DUAL MIXED CONVECTION FLOW OF A COUPLE STRESS FLUID IN A VERTICAL CHANNEL

PADMAVATHI R, SRIKRISHNA C V, INDIRA R RAO

Abstract : The problem of the fully developed mixed convection of a couple stress fluids with frictional heat generation in a vertical channel bounded by isothermal plane walls having the same temperature is considered. The resulting equations are analytically solved using power series method. The effects of Gebhart, Prandtl and Reynolds number on the flow as well as temperature are analysed by considering the product Ge Re Pr = Ξ. The effects of couple stress parameter on the flow and temperature is analysed and graphically depicted. The existence of dual solution of the local balance equations is pointed out. The effect of i increase in couple stress is to decrease the velocity as the couple stress parameter is inversely proportional to spin of the particulate matter present.

Keywords : mixed convection, couple stress fluid, vertical channel, Heat transfer

Introduction : A quantitative study is essential to understand the physics of heat and mass transfer and for obtaining in valuable scale-up information in industry application. Recently there is a rapid change in technology which demands a thorough understanding of principles of fluid mechanics and knowledge of how to apply them. Bio-medical, aeronautics, civil, mechanical and chemical engineers as well as space researcher, meteorologist, geophysicist, astrophysicists and physical oceanographer encounter multitude of complex phenomenon. In particular, it is of interest in many varied practical situations such as material science processing, geothermal energy extraction, energy store device and many industrial problems.

Buoyancy induced flows in ducts deserve wide attention mainly for their applications in several thermal control devices ranging from electronics to nuclear plants. Indeed, in passive or semi-passive thermal control systems, either purely free convection flows or mixed convection flows are involved. Mixed convection flows in a vertical channel has been the subject of many previous investigations due to its possible application in many industrial and engineering processes.

The theoretical investigations on fully developed mixed convection in vertical or inclined ducts are often devoted to a description of changes on the velocity profiles induced by buoyancy as well as to the determination of the conditions for the onset of flow reversal (crossover from a columnar to a cellular flow).Indeed, the flow reversal phenomenon arises when buoyancy forces are so strong that exists a domain within the duct where the local fluid velocity has a direction opposite to the mean fluid flow. These studies are often based on assumption that effect of viscous dissipation in fluid is negligible. This assumption holds whenever the fluid has a sufficiently high thermal conductivity, a sufficiently small prandtl number and sufficiently high wall heat fluxes are present. On the other hand, the other theoretical investigations have been devoted to the analysis of the interplay between the effect of viscous dissipation and the effect of buoyancy [11-16].

Tao[1] analysed the laminar fully developed mixed convection flows in a vertical parallel- plate Channel with uniform wall temperatures. Aung and Worku [2, 3] discussed the theory of combined and free convection in a vertical channel with a flow reversal conditions for both developing and fully developed flows. The case of developing mixed convection flow in ducts with asymmetric wall heat flux was analysed by the same author [4]. Oreper and Szekely [5] analysed the buoyancy driven flow in a rectangular cavity under the action of externally imposed magnetic field. Ruchu Chaturvedi, Rajesha Kumar, and R.K.Shrivastava [6] analysed the mixed convective heat transfer of non-Newtonian fluids through porous medium with magnetic field on a flat plate has been investigated using a modified power-law viscocity model.

The interest in heat transfer problems involving power-law non-Newtonian fluids in the past half century. An excellent sequence of lectures on non-Newtonian fluids was given by Hinch [8]. It appears that Acrivos [9], a frequently cited paper, was the first to consider boundary-layer flows for such fluids. Since then a large number of related papers have been published due to their wide relevance in chemicals, foods, polymers, molten plastics and petroleum production, and other natural phenomena.

Two widespread mistakes appear continuously in papers studying boundary-layers involving the traditional two-parameter power-law model of non-Newtonian fluids(see Bird et al.[10]). Barletta et.al (2005) have considered dual mixed convection flows of Newtonian fluids in a vertical channel. Analytical solution of mixed convection flow of couple stress fluid between two circular cylinders with Hall and ion-slip effects was analysed by

Srinivasacharya and Kaladhar in 2011.

In the present analysis, combined forced and free flow of couple stress fluid in the fully developed region of a vertical channel with isothermal walls kept some temperature is considered, the fluid properties being assumed constant.

Mathematical Formulation: We consider the laminar fully developed flow in a vertical channel bounded by isothermal plane kept at a temperature T° (Fig. 1). The effect of viscous dissipation is taken into account and the Boussinesq approximation is adopted. A steady parallel flow regime is assumed, so that the velocity field is given by V = (U, 0, 0). The local mass balance equation is reduced in this case to $\frac{\partial U}{\partial X} = 0$ and U=U(Y). The momentum balance equation according to Boussinesq approximation yields

$$-\frac{\partial P}{\partial x} + \mu \frac{d^2U}{dy^2} - \eta \frac{d^4U}{dy^4} + \rho\,\beta\,g\left(T - T_r\right) = 0 \qquad (1)$$

where P = p + gρX is the difference between the pressure p and the hydrostatic pressure - gρX. The density ρ, the dynamic viscosity μ and the thermal expansion co-efficient β are evaluated at a reference temperature T_r.

The energy balance equation for temperature

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{dU}{dy}\right)^2 = 0 \qquad (2)$$

The no-slip conditions, vanishing of couple stress and thermal boundary conditions are

$$U(\pm L) = 0, \quad \frac{d^2U(\pm L)}{dY^2} \qquad T(\pm L) = T_0 \qquad (3)$$

On the mid plane y = 0 the symmetry conditions are

$$\frac{du}{dy} = 0 \quad at\ y = 0 \qquad \text{and}$$

$$\frac{\partial T}{\partial y} = 0 \quad at\ y = 0 \qquad (4)$$

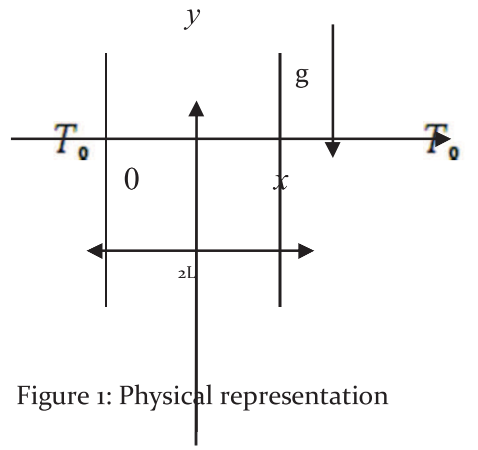

Figure 1: Physical representation

Rearranging equation (1) we get

$$T = T_r + \frac{1}{\rho g \beta}\frac{\partial P}{\partial x} - \frac{\mu}{\rho g \beta}\frac{d^2U}{dy^2} + \frac{\eta}{\rho g \beta}\frac{d^4U}{dy^4} \qquad (5)$$

$\frac{\partial p}{\partial x}$ is a constant due to the boundary condition (3).

Both temperature and velocity depend only on the transverse coordinate Y. Therefore the governing equations reduce to

$$\frac{d^2T}{dy^2} = -\frac{\mu}{\rho g \beta}\frac{d^4U}{dy^4} + \frac{\eta}{\rho g \beta}\frac{d^6U}{dy^6} \qquad (6)$$

Using equation (6) equation(2) becomes

$$\frac{d^6u}{dy^6} - \frac{\mu}{\eta}\frac{d^4u}{dy^4} + \frac{\mu\rho g\beta}{k\eta}\left(\frac{du}{dy}\right)^2 = 0 \quad (7)$$

The boundary conditions (3), (4) and equation (5) lead to following boundary conditions on U(Y) , i.e.

$$U(L) = 0, \quad \left.\frac{dU}{dY}\right|_{Y=0}, \quad \left.\frac{d^3U}{dY^3}\right|_{Y=0} \quad \text{and} \quad \left.\frac{d^5U}{dY^5}\right|_{Y=0}$$

The missing boundary conditions are replaced by Constraint given by the average fluid velocity and the average spin in the channel section.

Non-Dimensionalising the above equations using

$$\Xi = Ge\Pr\mathrm{Re} = \frac{16L^2 g\beta\rho\mu_m}{k}$$

$$u = \frac{U}{u_m}, \quad y = \frac{Y}{L}, \quad Ge = \frac{4Lg\beta}{c_p}, \quad \Pr = \frac{\mu c_p}{k},$$

$$\mathrm{Re} = \frac{4Lu_m}{v}, \quad a_0 = \frac{\mu 1^2}{\eta}$$

where a_0 is the couple stress parameter and the Gebhart (Ge) and Reynolds (Re) Numbers are defined with respect to the hydraulic diameter 4L of the channel .

By employing dimensionless quantities defined as above, the boundary value problem from eqn (4) can be written as

$$\frac{d^6u}{dy^6} - \frac{\mu}{\eta}l^2\frac{d^4u}{dy^4} + \frac{\mu l^2 \Xi}{\eta\ 16}\left(\frac{du}{dy}\right)^2 = 0 \quad (8)$$

The limiting case of $\Xi \to 0$ corresponds to very small dissipation heating or negligible buoyancy effects.

$$u = c_1 + c_2 y + c_3 y^2 + c_4 y^3 + c_5 e^{a_0 y} + c_6 e^{-a_0 y} \qquad (9)$$

Using BC's

$$u(0) = 0\ ; \quad \frac{du}{dy} = 0, \quad \frac{d^3u}{dy^3} = 0,$$

$$\frac{d^5u}{dy^5} = 0 \quad at \quad y = 0$$

(10)

To make the D.E well poised, we take $\frac{d^2u}{dy^2}=\alpha$ and $\frac{d^4u}{dy^4}=\alpha_1$ at $y=0$ (11)

On solving the above equations we get,

$c_1=\frac{\alpha}{2a_0^2}-\frac{\alpha\cosh a_0}{a_0^4}-\frac{\alpha_1}{2}$, $c_2=c_4=0$,

$c_3=-\frac{\alpha}{2a_0^2}+\frac{\alpha_1}{2}$, $c_5=\frac{\alpha}{2a_0^4}=c_6$. Using these constants we get the velocity component as

$$u=c_1+c_3y^2+c_5e^{a_0y}+c_6e^{-a_0y} \quad (12)$$

The dimensionless teperature coefficient is given by

$$\theta(y)=\frac{1}{256}\int_y^1\int_0^y\left(\frac{du}{d\bar{y}}\right)^2 d\bar{y}dy$$

and can be computed as

$$\theta(y)=\frac{1}{256}\left[\begin{array}{l}\frac{1}{12}\left(\alpha_1-\frac{\alpha}{a_0^2}\right)^2\left(1-y^4\right)+\\ \frac{\alpha^2}{a_0^6}\left\{\frac{1}{8a_0^2}\left(\cosh 2a_0-\cosh 2a_0y\right)-\frac{\left(1-y^2\right)}{4}\right\}+\\ \frac{2\alpha}{a_0^6}\left(\frac{\alpha}{a_0^2}-\alpha_1\right)\left\{\begin{array}{l}a_o\sinh\left(a_0\right)-a_0y\sinh\left(a_0y\right)-\\ 2\cosh a_0+2\cosh a_0y\end{array}\right\}\end{array}\right. \quad (13)$$

The dimensionless pressure gradient can be calculated using

$$\lambda=-\frac{1}{16}\frac{d^2u}{dy^2}\bigg|_{y=1}=-\frac{1}{16}\left\{\left(\alpha_1-\frac{\alpha}{a_0^2}\right)-\frac{\alpha\cosh a_0}{a_0^2}\right\} \quad (14)$$

The effect of viscous dissipation is characterized by the Brinkman number Br which can be defined as $Br=\frac{1}{256\theta(0)}$

For $\Xi\neq 0$, the solutions of the governing equations (6) – (8) can be obtained analytically following Barletta et.al by assuming $\phi(y)=\frac{\Xi}{16}\frac{du(y)}{dy}$. The governing equations (6)-(8) reduce to

$$\frac{d^5\phi}{dy^5}-a_0\frac{d^3\phi}{dy^3}+a_0\phi^2=0 \quad (15)$$

subject to boundary conditions $\phi(0)=0$, $\frac{d^2\phi}{dy^2}\bigg|_{y=0}=0$, $\frac{d^4\phi}{dy^4}\bigg|_{y=0}=0$, $\frac{d\phi}{dy}\bigg|_{y=1}=0$

and to make the problem well poised we assume $\frac{d\phi}{dy}\bigg|_{y=0}=\alpha$ along with $\frac{d^3\phi}{dy^3}\bigg|_{y=0}=\alpha_1$

Assume a power series solution for ϕ in the form $\phi(y)=\sum_{n=0}^{\infty}A_ny^n$. Using the initial conditions the even coefficients assume a value zero. The non vanishing coefficients are given by $A_1=\alpha$,

$A_3=\frac{\alpha_1}{3!}$, $A_5=\frac{\alpha_1 a_0}{3!}$, $A_7=\frac{\left(a_0\alpha_1-2\alpha^2\right)a_0}{7!}$,

$A_9=\frac{a_0}{9!}\left(a_0^2\alpha_1-2a_0\alpha^2-8\alpha\alpha_1\right)$,......... The solution for ϕ is obtained as

$$\phi(y,\alpha,\alpha_1)=A_1y+A_3y^3+A_5y^5+A_7y^7+A_9y^9+.......... \quad (16)$$

The dimensionless temperature co-efficient using (16) can be calculated by $\theta(y)=\frac{1}{\Xi^2}\int_y^1\int_0^y\phi^2(\bar{y})d\bar{y}dy$ and we get

$$\theta(y)=\frac{1}{\Xi^2}\left\{\begin{array}{l}\frac{A_1^2}{12}\left(1-y^4\right)+\frac{A_3^2}{56}\left(1-y^8\right)+\\ \frac{A_5^2}{132}\left(1-y^{12}\right)+\frac{A_7^2}{240}\left(1-y^{16}\right)+\frac{A_9^2}{380}\left(1-y^{20}\right)\end{array}\right\}$$

The dimensionless axial pressure gradient

$$\lambda=-\frac{1}{\Xi}\left(\frac{d\phi}{dy}\right)\bigg|_{y=1}=-\frac{1}{\Xi}\left(A_1+3A_3+5A_5+\ldots\right)$$

Results And Discussion : Mixed convection flow of couple stress fluid in a vertical Channel with isothermal Walls at a given temperature T_o has been analyzed by taking into account the effect of viscous heating. The local mass momentum and energy balance equations have been written according to Boussinesq approximation. The governing 6th orders ordinary differential equations are solved using power series and shooting methods. The case Ξ = 0 corresponds to negligible Gebhart number, i.e. to a negligible buoyancy effect. The velocity profile, dimensionless temperature and Brinkmann number are graphically depicted.

Figures 2-3 shows graphs for case Ξ = 0. Figure 1 show velocity profile for different values of couple stress parameter. The couple stress parameter varies as reciprocal of the spin of the particulate matter. As couple stress parameter increases the velocity decreases. This is due to the increase of resistance to flow due to decrease in spin of the particles.

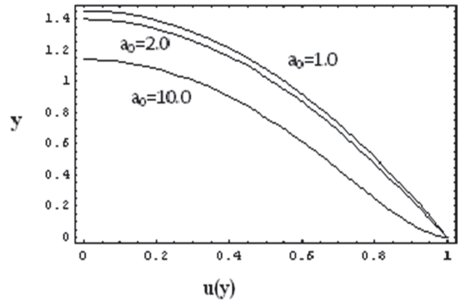

Figure 2. Velocity Profile for different valus of couple stress parameter

Figure 3. shows Temperature Profile for different values of couple stress parameter a_o. The temperature profile shows similar pattern as in case of velocity. As a_o increases velocity decreases hence convection decreases and temperature also decreases.

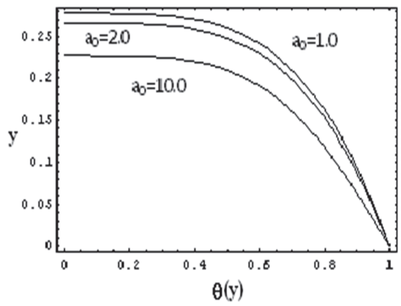

Figure 3. Temperature profile for different values of couple stress parameter

Figures 4 -7 Shows graph for case $\Xi \neq 0$. Ξ is the important non dimensional parameter which influences velocity as well as the temperature gradient. The parameter Ξ has a maximum bound above which no solution is allowed . the series solution obtained is convergent and few terms are enough to compute the parameters correctly.

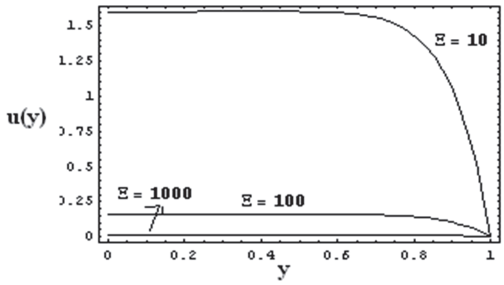

Figure 4. Velocity profile for $\Xi \neq 0$ with $a_o = 1.0$

Figure 4. shows velocity profile for different values of Ξ. As Ξ increases velocity decreases evidently due to the presence of Reynolds number in the denominator. The flow almost reduces to zero for large Ξ. Showing there exist a maximum limit for Reynolds number. Comparison between figure 4 and figure 5 shows that the effect of a_o is not very significant as the values of each curve matches in both the figures.

Figure 6. shows temperature profile for different values of couple stress parameter. The effect of couple stress is dominant in the middle region between y = 0.6 to 0.85. For a very small couple stress parameter the temperature decrease gradually starting from the value y = 0.2 showing more or less parabolic structure. But for larger couple stress the temperature remains constant initially and decreases suddenly after y = 0.6. This is again due to the particle spin which is very lager for small ao increasing the convection of heat resulting in steady decrease of temperature along y.

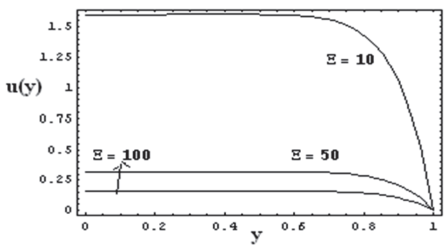

Figure 5. Velocity profile for $\Xi \neq 0$ with $a_0 = 10.0$

Figure 7 Shows variation of Brinkmann number with the Reynolds number. Br varies inversely as the Reynolds number.

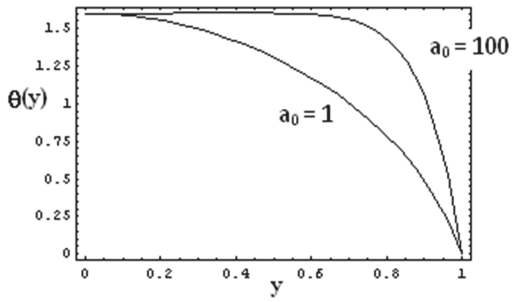

Figure 6. Temperature profile for $\Xi \neq 0$

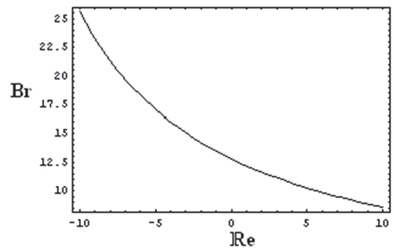

Figure 7. Variation of Brinkmann number with Reynolds number

Conclusion : Reynolds number and couple stress parameter are the two parameters affecting the flow as well as temperature profile. Initial temperature does not affect the analysis. The Reynolds number has an upper bound beyond which solutions do not exist. Reynolds number influences the effect of the couple stress parameter. At very high Reynolds number the effect of couple stress parameter is negligible. The results reduce to Newtonian case as $a_0 \to \infty$.

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Sr. Lecturer/ Dept of Mathematics/NMIT/prakruthi500@gmail.com
/Professor/Dept. of MCA/PESIT/cvsrikrishna@yahoo.co.in,
Professor/ Dept. of Mathematics/NMIT/ indira_rao09@rediffmail.com