A STUDY ON BAYESIAN MODIFIED CHAIN SAMPLING PLAN -1

M.LATHA, M.RAJESWARI

Abstract : The average probability of acceptance for Modified Chain Sampling Plan based on gamma prior distribution is given and the selection procedure for Bayesian Modified Chain Sampling Plan(BMChSP-1) is given with illustrations. Comparison is made with Bayesian Chain Sampling Plan -1.

Keywords: Average probability of acceptance, gamma distribution.

Introduction: In Dodge's (1955) approach, chaining of past lot results does not always occur. It occurs only when a nonconforming unit is observed in the current sample. This means that the available historical evidence of quality is not fully utilized. Govindaraju and Lai(1998) developed a modified chain sampling plan (MChSP-1) that always utilizes the recently available lot-quality history. In this paper, Overall Average outgoing quality limit(OAOQL) values are provided for BMChSP-1 when the product quality follows gamma prior distribution.

The operating procedure of the MChSP-1 plan is given below.

From each of the submitted lots, draw a random sample of size n. Reject the lot if one or more nonconforming units are found in the sample.

Accept the lot if no nonconforming units are found in the sample, provided that the preceding i samples also contained no nonconforming units except in one sample, which may contain at most one nonconforming unit. Otherwise, reject the lot.

The OC function $P_a(p)$ of the MChSp-1 plan was derived by Govindaraju and Lai(1998) as

$$P_a(p) = P_{0,n}(P_{0,n}^{i} + iP_{0,n}^{i-1}P_{1,n})$$

With gamma as prior distribution,

$$w(P) = e^{-np}P^{s-1}t^{s}/s \quad s,t,p>0$$

with parameters s and t.

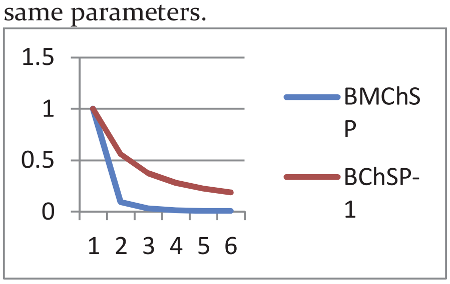
The average probability of acceptance is given as

$$\overline{P} = \int_0^{\infty} P_a(p)w(p)dp$$

$$= \frac{s^s}{(s+n\mu(1+i))^s} + \frac{s^{s+1}n\mu i}{(s+n\mu(1+i))^{s+1}} \qquad (1)$$

where μ = s/t the mean value of the product quality p.

Figure 1 OC Curve for BChSP-1 and BMChSP for the same parameters.



Suresh and Latha(2002) has given the OAOQL values for Bayesian Chain Sampling Plan-1. The Overall Average Outgoing Quality(OAOQ) for Bayesian sampling plan is given as,

$$OAOQ = \int pP_a(p)dw(p)$$

For BMChSP,

$$nOAOQ = \frac{n\mu s^{s+1}}{(n\mu+n\mu i+s)^{s+1}} + \frac{i(n\mu)^2 s^s(1+s^2)}{(n\mu+n\mu i+s)^{s+2}} \qquad (2)$$

Differentiating (2) with respect to μ and equating to zero will results with,

$$\frac{n\mu s^{s+2}(1-n\mu-n\mu i)}{(n\mu+n\mu i+s)^2} + \frac{i(n\mu)^2 s^{s+1}(1+s^2)(2-n\mu-n\mu i)}{(n\mu+n\mu i+s)^{s+2}} = 0 \qquad (3)$$

Equation(3) is solved using Newton's method of successive approximation and the values of $n\mu(= n\mu_m)$ are listed out for different values of sand i in Table 1. Substituting $n\mu_m$ in (2) gives OAOQL as in Table 2.

Differentiating (1) twice with respect to μ and equating to zero results with,

$$s^{s+1}n^2(1+i)\begin{bmatrix}(s+1)(1+i)(n\mu+n\mu i+s)+\\ si(n\mu+n\mu i-2s-2+n\mu s+n\mu si)\end{bmatrix} = 0 \qquad (4)$$

Solving (4) for $n\mu$ with Newton's method of approximation gives various values for $n\mu_*$ listed in Table 2.

Comparison with Bayesian Chain Sampling Plan -1

From Table 2 it is observed that $n\mu_1$ values of BMChSP are more or less equal to the $n\mu_1$ values of BChSP-1, but the $n\mu_2$ values of BChSP-1 is much greater than the $n\mu_2$ values of BMChSP for the same parameters. So, when the recent available lot quality history gives good result, more lots will be accepted under current process of production. The BMChSP-1 plan gives a psychological protection to the consumer and on the producer side BChSP-1 would be the better.

Example 1

For the given s=1 and i=3, the nOAOQL value for BMChSP is 0.5115, for BChSP-1 .nOAOQL for the same s and i value is 0.3184.

Table 1 $n\mu$ values for Bayesian Modified Chain Sampling Plan for the given average probability of acceptance.

s	$\bar{p}$ / i	0.99	0.95	0.90	0.50	0.10	0.05
1	0	1.0101	1.0526	1.1111	2.0000	10.0000	20.0000
	1	0.4489	0.4678	0.4938	0.8889	4.4444	8.8889
	2	0.2828	0.2947	0.3111	0.5600	2.8000	5.6000
	3	0.2061	0.2148	0.2268	0.4082	2.0408	4.0816
	4	0.1621	0.1689	0.1783	0.3210	1.6049	3.2099
	5	0.1336	0.1392	0.1469	0.2645	1.3223	2.6446
	6	0.1136	0.1183	0.1249	0.2249	1.1243	2.2485
	7	0.0988	0.1029	0.1086	0.1956	0.9778	1.9556
	8	0.0874	0.0911	0.0961	0.1730	0.8651	1.7301
	9	0.0783	0.0816	0.0862	0.1551	0.7756	1.5512
	10	0.0710	0.0739	0.0781	0.1406	0.7029	1.4059
2	0	2.0101	2.0520	2.1082	2.8284	6.3246	8.9443
	1	0.6155	0.6283	0.6455	0.8660	1.9365	2.7386
	2	0.3600	0.3675	0.3775	0.5065	1.1326	1.6017
	3	0.2543	0.2596	0.2667	0.3578	0.8000	1.1314
	4	0.1965	0.2006	0.2061	0.2765	0.6183	0.8745
	5	0.1601	0.1635	0.1680	0.2253	0.5039	0.7126
	6	0.1351	0.1379	0.1417	0.1901	0.4252	0.6013
	7	0.1169	0.1193	0.1226	0.1645	0.3677	0.5201
	8	0.1030	0.1051	0.1080	0.1449	0.3240	0.4582
	9	0.0920	0.0939	0.0965	0.1295	0.2895	0.4094
	10	0.0832	0.0849	0.0872	0.1170	0.2617	0.3701
3	0	3.0101	3.0517	3.1072	3.7798	6.4633	8.1433
	1	0.7041	0.7139	0.7268	0.8842	1.5119	1.9049
	2	0.3965	0.4020	0.4093	0.4979	0.8515	1.0728
	3	0.2759	0.2797	0.2848	0.3465	0.5925	0.7465
	4	0.2116	0.2145	0.2184	0.2657	0.4543	0.5724
	5	0.1715	0.1739	0.1771	0.2154	0.3684	0.4641
	6	0.1443	0.1463	0.1489	0.1811	0.3098	0.3903
	7	0.1245	0.1262	0.1285	0.1563	0.2672	0.3367
	8	0.1094	0.1110	0.1130	0.1374	0.2350	0.2961
	9	0.0977	0.0990	0.1008	0.1226	0.2097	0.2642
	10	0.0882	0.0894	0.0910	0.1107	0.1893	0.2385
4	0	4.0101	4.0516	4.1068	4.7568	7.1131	8.4590
	1	0.7594	0.7673	0.7777	0.9008	1.3470	1.6019
	2	0.4179	0.4223	0.4280	0.4958	0.7413	0.8816
	3	0.2883	0.2913	0.2952	0.3419	0.5113	0.6081
	4	0.2200	0.2223	0.2253	0.2610	0.3902	0.4641
	5	0.1779	0.1797	0.1822	0.2110	0.3155	0.3752
	6	0.1493	0.1508	0.1529	0.1771	0.2648	0.3149
	7	0.1286	0.1300	0.1317	0.1526	0.2282	0.2713
	8	0.1130	0.1142	0.1157	0.1340	0.2004	0.2383
	9	0.1007	0.1018	0.1032	0.1195	0.1787	0.2125
	10	0.0909	0.0918	0.0931	0.1078	0.1612	0.1917

5	0	5.0101	5.0516	5.1065	5.7435	7.9245	9.1028
	1	0.7972	0.8038	0.8125	0.9139	1.2609	1.4484
	2	0.4320	0.4356	0.4403	0.4952	0.6833	0.7849
	3	0.2962	0.2987	0.3019	0.3396	0.4686	0.5382
	4	0.2254	0.2273	0.2297	0.2584	0.3565	0.4095
	5	0.1819	0.1834	0.1854	0.2085	0.2877	0.3305
	6	0.1525	0.1537	0.1554	0.1748	0.2412	0.2770
	7	0.1313	0.1323	0.1338	0.1505	0.2076	0.2385
	8	0.1152	0.1162	0.1174	0.1321	0.1822	0.2093
	9	0.1027	0.1035	0.1046	0.1177	0.1624	0.1865
	10	0.0926	0.0933	0.0944	0.1061	0.1464	0.1682

Table 2 Parametric values of Bayesian Modified Chain Sampling Plan

s	$\bar{p}$ / i	$n\mu_1$	$n\mu_2$	$n\mu_*$	$n\mu_m$	$nOAOQL$	μ_2/μ_1	$OAOQL/\mu$	$OAOQL/\mu_*$
1	2	0.2947	2.8000	0.066	0.4801	0.1441	9.5012	0.4890	2.1614
	3	0.2148	2.0408	0.0714	0.367	0.1140	9.5010	0.5307	1.5966
	4	0.1689	1.6049	0.066	0.297	0.0941	9.5023	0.5571	1.4114
	5	0.1392	1.3223	0.060	0.250	0.0800	9.4994	0.5747	1.3201
	6	0.1183	1.1243	0.054	0.2154	0.0696	9.5035	0.5879	1.2668
	7	0.1029	0.9778	0.050	0.1891	0.0615	9.5022	0.5977	1.2300
	8	0.0911	0.8651	0.045	0.1686	0.0551	9.4956	0.6048	1.2031
	9	0.0816	0.7756	0.0421	0.1521	0.0499	9.5052	0.6115	1.1853
	10	0.0739	0.7029	0.038	0.1385	0.0456	9.5121	0.6171	1.1722
2	2	0.3675	1.1326	0.095	0.442	0.5474	3.0819	1.4895	5.7477
	3	0.2596	0.8000	0.1000	0.343	0.4284	3.0817	1.6502	4.2840
	4	0.2006	0.6183	0.092	0.280	0.3509	3.0823	1.7493	3.8014
	5	0.1635	0.5039	0.083	0.236	0.2969	3.0820	1.8159	3.5628
	6	0.1379	0.4252	0.303	0.204	0.2572	3.0834	1.8651	0.8488
	7	0.1193	0.3677	0.068	0.1805	0.2268	3.0821	1.9011	3.3264
	8	0.1051	0.3240	0.062	0.1613	0.2028	3.0828	1.9296	3.2593
	9	0.0939	0.2895	0.0571	0.1458	0.1833	3.0831	1.9521	3.2078
	10	0.0849	0.2617	0.052	0.1331	0.1673	3.0824	1.9706	3.1694
3	2	0.4020	0.8515	0.1111	0.432	1.4738	2.1182	3.6662	13.2642
	3	0.2797	0.5925	0.1154	0.3371	1.0744	2.1183	3.8413	9.3115
	4	0.2145	0.4543	0.1059	0.276	0.8436	2.1179	3.9329	7.9673
	5	0.1739	0.3684	0.095	0.233	0.6938	2.1185	3.9896	7.2849
	6	0.1463	0.3098	0.085	0.202	0.5889	2.1176	4.0253	6.8705
	7	0.1262	0.2672	0.077	0.1791	0.5115	2.1173	4.0531	6.5927
	8	0.1110	0.2350	0.070	0.1604	0.4519	2.1171	4.0712	6.3912
	9	0.0990	0.2097	0.064	0.1451	0.4048	2.1182	4.0889	6.2407
	10	0.0894	0.1893	0.059	0.1326	0.3665	2.1174	4.0996	6.1219

4	2	0.4223	0.7413	0.1212	0.428	2.4655	1.7554	5.8383	20.3404
	3	0.2913	0.5113	0.1250	0.334	1.7838	1.7552	6.1236	14.2704
	4	0.2223	0.3902	0.1143	0.274	1.3920	1.7553	6.2618	12.1800
	5	0.1797	0.3155	0.1026	0.2333	1.1393	1.7557	6.3400	11.1082
	6	0.1508	0.2648	0.092	0.202	0.9633	1.7560	6.3879	10.4518
	7	0.1300	0.2282	0.083	0.1791	0.8339	1.7554	6.4146	10.0068
	8	0.1142	0.2004	0.075	0.1604	0.7350	1.7548	6.4361	9.6863
	9	0.1018	0.1787	0.069	0.1452	0.6569	1.7554	6.4528	9.4429
	10	0.0918	0.1612	0.064	0.1328	0.5937	1.7560	6.4673	9.2518
5	2	0.4356	0.6833	0.1282	0.4261	65.4602	1.5686	150.2759	510.5896
	3	0.2987	0.4686	0.1316	0.3338	45.9706	1.5688	153.9022	349.3766
	4	0.2273	0.3565	0.1200	0.274	35.1821	1.5684	154.7827	293.1842
	5	0.1834	0.2877	0.1075	0.2332	28.4033	1.5687	154.8708	264.1507
	6	0.1537	0.2412	0.096	0.202	23.7741	1.5693	154.6786	246.2997
	7	0.1323	0.2076	0.087	0.1792	20.4222	1.5692	154.3628	234.1746
	8	0.1162	0.1822	0.079	0.1607	17.8877	1.5680	153.9389	225.3850
	9	0.1035	0.1624	0.072	0.1456	15.9065	1.5691	153.6860	218.7144
	10	0.0933	0.1464	0.0671	0.1331	14.3166	1.5691	153.4469	213.4764

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Associate Professor, Government Arts College, Udumalpet, Tirupur, drlathamadan@gmail.com

P.hD, Research Scholar, Karpagam University, Coimbatore, mrajimay7@gmail.com