CREEPING FLOW OF MICROPOLAR FLUID PAST A FLUID SPHERE

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Abstract: The Paper concerns the creeping flow of micropolar fluid past a fluid sphere, assuming uniform stream far away from the body along its axis of symmetry. For outside the fluid sphere we consider micropolar fluid and inside the fluid sphere we consider Stokes flow. The stream function is determined by matching the solution of micropolar field equation for the flow outside the fluid sphere with that of the Stokes equation for the flow inside the fluid sphere. Two known boundary condition are considered No spin and spin boundary condition. The drag force experienced by the fluid sphere is determined. The variation of drag for different values of the permeability parameter(η), the coupling number N and the micropolar parameter (m) is studied. Some well-known result then deduced as a limiting case from present analysis.

Keywords: Drag force ,Micropolar fluid, Modified Bessel function

Introduction : The model of micropolar fluid is introduced by Eringen. They suggested that the theory might serve as a satisfactory model for a description of the flow properties of polymeric fluids and possibly animal fluid, for which the Navier-Stokes theory is not good enough. In the theory of micropolar fluids, rigid particles contained in a small volume element can rotate about the centroid of the volume element.The micropolar fluids can support couple stress and body couples only. Physically they may represent adequately the fluid consisting of dipole elements.

Many researchers have made the useful investigation in the field of micropolar fluid with different geometries. Rao & Rao studied "the slow stationary flow of a micropolar liquid past a sphere". In this paper he concludes that the drag of sphere is more in comparison to micropolar fluid to non polar fluids. he also say that streamlines in the polar case have greater deflection towards the sphere than in the non polar(classical)case. the creeping flow of micropolar fluid past a rigid sphere,fluid sphere, approximate sphere and porous sphere have been considered by Rao & Rao, Ramkisson, Iyenger & Srinivascharya and Srinivascharya & Rajlakshmi respectively.they evaluate the drag force experienced by the corresponding body. Drag on a sphere in micropolar fluids with non-zero spin boundary condition for micro-rotations was studied by Hoffman et al.[7] and he extended the Stokes formula for the resistance force exerted on a sphere moving with constant velocity in a fluid for micropolar fluids.

In this note we consider the creeping flow of micropolar fluid past a fluid sphere, assuming uniform stream far away from the body along its axis of symmetry. For outside the fluid sphere we consider micropolar fluid and inside the fluid sphere we consider Stokes flow. The stream function is determined by matching the solution of micropolar field equation for the flow outside the fluid sphere with that of the Stokes equation for the flow inside the fluid sphere. Two known boundary condition are considered and compared. No spin and spin boundary condition. The drag force experienced by the fluid sphere is determined.

1. Micropolar field equations:

The field equations of the micropolar fluid dynamics (Eringen [1, 2]) are

$$\frac{\partial \rho}{\partial t} + div \rho \, \mathbf{v} = 0\,, \qquad (1.1)$$

$$\rho \frac{d\mathbf{v}}{dt} = \rho \mathbf{f} - \nabla p + \kappa \nabla \times \boldsymbol{\omega} - (\mu+\kappa)\nabla \times \nabla \times \mathbf{v} + (\lambda + 2\mu + \kappa)\nabla(div\,\mathbf{v}) \qquad (1.2)$$

$$\rho J \frac{d\boldsymbol{\omega}}{dt} = \rho \mathbf{l} - 2\kappa\boldsymbol{\omega} + \kappa \nabla \times \mathbf{v} - \gamma \nabla \times \nabla \times \boldsymbol{\omega} + (\alpha+\beta+\gamma)\nabla(div\,\boldsymbol{\omega}) \qquad (1.3)$$

where ρ is the density, $\mathbf{v}$ the velocity field, $\boldsymbol{\omega}$ the micro-rotation field, J the gyration parameter, $\mathbf{f}$ body forces per unit mass, $\mathbf{l}$ micro-rotation driving forces per unit mass, p the hydrostatical pressure, μ the classical viscosity coefficient, κ, λ the vortex viscosity coefficient and α, β, γ are gyroviscosity coefficients satisfying the following inequalities

$$3\alpha+\beta+\gamma \ge 0\,, \quad 2\mu+\kappa \ge 0\,,\ 3\lambda + 2\mu + \kappa \ge 0\,, \quad \gamma \ge |\beta|\,, \quad \kappa \ge 0\,,\ \gamma \ge 0\,. \qquad (1.4)$$

Let us consider a uniform, axi-symetric slow viscous flow of an unbounded incompressible micropolar fluid past a Newtonian fluid sphere. The governing equations for creeping flow around and through the fluid sphere written for two regions separated by the interface. The flow of fluid for outside region is assumed to be Stokesian, i.e. it is assumed that the inertial terms in the momentum equation and bilinear terms in balance of first stress moments can be neglected. Further, for outside region (1) we assume that the body force and body couple terms are absent. Therefore, the governing equations for outside flow are given by

$$div\,\mathbf{v}^{(1)} = 0, \quad (1.5)$$

$$-\nabla p^{(1)} + \kappa\nabla\times\boldsymbol{\omega}^{(1)} - (\mu_1+\kappa)\nabla\times\nabla\times\mathbf{v}^{(1)} = 0, \quad (1.6)$$

$$-2\kappa\boldsymbol{\omega}^{(1)} + \kappa\nabla\times\mathbf{v}^{(1)} - \gamma\nabla\times\nabla\times\boldsymbol{\omega}^{(1)} + (\alpha+\beta+\gamma)\nabla(\nabla. \quad (1.7)$$

For the region (2), inside the fluid sphere governed by Stokes equations (Happel and Brenner, [10]) as

$$\mu_2\nabla^2\mathbf{v}^{(2)} = \nabla p^{(2)}, \qquad div\,\mathbf{v}^{(2)} = 0. \quad (1.8)$$

Stream Function Formulation:

Since, the flow generated is axially symmetric; all the flow functions are independent of azimuthal angle ϕ. Hence the velocity and microrotation can be chosen in the spherical polar coordinates (r,θ,ϕ) as

$$\mathbf{v}^{(i)} = v_r^{(i)}(r,\theta)\hat{e}_r + v_\theta^{(i)}(r,\theta)\hat{e}_\theta \quad (2.1)$$

and $$\boldsymbol{\omega}^{(i)} = \upsilon_\phi^{(i)}(r,\theta)\hat{e}_\phi. \quad (2.2)$$

To non-dimensionalize the equations and variables, we put

$$r = a\tilde{r}, \quad \psi^{(i)} = Ua^2\tilde{\psi}^{(i)}, \quad p^{(i)} = \frac{\mu U}{a}\tilde{p}^{(i)},$$

$$\upsilon_\phi^{(i)} = \frac{U}{a}\tilde{\upsilon}_\phi^{(i)}$$

and dropping tildes subsequently in further analysis. Introducing the stream functions for both regions through

$$v_r^{(i)} = -\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta}; \qquad v_\theta^{(i)} = \frac{1}{r\sin\theta}\frac{\partial\psi^{(i)}}{\partial r}.$$

$$i = 1,2. \quad (2.3)$$

These velocity components satisfy the equations of continuity. Eliminating pressure from equation (1.8) and using equation (2.3), we get

$$E^4\psi^{(1)} - NE^2(\mathrm{r}\sin\theta\,\upsilon_\phi^{(1)}) = 0. \quad (2.4)$$

Using above equation (2.4) in equation (1.9), we find that

$$\upsilon_\phi^{(1)} = \frac{1}{2r\sin\theta}[E^2\psi^{(1)} + \frac{2-N}{Nm^2}E^4\psi^{(1)}]. \quad (2.5)$$

Therefore, the stream function formulation for the outside flow can be found by eliminating $\upsilon_\phi^{(1)}$ from equations (2.4) and (2.5) as

$$E^4(E^2 - m^2)\psi^{(1)} = 0. \quad (2.6)$$

Similarly, eliminating the pressure from equations (1.10) and using equations (2.3), we obtain the following equation as

$$\mathrm{E}^2(\mathrm{E}^2\psi^{(2)}) = 0, \quad (2.7)$$

where $E^2 = \frac{\partial^2}{\partial r^2} + \frac{(1-\zeta^2)}{r^2}\frac{\partial^2}{\partial\zeta^2}$, $\zeta = \cos\theta$,

$m^2 = \frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}a^2$ and $N = \frac{\kappa}{\mu+\kappa}$ being the coupling number $(0 \le N < 1)$.

the general solution which is regular on symmetry axis of the equation (2.6) which satisfies the uniform condition at infinity $\psi_\infty(r,\varsigma) = r^2 G_2(\zeta)$ and requirement for spherical case will be

$$\psi^{(1)}(r,\zeta) = [r^2 + A_2 r^{-1} + B_2 r + C_2\sqrt{r}K_{3/2}(mr)]G_2(\zeta) \quad (2.9)$$

Substituting the value of $\psi^{(1)}$ from equation (2.9) in equation (2.5), we get microrotation components, respectively as

$$\upsilon_\phi^{(1)}(r,\zeta) = \frac{1}{r\sin\theta}[-B_2 r^{-1} + \frac{m^2(\mu_1+\kappa)}{\kappa}C_2\sqrt{r}K_{3/2}(mr)]G_2(\zeta) \quad (2.10)$$

For the inside region of the fluid sphere, the regular solution of Stokes equation (Happel and Brenner [12]) will be

$$\psi^{(2)}(r,\zeta) = [A_2^* r^2 + D_2^* r^4]G_2(\zeta). \quad (2.11)$$

Boundary Conditions:

The boundary conditions those are physically realistic and mathematically consistent for this proposed problem can be taken as:

The kinematical conditions of mutual impenetrability at the surface requires that we take

$$\psi^{(1)} = 0 \quad \text{on} \quad r = 1, \quad (3.1)$$

$$\psi^{(2)} = 0 \quad \text{on} \quad r = 1. \quad (3.2)$$

Also, since the tangential velocity is continuous across the surface, so that we may take

$$\frac{\partial\psi^{(1)}}{\partial r} = \frac{\partial\psi^{(2)}}{\partial r} \quad \text{on} \quad r = 1. \quad (3.3)$$

Now from the theory of interfacial tension, the presence of interfacial tension only produces a discontinuity in the normal stress T_{rr} and does not in anyway affect the tangential stress $T_{r\theta}$. Therefore, the latter is continuous across the surface and so that we may take

$$T_{r\theta}^{(1)} = T_{r\theta}^{(2)} \quad \text{on} \quad r = 1. \quad (3.4)$$

For No spin boundary condition:

$$\upsilon_\phi^{(1)} = 0$$

For spin boundary condition:

The micro-rotation on the boundary of the sphere is assumed proportional to the rotation rate of the velocity field on the boundary (Lukaszewicz, [14]), i.e.

$$\boldsymbol{\omega}^{(1)} = \frac{\tau}{2} curl\, \mathbf{v}^{(1)}$$

which on simplification provides

$$\upsilon_{\phi}^{(1)} = \frac{\tau}{2r\sin\theta} E^2 \psi^{(1)}, \qquad \text{on } r = 1. \quad (3.5)$$

Additionaly, we have the regularity conditions at infinity and the condition that velocity and pressure must be nonsingular everywhere in the flow of field

i.e $\psi^{(1)} = \frac{1}{2} r^2 \sin^2\theta$ as $r \to \infty$ (3.6)

4.Evaluation of the drag force:

The drag force experienced by a fluid sphere can be evaluated by applying the elegant formula provided by Ramkissoon and Mazumdar [4]

$$D = 4\pi(2\mu_1 + \kappa)Ua \lim_{r\to\infty} \left[\frac{r(\psi^{(1)} - \psi_{\infty})}{\varpi^2} \right], \quad (4.1)$$

For non zero spin boundary condition

Thus for present case, substituting these above values in (4.1) and taking the limit, we found that

$$D = 2\pi(2\mu_1 + \kappa)UaB_2$$
$$= -\frac{3\pi Ua(2\mu_1 + \kappa)[(1+m)\{\kappa(-2+\tau) - 2\mu_1\}(\kappa + 2\mu_1 + 3\mu_2)]}{\Delta_1} \quad (4.2)$$

Hence, the non dimensional drag $D_N = D/(-2\pi\mu_1 Ua)$ is given by

$$D_N = -B_2(2-N)/(1-N).$$

$$= \frac{3(2-N)}{2(1-N)} \frac{[(1+m)\{\kappa(-2+\tau) - 2\mu_1\}(\kappa + 2\mu_1 + 3\mu_2)]}{\Delta_1}, \quad (4.3)$$

$$\Delta_1 = 2[\{-6(-1+m)\mu_1^2 + \mu_1\{\kappa(-7-9m+2m\tau) - 6(-1 \ldots + \kappa\{\kappa(-2+m(-3+\tau)) + 3(-1+m(-2+\tau))\mu_2\}\}]$$

For zero spin boundary condition:

$$D = -\frac{6\pi Uam^2(2\mu_1 + \kappa)(\mu_1 + \kappa)[(1+m)(\kappa + 2\mu_1 + 3\mu \ldots}{\Delta_2}, \quad (4.4)$$

The following special results can be deduced immediately:

Case I: Drag on a fluid sphere embedded in another fluid:

If $\kappa \to 0$, i.e. $m \to 0$ then micropolar fluid becomes a Newtonian fluid. Therefore, the drag force on the fluid sphere by equation (4.3) reduces to

$$D = -6\pi\mu_1 Ua \frac{(1 + \frac{2}{3}\eta)}{(1+\eta)}, \quad (4.6)$$

where $\eta = \mu_1/\mu_2$ being the viscosity ratio.

This result agrees with the result reported earlier in [11, 12] for the drag force experienced by a fluid sphere.

Case II: If $\eta \to 0$, then fluid sphere behaves like a solid sphere, then the drag force from the equation (4.6) comes out as

$$D = -6\pi\mu_1 Ua. \quad (4.7)$$

This well-known result for the drag experienced by a rigid sphere in an unbounded medium is reported by Stokes.

Case III: if $\mu_2 << \mu_1$ *i.e* $\eta \to \infty$ then fluid sphere behaves like a gaseous spherical bubble, in this case drag force can be found as

$$D = -4\pi\mu_1 Ua. \quad (4.8)$$

This result is identical to that previously given for a sphere at whose surface perfect slip occurs (Happel and Brenner [12]).

Conclusion:

The dependence of the drag force D for a fluid sphere embedded in a micropolar fluid on spin parameter τ is depicted in figure-1 for various values of vortex viscosity coefficient κ and micropolar parameter m. It shows that the drag decreases as spin parameter τ increases and increases with increasing values of vortex viscosity coefficient κ and micropolar parameter m. The variation of drag D on vortex viscosity coefficient κ for various values of τ and m is shown in figure -2. It can be observed that the drag force increases linearly on vortex viscosity coefficient κ for different values of τ and m.

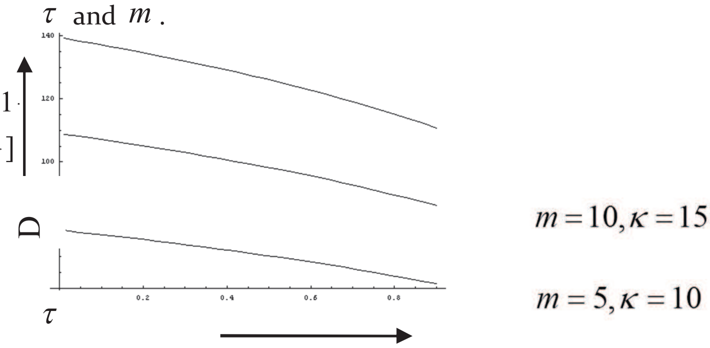

Figure-1: The variation of drag force D on τ for various values of vo rtex viscosity coefficient κ and m,

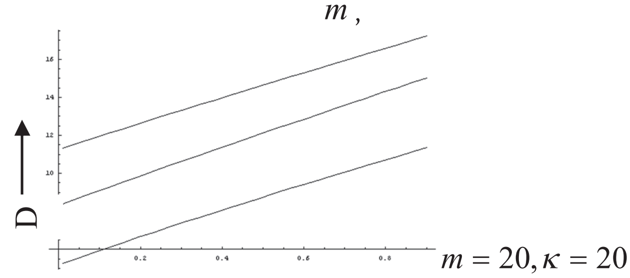

Figure-2: The variation of drag D on vortex viscosity coefficient κ for various values of τ and m.

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