A REVIEW ON BAYESIAN CHAIN SAMPLING PLAN

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Abstract: Acceptance Sampling Plans by attributes involving binomial distribution and the process average or fraction non-conformities following a Beta distribution. A formula to compute the Average Probability of Acceptance of chain sampling plans (ChSP-1) by attributes under Beta-Binomial distribution is provided. The performance/discriminating power of Beta-Binomial sampling plans is also discussed by determining the operating characteristic curve. These probabilities are compared with conventional probabilities.

Key Words: Acceptance Sampling Plan, Bayesian Chain Sampling, Plan, Beta Binomial distribution, Average Probability of Acceptance

Introduction To Acceptance Sampling:

According to schilling (1984) "In a very real sense, the goal of acceptance sampling is to reduce the need for acceptance sampling" A sampling plan indicates the number of units of product from each lot or batch which are to be inspected (sample size or series of sample sizes) and the criteria for determining the acceptability of the lot or batch (acceptance and rejection numbers).

Vardeman (1986) states that Acceptance Sampling is best used in situations either where control charts cannot be applied or where special causes still abound. In particular acceptance sampling is of value.

- ❖ For processes that are not yet in control.
- ❖ As a means of production against gross production mistakes.
- ❖ For volatile processes with uneven quality.
- ❖ For correcting problems in already created lots in order to meet specified quality levels.
- ❖ For low yield processes or first article inspection.

Bayesian statistics: Classical statistics is directed towards the use of sample information. In addition to the sample information two other types of information are typically relevant. The first is knowledge of the possible consequences of the decision and the second source of non-sample information is prior information. Thomas Baye's (1702-1761) was first to use the prior information in inductive inference and the approach to statistics, which formally seeks to utilize prior information, is called Bayesian analysis. Bayesian Acceptance sampling:

Bayesian Acceptance sampling Approach is associated with utilization of prior process history for the selection of distribution (Viz., Gamma-Poisson, Beta-Binomial distribution) to describe the random fluctuations involved in Acceptance Sampling. Bayesian Sampling plans requires the user to specify explicitly the distribution of defectives from lot to lot.

The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior because it is formulated prior to the taking of sequence of samples. The combination of prior knowledge represented with the prior distribution and the empirical knowledge based on the sample leads to the decision of the lot.

Beta Binomial distribution.

Let 'x' be the outcome of 'n' Bernoulli trials with a fixed probability 'p' and let the corresponding binomial probability be denoted by

$$b(x,n,p) = nc_x\, p^x\, q^{n-x}$$

Assuming that 'p' has a prior distribution with density w (p), the marginal distribution of x, the mixed binomial distribution is given as

$$b_w(x,n) = \int_0^1 b(x,n,p)\; w(p)\, dp$$

Lauer (1978) has studied Acceptance probability for sampling inspection by Attributes with Beta prior distribution for single sampling plan.

Chain sampling plan (ChSP-1)

Sampling inspection in which the criteria for acceptance and non-acceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in chain sampling plans. Chain sampling plan (ChSP-1) is proposed by Dodge (1955) makes use of cumulative results of several samples helps to overcome the short comings of the single sampling plan.

Conditions for application of ChSP-1

The cost of destructiveness of testing is such that a relatively small sample sizes necessary, although others factors make a large sample desirable.

The product comprises a series of successive lots produced by a continuing process.

Normally lots are expected to be of essentially the same quality.

The consumer has faith in the integrity of the producer.

Operating procedure:

The plan is implemented in the following way.

For each lot, select a sample of 'n' units and test each unit for conformance to the specified requirements.

Accept the lot if'd' (the observed number of defectives) is zero in the series of sample of size

'n'units and reject if d >1.
Accept the lot if'd' is equal to 1 and if no defectives are found in the immediately preceding 'i'series of sample of size 'n'.
Dodge (1955) has given the operating characteristics function of ChSP-1 as

$$P_a(p) = P_o + P_1 (P_o)^i$$

The chain sampling plan is characterized by the parameters n & i where 'n' is the sample size and 'i' is the number of preceding samples with zero defectives.

Latha (2002) has studied the average probability of Acceptance for chain sampling plan with gamma prior distribution. Rajagopal, A. Loganathan and Vijayaraghavan (2009) have studied the selection of Bayesian Single sampling plans with Beta distribution as prior distribution.

Bayesian chain sampling plan:

According to Dodge (1955) the operating characteristic function of ChSP-1 is

$$Pa\ (p) \quad = \quad P_0 \ + \ P_1 \ (P_0)i$$

The Probability of Acceptance of BChSP-1 is based on binomial distribution is provided as

$$P\left(n,\frac{i}{p}\right) = q^n + npq^{n-1}\, q^{ni}$$

Using the past history of inspection, it is observed that 'p' follows a Beta distribution with density function

$$\text{w (p)} = \frac{1}{\beta(s,t)} \int_0^1 p^{s-1}\,(1-p)^{t-1}dp$$

The Average Probability of Acceptance is given by

$$\overline{P} = \int_0^1 p(n,i/p)\ \text{w (p)}\ dp$$

$$\overline{P}\,\frac{1}{\beta(s,t)}\left(\ \beta\ (s,n+t) + n\beta\ (s+1,\ n(1+i) + t-1)\ \right)$$

The Average probability of acceptance for Bayesian Chain Sampling Plan (BChSP-1) using Beta Binomial distribution is given by the above function

If i=1,s=1, $\overline{P}$ can be reduced to

$$\overline{P} = \frac{(1-\mu)}{(n\mu+1-\mu)} + \frac{n\mu(1-\mu)}{(2n\mu+1-\mu)\ (2n\mu+1-2\mu)} \qquad \text{where} \quad \mu = s\ /\ s+t$$

if i = 1, s = 2, then $\overline{P}$ reduced to

$$\overline{P} = \frac{(2-2\mu)(2-\mu)}{(n\mu+2-\mu)\ (n\mu+2-2\mu)} + \frac{2n\mu(2-2\mu)\ (2-\mu)}{(2n\mu+2-\mu)\ (2n\mu+2-2\mu)(2n\mu+2-3\mu)}$$

If i=1, s=3, then $\overline{P}$ reduces to

$$\overline{P} = \frac{(3-3\mu)(3-2\mu)(2-\mu)}{(n\mu+s-\mu s+2\mu)\ (n\mu+s-\mu s+\mu)(n\mu+s-\mu s)} + \frac{3n\mu(3-3\mu)\ (3-2\mu)(3-\mu)}{(2n\mu+3-\mu)\ (2n\mu+3-2\mu)(2n\mu+3-3\mu)(2n\mu+3-4\mu)}$$

If i = 1, s = 4, then $\overline{P}$ reduced to
In the similar way

$$\overline{P} = \frac{(4-4\mu)(4-3\mu)(4-2\mu)(4-\mu)}{(n\mu+4-\mu)\ (n\mu+4-2\mu)(n\mu+4-3\mu)(n\mu+4-4\mu)} + \frac{4n\mu(4-4\mu)\ (4-3\mu)(4-2\mu)(4-\mu)}{(2n\mu+4-\mu)\ (2n\mu+4-2\mu)(2n\mu+4-3\mu)(2n\mu+4-4\mu)(2n\mu+4-5\mu)}$$

i = 2, s=1, then $\overline{P}$ reduces to

$$\overline{P} = \frac{(1-\mu)}{(n\mu+1-\mu)} + \frac{n\mu(1-\mu)}{(3n\mu+1-\mu)\ (3n\mu+1-2\mu)}$$

i = 2, s =2, $\overline{P}$ reduces to

$$\overline{P} = \frac{(2-2\mu)(2-\mu)}{(n\mu+2-2\mu)(n\mu+2-\mu)} + \frac{2n\mu(2-2\mu)(2-\mu)}{(3n\mu+2-\mu)\ (3n\mu+2-2\mu)(3n\mu+2-3\mu\mu)}$$

i =2, s=3, the $\overline{P}$ reduces to

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In the similar way

i = 3, s = 1, $\overline{P}$ reduces to

$$\overline{P} = \frac{(1-\mu)}{(n\mu+1-\mu)} + \frac{n\mu(1-\mu)}{(4n\mu+1-\mu)\ (4n\mu+1-2\mu)}$$

i = 3, s = 2, $\overline{P}$ reduced to

$$\overline{P} = \frac{(2-2\mu)(2-\mu)}{(n\mu+2-2\mu)(n\mu+2-\mu)} + \frac{2n\mu(2-2\mu)(2-\mu)}{(4n\mu+2-\mu)\ (4n\mu+2-2\mu)(4n\mu+2-3\mu)}$$

i = 3, s = 3, $\overline{P}$ reduces

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Comparison with conventional plans

The values obtained in BChSP-1 are compared with conventional sampling plan.

Illustration-1

For i = 1, s = 1, n= 100, the average probability of acceptance in BChSP-1 is 0.9785 where as in conventional plan it is 0.98675

Illustration-2

For i = 2, s = 1, n=100 the average probability of acceptance in BChSP-1 is 0.96825 where as in conventional plan it is 0.97893

Illustration-3

For i = 3, s = 1, n=100 the average probability of acceptance in BChSP-1 is 0.96009 where as in conventional plan it is 0.97188

On comparison it is observed that Bayesian chain sampling plan is more advantageous to the consumer than the conventional sampling plan.

Conclusion: Bayesian Acceptance Sampling is a technique which deals with procedures in which decision to accept or reject the lot or process is based in the examination of past history or knowledge of samples. The present work mainly relates to the construction of probability of acceptance of chain sampling with beta binomial distribution and which are compared to the conventional sampling plan. Bayesian chain Sampling Plan using Beta-Binomial distribution more accurate and advantageous to the consumer.

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