_1\rangle$$

$$C_{11} = DW_1(1)T_1(1)$$

Where the angle bracket < > denotes the integration with respect to z from 0 to 1.

We select the following trial functions

$W_1 = z^2(1-z^2)$, $G_1 = z(1-z)$, $T_1 = z(2-z)$, $\phi_1 = z(1-z)$.. (25)

Such that they satisfy all the boundary conditions (23) except the one given by $D^2W + a^2MT = 0$ at z = 1, but the residual from this is included in the residual from the differential equations. Substituting equation (25) in (24) and performing the integration we can calculate the critical Marangoni number M_c which attains its minimum at a_c^2.

Results And Discussions : In this paper we study the effect of six non-uniform basic temperature gradients on the onset of Rayleigh-Benard-Marangoni convection in a micropolar dielectric fluid . It is observed that the step function is the most destabilising basic temperature distribution, because the jump in temperature occurs nearer the free surface and inverted parabolic is the most stabilising basic temperature distribution. In the case of piecewise linear profiles heating from below, cooling from above and step function profiles the critical Marangoni number M_c depends also on the thermal depth ε which is attained at ε = 0.9323, ε = 0.4276 and ε = 0.7394 respectively. Eringen [19] presented certain thermodynamic restrictions which lead to non-negativeness of N_1, N_3 and N_5. For $\varepsilon = 0$ ($N_1 = 0$) it is clear that equation (14) for W becomes independent of G, i.e. it is uncoupled. As $\varepsilon \to \infty$ we see that N_1 tends to 1 and N_3 tends to 0. This is the Stokesian description of suspension. Thus $0 \le N_1 \le 1$ and N_3 is a small positive real number. N_5 is a positive, finite real number because the increasing of concentration has to be finite. This typical order of magnitudes of N_1, N_3 and N_5 mentioned above applies to fluid systems encountered in material processing under microgravity in space. From Table 1 we see that the critical wave number is sensitive to changes in the electric Rayleigh number L and the micropolar parameter N_1 but is insensitive to changes in the micropolar parameters N_3 and N_5. Fig. (1) is the plot of M_c versus the electric Rayleigh number L for different non-uniform basic temperature profiles. Clearly M_c decreases with increase in the electric Rayleigh number. Thus L destabilizes the system. Fig.

(2) is the plot of M_c versus the coupling parameter N_1 for different non-uniform basic temperature gradients. M_c increases with increase in N_1. Increase in N_1 indicates the increase in the concentration of micro elements. Since micro elements increase in number with increasing N_1; a greater part of the energy of the system is consumed by these elements in developing gyrational velocities of the fluid, and as a result onset of convection is delayed. The increase in N_1 stabilizes the system. Fig (3) is a plot of M_c versus the couple stress parameter N_3 for six different non-uniform temperature profiles. Clearly M_c decreases with increasing N_3 and ultimately levels off to the Newtonian value. Increase in N_3, increase the couple stress of the fluid which causes a decrease in micro rotation and hence makes the system more unstable. At only small values of N_3 couple stress are operative and hence we observe that micro rotations (small values of N_3) stabilize the system in comparison with the Newtonian problem. Fig. (4) is a plot of M_c versus the micropolar heat conduction parameter N_5. When N_5 increases, the heat induced into the fluid due to the microelements is also increasing, thus reducing the heat transfer from the bottom to the top. The decrease in heat transfer is responsible for delaying the onset of instability. This result can also be anticipated because equation (4) clearly shows that the effect of the suspended particles is to deduct from the velocity. Thus increasing N_5 leads to increase in M_c. In other words N_5 stabilizes the system.

Fig. (5) are the plots of M_c versus the Rayleigh number R for the various non-uniform temperature profiles with N_1 varying. The critical Marangoni number M_c decreases with increase in the Rayleigh number R. Thus surface tension becomes negligible when the buoyancy effect is predominant.

Conclusion : Rayleigh-Benard-Marangoni convection in a system can be controlled by considering

a dielectric fluid with micron sized suspended particles. A suitable non-linear temperature profile suppresses or augments convection.

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Fig 1.- M_c versus L

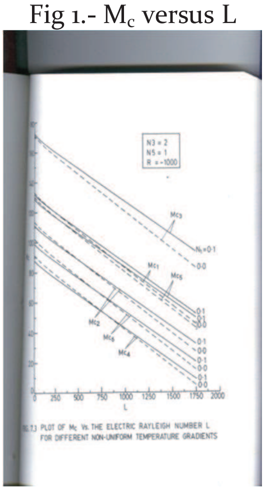


Fig 2 – M_c versus N_1

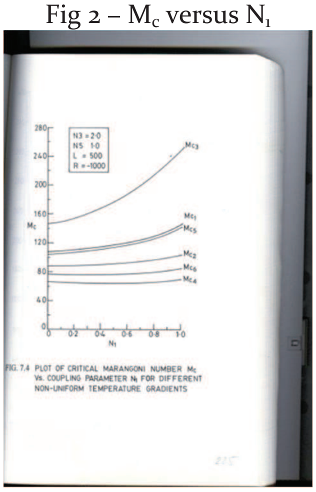


Fig. 3 – M_c versus N_3

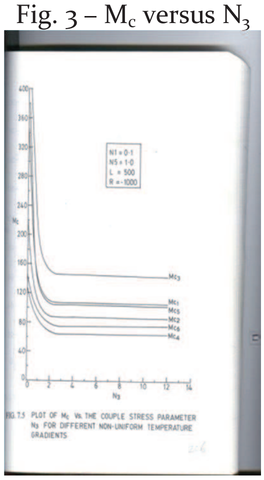


Fig. 4– M_c versus N_5

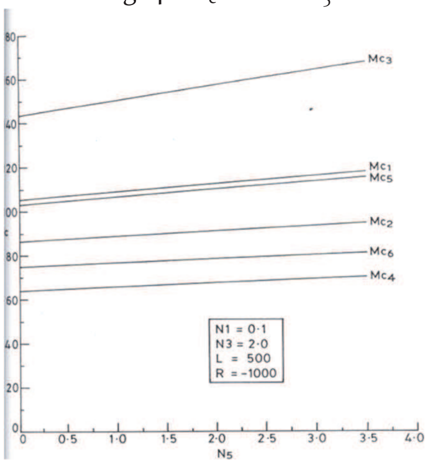
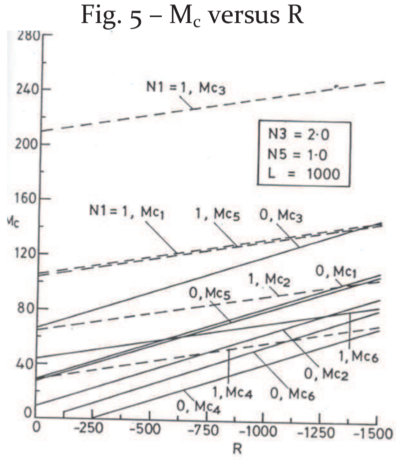


Fig. 5 – M_c versus R



Where the temperature profiles indicated in the graph are
M_{c1} – Linear M_{c2} – Heated from Below
M_{c3} – Cooled from Above M_{c4} – Step Function
M_{c5} – Inverted Parabola M_{c6} – Parabola

L	N_1	N_3	N_5	R	a_c^2
0 500 1000 1500	0.1	2	1	-1000	5.91 6.04 6.18 6.31
500	0 0.2 0.6 1	2	1	-1000	6.06 6.03 5.95 5.82
500	0.1	2 4 6 8	1	-1000	6.04 6.04 6.05 6.05
500	0.1	2	0 1 1.5 2	-1000	6.05 6.04 6.04 6.04
1000	0.1	2	1	0 -500 -1000 -1500	6.18 6.18 6.18 6.18

TABLE 1 : THE VALUES OF a_c^2 FOR VARIOUS VALUES OF L, N_1, N_3 , N_5 AND R FOR f(z) =1

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