DETERMINATION OF SURFACE PRESSURE OF AN AXISYMMETRIC OGIVE IN HYPERSONIC FLOW

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Abstract: The application of Ghosh’s hypersonic equivalence principle shows that the analysis of flow in the direction transverse to a near planar surface is sufficient to get the solution for the entire flow. Ghosh ‘s large deflection similitude is used to find out the pressure distribution over an ogive at zero angle of attack and the results have been compared with the results of Rossow’s method of rotational characteristics and the tangent cone method. Both show good agreement with each other.

Keywords: Ogive, Surface Pressure, Tangent cone , zero Angle of attack

Introduction

The study of hypersonic flow over an oscillating wedge is a step towards solving various problems concerned with the current interest in the space shuttle programme and re-entry vehicle. Theoretical study of unsteady flow with a small perturbation approach shows good agreement generally with the experimental results. Since the capital investment for the experiments is considerably high, it is quite logical to deal with the problem theoretically rather than perform capital and time consuming experiments.

In the analysis of incompressible, subsonic, transonic, and supersonic flows, simplified theory based on the assumption of small disturbances due to thin bodies has proved to be of great value. It is also true that a hypersonic small-disturbance theory is of considerable value in various applications. An adequate description of the in viscid hypersonic flow past a thin body requires non-linear equations. The hypersonic small disturbance theory is inherently connected with classical hypersonic similitude. The establishment of hypersonic similitude does not provide hypersonic flow solutions. The appropriate equations of motion must be written down and solutions obtained.

Ghosh & Mistry [2] have given a theory for the quasi steady flow over the oscillating 2-D wedge which has been further extended by Ghosh [3] to solve the axi- symmetric flow problem. Appleton [1] has analyzed the 2- D oscillating wedge problem where he has included the secondary wave effects. In this paper Ghosh’s large deflection hypersonic similitude [3] has been used to estimate the pressure distribution over an axi-symmetric ogive.

Method For Ogive Surface Pressure Determination

The work consists of the application of Ghosh’s [3] large- deflection hypersonic similitude for the estimation of the pressure distribution over the axi-symmetric ogive and the comparison of this quasi steady result with the results of tangent cone approximation [6] and J. Rossow’s methods of characteristics [7].

In Ghosh’s work [3], an axially symmetric piston motion in a conico- annular space has been shown to be equivalent to an axi-symmetric body of large nose angle in hypersonic flow. The partial differential equations for the piston driven 1-D unsteady gas motion are reduced to a set of ordinary differential equations by introducing similarity variables. The wedge and cone flow solutions obtained predict constant density in the shock layer. The density ratios are given in closed form as function of piston Mach number, dispensing with the successive approximation procedure of the earlier constant density theory.

By applying the unsteady Bernoulli’s equation and considering the boundary conditions at the shock Ghosh [3] has found out the pressure ratio over the cone as

$$\frac{P_b}{P_1} = 1 + \gamma\, M_P^2 \left[1 + \frac{1}{4} \qquad \frac{2+ (\gamma-1)M_P^2}{2+ (\gamma+1)M_P^2}\right]$$

The flow over the ogive is considered quasi steady, as the change of piston velocity with respect to time is very small in comparison to steady state piston velocity, such that the pressure at a point is a function of instantaneous piston Mach number and the piston Mach number is the function of the slope at the point under consideration over the ogive.

The idea of tangent cone approximation, as applied to determining the pressure distribution on a pointed un yawed body of revolution in supersonic or hypersonic flow, is to replace each transverse section of the body by a section of circular cone tangent to the body surface. The pressure is then assumed to be that which exists on the cone, at the same free stream Mach number. According to hypersonic similarity, the pressure on the surface of a series of slender un yawed bodies of revolution, with same thickness distribution along their axes, may be written as,

$$\frac{P}{P_1} = G(K_C, \xi), where\ K_C = M_\infty/(l/d)$$

is the hypersonic similarity parameter, l is the body length, d is the body diameter, M_∞ is the undisturbed free stream Mach number, and $\xi = x/l$, where x is the axial distance from the nose.

Lees[7] showed for slender cones when the conical shock is not too far from the cone surface, and when the hypersonic similarity applies, that the surface pressure is given by

$$\frac{P}{P_1} - 1 \frac{2\gamma}{\gamma+1}(K_C^2 - 1) + \gamma (K_S - K_C)^2 \left[\frac{\gamma+1}{\gamma-1+(2/K_S^2)}\right]$$

Where

$$K_S = \left(\frac{\gamma+1}{\gamma+3}\right) K_C \pm \sqrt{\frac{(\gamma+1)^2}{(\gamma+3)} K_C^2 + \frac{2}{\gamma+3}}$$

Where $K_C = M_\infty/(l/d)$ is the hypersonic similarity parameter, these relations were obtained by considering the limiting forms of the Taylor series expansions for the solution of the differential equation for conical flow behind the shock, the boundary conditions at the shock surface and the oblique shock relation. As the flow properties are constant along the rays the pressure distribution for different cones corresponding to different points on the ogive can be found out by using Taylor series expansion provided the pressure at the shock surface is known.

Another important method taken for comparison is Rossow's method of rotational characteristics. Applicability of the hypersonic similarity rule to pressure distribution which includes the effect of rotation for bodies of revaluation at zero angle of attack has been shown by j. Rossow [7]. Rossow has included the effect of entropy gradient due to the presence of curved shock. Rotation enters the computation when entropy gradient exists normal to the stream lines. The stronger the entropy gradient the greater would be the rotation in the flow field. The entropy gradient results from the curved shock wave. The following equation shows that for a given body in uniform free stream the shock wave angle ψ must change to produce an entropy gradient ΔS which is equal to total head ratio across the shock wave.

$$e^{\frac{-\Delta S}{R}} = F(M_o \sin\psi) = \left[\frac{(\gamma+1)M_0^2\sin^2\psi}{(\gamma-1)M_0^2\sin^2\psi+2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_0^2\sin^2\psi-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

(Where R is the Universal Gas Constant)

The increment of surface velocity due to rotation has been linked with the entropy gradient. These increments are added algebraically to the velocity determined by ignoring rotation and the resulting velocities are then summed to compute a new pressure distribution over the ogive.

The circular arc ogive has been taken for comparison (Fig .1). The equation to the generator of the curve is

$$(x - 1/2)^2 + (y + R_1 - d)^2 = R_1^2$$

$$\theta = \tan^{-1}(1 - x/l)$$

The corresponding values of slopes, θ, have been found out for the following axial positions :X=11,21,31,41,51,61…….

By applying the formula developed by Ghosh the pressure at these points are determined and compared with the results given by-Probstein and Rossow.

Results And Discussion

The results of the extension of Ghosh's [3] large deflection similitude to the axisymmetric ogives of different similarity parameter have been plotted out in Fig. (2) & Fig (3).The results of Rossow's method [7] of rotational characteristics and tangent cone method [6] have been compared with that of the present method in the figures such as Fig (2) & Fig. (3) Fig1: Slope calculation of an axisymmetric ogive

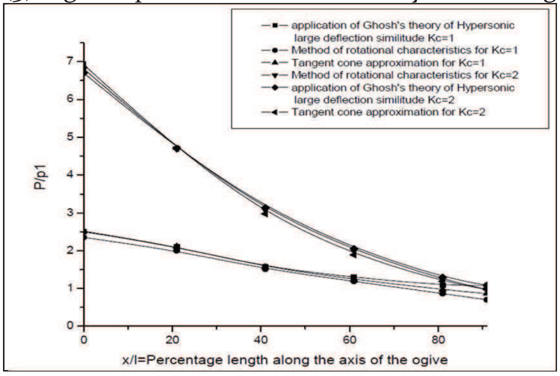

Fig 2: Comparison of Pressure Distribution over an ogive at Zero angle of attack

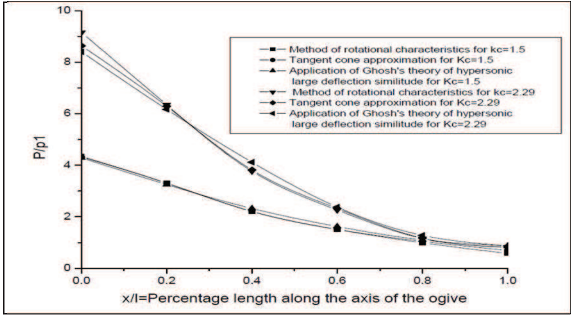

Fig3: Comparison of Pressure distribution over an ogive at zero angle of attack.

Conclusion:

The present theory provides a practical tool in the field of unsteady hypersonic flow for large flow deflection angles. Hui's theory [5] is much more involved and takes large computational time where as the present theory is simple, takes less time and yet gives good results.

The extension of Ghosh's large deflection similitude [4] to the axisymmetric ogive has shown a very good agreement with the established theories like Rossow's method of rotational characteristics [7] and the tangent cone method [6]. In the present method flow is considered as quasisteady flow and the effect of the expansion wave due to the curved surface has been ignored. In the future research the study of inclusion of expansion wave effect may give more appropriate results.

Nomenclature

l = Length of the body

p_1 = Pressure ahead of the Shock .

p = Surface Pressure

p_b = pressure at body

ψ = Shock angle

M_o = Mach number of the unperturbed shock

M_∞ = Undisturbed Free stream Mach number

M_p = Piston Mach number

x = Axial distance from the Nose.

γ = Specific heat ratio.

S = the subscript which Identifies the condition Immediately behind the Shock

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