A NEW CLASS OF CONTRA CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract : In this paper we have introduced intuitionistic fuzzy contra π - generalized semi continuous mappings and some of their basic properties are studied.

Keywords : intuitionistic fuzzy π - generalized semi closed sets,

Introduction : The concept of fuzzy sets was introduced by Zadeh [13] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduce intuitionistic fuzzy contra π - generalized semi continuous mappings and studied some of their basic properties. We arrive at some characterizations of intuitionistic fuzzy contra π - generalized semi continuous mappings.

Preliminaries :

Definition 2.1 : [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by IFS (X).

Definition 2.2 : [1] Let A and B be IFS's of the form
$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and
$B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$. Then
(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$,
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$.
For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in$
(iii) intuitionistic fuzzy regular open set (IFROS in short) if $A = \text{int}(\text{cl}(A))$.
The family of all IFOS (respectively IFSOS, IFαOS, IFROS) of an IFTS (X, τ) is denoted by IFO(X) (respectively IFSO(X), IFαO(X), IFRO(X)).

Definition 2.6 : [7] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an (i) intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$, (ii) intuitionistic fuzzy α-closed set (IFαCS in short) if $\text{cl}(\text{int}(\text{cl}(A)) \subseteq A$,
(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = \text{cl}(\text{int}((A)$. The family of all IFCS (respectively IFSCS, IFαCS, IFRCS) of an IFTS (X, τ) is X}. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3 : [3] An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFS in X satisfying the following axioms:
(a) $0_\sim, 1_\sim \in \tau$
(b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4 : [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by
$\text{int}(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,
$\text{cl}(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.
Note that for any IFS A in (X, τ), we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$.

Definition 2.5 : [7] An IFS $A = \{ \langle x, \mu_A, \nu_A \rangle \}$ in an IFTS (X, τ) is said to be an
(i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$,
(ii) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,
denoted by IFC(X) (respectively IFSC(X), IFαC(X), IFRC(X)).

Definition 2.7 : [12] Let A be an IFS in an IFTS (X, τ). Then $\text{sint}(A) = \cup \{ G / G$ is an IFSOS in X and $G \subseteq A \}$, $\text{scl}(A) = \cap \{ K / K$ is an IFSCS in X and $A \subseteq K \}$.Note that for any IFS A in (X, τ), we have $\text{scl}(A^c)=(\text{sint}(A))^c$ and $\text{sint}(A^c) = (\text{scl}(A))^c$ Definition 2.8:[11] An IFS A in an IFTS (X, τ) is an (i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.
(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Definition 2.9 : [10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Definition 2.10 : [10] An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X. The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition 2.11 : [8] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy ***π***- generalized semi closed set (IF***π***GSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IF***π***OS in (X, τ). if the complement A^c is an IF***π***GSOS in (X, τ).

Result 2.12 : [8] Every IFCS, IFGCS, IFRCS, IFαCS , IFGSCS is an IF***π***GSCS but the converses may not be true in general. Every IFαGCS is IFGSCS but the converse is need not be true.

Definition 2.13 : [9] An IFS A is said to be an intuitionistic fuzzy alpha generalized open set (IFαGOS in short) in X if the complement A^c is an IFαGCS in X.

The family of all IFαGCSs (IFαGOSs) of an IFTS (X, τ) is denoted by IFαGC(X) (IFαGO(X)).

Definition 2.14 : [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in$ IFO(X) for every B ∈ σ.

Definition 2.15 : [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be (i) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B) \in$ IFSO(X) for every B ∈ σ.

(ii) intuitionistic fuzzy α- continuous (IFα continuous in short) if $f^{-1}(B) \in$ IFαO(X) for every B ∈ σ.

(iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B) \in$IFPO(X) for every B∈ σ

Result 2.16 : [7] Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping.

Definition 2.17 : [6] A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy γ continuous (IFγ continuous in short) if $f^{-1}(B)$ is an IFγOS in (X, τ) for every B ∈ σ.

Definition 2.18 : [10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in$ IFGCS(X) for every IFCS B in Y.

Result 2.19 : [10] Every IF continuous mapping is an IFG continuous mapping. A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.21 : [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy contra-continuous (IF C continuous in short) mapping if $f^{-1}(B)$ is an IFOS in X for each IFCS B in Y.

Definition 2.21 : [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be (i)An intuitionistic fuzzy contra α-continuous (IFCα continuous in short) mapping if $f^{-1}(B)$ is an IFαOS in X for each IFCS B in Y.

(ii)An intuitionistic fuzzy contra pre-continuous (IFCP continuous in short) mapping if $f^{-1}(B)$ is an IFPOS in X for each IFCS B in Y.

Result 2.22 : [11] Every IF contra-continuous mapping is an IFCα continuous mapping and IFCP continuous mapping.

Definition 2.23 : [11] A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy contra γ continuous (IFCγ continuous in short) if $f^{-1}(B)$ is an IFγOS in (X, τ) for every B ∈ σ.

Definition 2.24 : [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi T_{1/2}$ (IF$\pi T_{1/2}$ in short) space if every IF***π***GSCS in X is an IFCS in X.

Definition 2.25 : [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi_g T_{1/2}$ (IF $\pi_g T_{1/2}$ in short) space if every IF***π***GSCS in X is an IFGCS in X.

Result 2.26 : [9] (i) Every IF***π***OS is an IFOS in (X, τ). (ii) Every IF***π***CS is an IFCS in (X, τ).

Intuitionistic Fuzzy Contra Π ¬Generalized Semi Continuous Mappings

In this section we have introduced intuitionistic fuzzy contra π - generalized semi continuous mapping and studied some of its properties.Definition 3.1 : A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy contra π - generalized semi continuous (IFCπGS continuous in short) if f -1(B) is an intuitionistic fuzzy π - generalized semi open set in (X, τ) for every intuitionistic fuzzy closed set B of (Y, σ).

Example 3.2 : Let X = {a, b}, Y = {u, v} and G1 = ⟨ x, (0.4, 0.2), (0.6, 0.7) ⟩, G2 = ⟨ y, (0.3, 0.2), (0.7, 0.8) ⟩. Then τ = {0~, G1, 1~} and σ = { 0~, G2, 1~} are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping f: (X, τ) → (Y,σ) by f(a) = u and f(b) = v. Then f is an intuitionistic fuzzy contra π - generalized semi continuous mapping.Theorem 3.3 : Every intuitionistic fuzzy contra continuous mapping is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not conversely.

Proof : Let f: (X, τ) → (Y,σ) be an intuitionistic fuzzy contra continuous mapping. Let A be an intuitionistic fuzzy closed set in Y. By hypothesis, f -1(A) is an intuitionistic fuzzy open set in X. Since every intuitionistic fuzzy open set is an intuitionistic fuzzy ***π*** - generalized semi open set , $f^{-1}(A)$ is an intuitionistic fuzzy ***π*** - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra ***π*** -

generalized semi continuous mapping.

Example 3.4 : Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.1, 0.1), (0.8, 0.6) ⟩, G_2 = ⟨ y, ,(0.1, 0.1),(0.8, 0.6) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The intuitionistic fuzzy set G_2^c = ⟨ y, (0.8, 0.6),(0.1, 0.1) ⟩ is intuitionistic fuzzy closed set Y. Then $f^{-1}(G_2^c)$ is intuitionistic fuzzy contra π - generalized semi open set in X but not intuitionistic fuzzy open set in X. Therefore, f is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not an intuitionistic fuzzy contra-continuous mapping.

Theorem 3.5 : Every intuitionistic fuzzy contra α continuous mapping is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not conversely.

Proof : Let f: (X, τ) → (Y,σ) be an intuitionistic fuzzy contra α continuous mapping. Let A be an intuitionistic fuzzy closed set in Y. Then $f^{-1}(A)$ is an intuitionistic fuzzy α open set in X. Since every intuitionistic fuzzy α open set is an intuitionistic fuzzy π - generalized semi open set (IFπGSOS), $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π - generalized semi continuous mapping.

Example 3.6 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.7, 0.8), (0.3, 0.1) ⟩, G_2 = ⟨ y, (0.5, 0.4), (0.2, 0.3) ⟩. Then τ = {$0_\sim$, G_1,$1_\sim$} and σ = { $0_\sim$, G_2,$1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. In the intuitionistic fuzzy topological space, G_2^c = ⟨ y, (0.2, 0.3) ,(0.5, 0.4) ⟩ is intuitionistic fuzzy closed set in Y. Then $f^{-1}(G_2^c)$ is intuitionistic fuzzy π - generalized semi open set in X but not intuitionistic fuzzy α open set in X. Then f is intuitionistic fuzzy contra π - generalized semicontinuous mapping but not an intuitionistic fuzzy contra α continuous mapping.

Theorem 3.7 : Every intuitionistic fuzzy contra regular continuous mapping is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not conversely.

Proof : Let f: (X, τ) → (Y,σ) be an intuitionistic fuzzy contra regular continuous mapping. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis $f^{-1}(A)$ is an intuitionistic fuzzy regular open set in X. Since every intuitionistic fuzzy regular open set is an intuitionistic fuzzy π - generalized semi open set, $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π - generalized semi continuous mapping.

Example 3.8 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.3, 0.4), (0.5, 0.6) ⟩, G_2 = ⟨ y, (0.3, 0.4), (0.5, 0.6) ⟩. Then τ = {$0_\sim$, G_1,$1_\sim$} and σ = { $0_\sim$, G_2,$1_\sim$} are intuitionistic fuzzy topological space on X and Y respectively. **Define a mapping f** : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The intuitionistic fuzzy set G_2^c = ⟨ y, (0.5, 0.6), (0.3, 0.4) ⟩ is intuitionistic fuzzy closed set in Y. Then, $f^{-1}(G_2^c)$ is intuitionistic fuzzy π - generalized semi open set in X but not intuitionistic fuzzy regular open set in X. Then f is intuitionistic fuzzy contra π - generalized semi continuous mapping but not an intuitionistic fuzzy contra regular continuous mapping.

Theorem 3.9 : Every intuitionistic fuzzy contra generalized continuous mapping is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not conversely.

Proof : Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy contra generalized continuous mapping. Let A be an intuitionistic fuzzy closed set in Y. Since f is an intuitionistic fuzzy contra generalized continuous mapping, $f^{-1}(A)$ is an intuitionistic fuzzy generalized open set in X. Since every intuitionistic fuzzy generalized open set is an intuitionistic fuzzy π - generalized semi open set, $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π - generalized semi continuous mapping..

Example 3.10 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.2, 0.8), (0.3, 0.1) ⟩, G_2 = ⟨ y, (0.5, 4) ,(0.2, 0.3)⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$} are intuitionistic fuzzy topological space on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an intuitionistic fuzzy contra π - generalized semi continuous mapping. Now consider the intuitionistic fuzzy closed set G_2^c = ⟨ y, (0.2, 0.3), (0.5, 0.4) ⟩ in Y. Then $f^{-1}(G_2^c)$ = ⟨ x, (0.2, 0.3), (0.5, 0.4) ⟩ is not an intuitionistic fuzzy generalized open set in X. Hence f is not an intuitionistic fuzzy contra-generalized continuous mapping.

Theorem 3.11: Every intuitionistic fuzzy contra generalized semi continuous mapping is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not conversely.

Proof : Let f: (X, τ) → (Y,σ) be an intuitionistic fuzzy contra generalized semi continuous mapping. Let A be an intuitionistc fuzzy closed set in Y. Then by hypothesis $f^{-1}(A)$ is an intuitionistic fuzzy generalized open set in X. Since every intuitionistic fuzzy generalized open set I is an intuitionistic fuzzy π - generalized semi open set, $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π - generalized semi continuous mapping.

Example 3.12 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.5, 0.6), (0.2, 0.2) ⟩, G_2 = ⟨ y, (0.3, 0.2), (0.6, 0.6) ⟩. Then τ = {$0_\sim$, G_1,$1_\sim$} and σ = { $0_\sim$, G_2,$1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The intuitionistic fuzzy set

G_2^c = ⟨ y, (0.6, 0.6), (0.3, 0.2) ⟩ is intuitionistic fuzzy closed set in Y. Then $f^{-1}(G_2^c)$ is intuitionistic fuzzy π - generalized semi open set in X but not intuitionistic fuzzy generalized semi open set in X. Then f is intuitionistic fuzzy contra π - generalized semi continuous mapping.but not an intuitionistic fuzzy contra-generalized semi continuous mapping.

Theorem 3.13 : Every intuitionistic fuzzy contra α generalized continuous mapping is an intuitionistic fuzzy contra π - generalized semi continuous mapping but not conversely.

Proof : Let f: (X, τ) → (Y,σ) be an intuitionistic fuzzy contra α generalized continuous mapping. Let A be an intuitionisticfuzzy closed set in Y. Then by hypothesis $f^{-1}(A)$ is an intuitionistic fuzzy contra-α generalized closesd set in X. Since every intuitionistic fuzzy α generalized open set is an intuitionistic fuzzy generalized semi open set and every intuitionistic fuzzy generalized semi open set is an an intuitionistic fuzzy π - generalized semi open set, $f^{-1}(A)$ is an an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π - generalized semi continuous mapping continuous mapping.

Example 3.14 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.4, 0.6), (0.2, 0.2) ⟩, G_2 = ⟨ y, (0.4, 0.3) ,(0.6, 0.2) ⟩.Then τ = {0~, G_1, 1~} and σ = { 0~, G_2, 1~} are intuitionistic fuzzy topological space on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The intuitionistic fuzzy set G_2^c = ⟨ y,(0.6, 0.2), (0.4, 0.3) ⟩ is intuitionistic fuzzy closed set in Y. Then $f^{-1}(G_2^c)$ is intuitionistic fuzzy π - generalized semi continuous mapping but not an intuitionistic fuzzy contra-pre continuous mapping since G_2^c = ⟨ y, (0.6, 0.7),(0.4, 0.2)⟩ is an intuitionistic fuzzy closed set in Y but $f^{-1}(G_2^c)$ = ⟨ x, (0.6, 0.7), (0.4, 0.2) ⟩ is not intuitionistic fuzzy pre open set in X.

Remark 3.18: Every intuitionistic fuzzy contra γ continuous mapping and intuitionistic fuzzy contra π - generalized semi continuous mapping are independent to each other.

Example 3.19 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.1, 0.3), (0.4, 0.3) ⟩, G_2 = ⟨ x, (0.0, 0.2), (0.2, 0.3) ⟩, G_3 = ⟨ x, (0, 0.2), (0.4, 0.3) ⟩, G_4 = ⟨ x, (0.1, 0.3), (0.2, 0.3) ⟩, G_5 = ⟨ x, (0.3, 0.3), (0.2, 0.3) ⟩, G_6 = ⟨ y, (0, 0.1),(0.3, 0.4) ⟩. Then τ = { 0~,G_1,G_2,G_3,G_4,G_5,1~ }and σ = {0~, G_6, 1~} are intuitionistic fuzzy topological space on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is intuitionistic fuzzy contra-γ continuous mapping but not an intuitionistic fuzzy contra π - generalized semi continuous mapping since G_6^c = ⟨ y, (0.3, 0.4),(0, 0.1) ⟩ is an intuitionistic fuzzy closed set in Y but $f^{-1}(G_6^c)$ = ⟨ x, (0.3, 0.4),(0, 0.1) ⟩ is not an intuitionistic fuzzy π - generalized semi open set in X.

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generalized semi open set in X but not intuitionistic fuzzy α generalized open set in X. Then f is intuitionistic fuzzy contra π - generalized semi continuous mapping but not an intuitionistic contra α generalized continuous mapping.

Remark 3.15: Every intuitionistic fuzzy contra pre continuous mapping and intuitionistic fuzzy contra π - generalized semi continuous mapping are independent to each other.

Example 3.16 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.2, 0.4), (0.5, 0.4) ⟩, G_2 = ⟨ x, (0.1, 0.3), (0.3, 0.4) ⟩, G_3 = ⟨ x, (0.1, 0.3), (0.5, 0.4) ⟩, G_4 = ⟨ x, (0.2, 0.4), (0.3, 0.4) ⟩, G_5 = ⟨ x, (0.4, 0.4), (0.3, 0.4) ⟩, G_6 = ⟨ y, (0, 0.3),(0.5, 0.4)⟩. Then τ = {0~,G_1,G_2,G_3,G_4,G_5,1~} and σ = {0~, G_6, 1~} are intuitionistic fuzzy topological space on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is intuitionistic fuzzy contra-pre continuous mapping but not an intuitionistic fuzzy contra π - generalized semi continuous mapping since G_6^c = ⟨ y, (0.5, 0.4), (0, 0.3) ⟩ is an intuitionistic fuzzy closed set in Y but $f^{-1}(G_6^c)$ = ⟨ x, (0.5, 0.4),(0, 0.3) ⟩ is not an intuitionistic fuzzy π - generalized semi open set in X.

Example 3.17 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.4, 0.2), (0.6, 0.7) ⟩, G_2 = ⟨ y, (0.4, 0.2),(0.6, 0.7) ⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are intuitionistic fuzzy topological space on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is intuitionistic fuzzy contra π -

Example 3.20 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.5, 0.1), (0.5, 0.9) ⟩, G_2 = ⟨ y, (0.7, 0.8),(0.2, 0.1) ⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are intuitionistic fuzzy topological space on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is intuitionistic fuzzy contra π - generalized semi continuous mapping but not an intuitionistic fuzzy contra γ continuous mapping since G_2^c = ⟨ y, (0.2, 0.1), (0.7, 0.8) ⟩ is an intuitionistic fuzzy closed set in Y but $f^{-1}(G_2^c)$ = ⟨ x, (0.2, 0.1), (0.7, 0.8) ⟩ is not intuitionistic fuzzy γ open set in X.

The relations between various types of intuitionistic fuzzy contra continuity are given in the following diagram. In this diagram 'IF' and 'cts' means intuitionistic fuzzy and continuous mappings

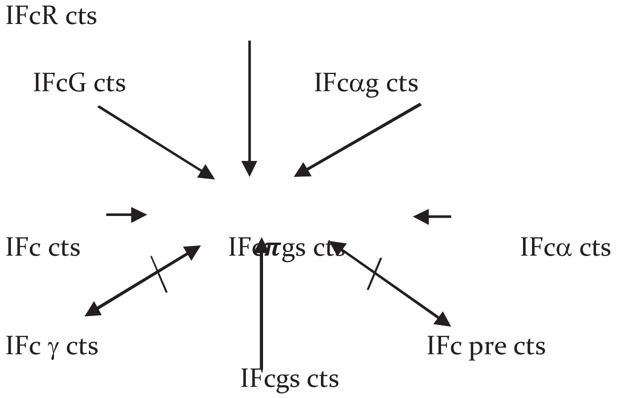

Fig.1 Relation between intuitionistic fuzzy contra π -generalized semi continuous mappings and other existing intuitionistic fuzzy contra mappings.

In this diagram by “A ⟶ B” we mean A implies B but not conversely and “A ⟷ B” means A and B are independent of each other.

Theorem 3.21 : A mapping f : X → Y is intuitionistic fuzzy contra π-generalized semi continuous if and only if the inverse image of each intuitionistic fuzzy open set in Y is an intuitionistic fuzzy π - generalized semi closed set in X .

Proof: Necessity: Let A be an intuitionistic fuzzy open set in Y. This implies A^c is an intuitionistic fuzzy closed set in Y. Since f is an is intuitionistic fuzzy contra π-generalized semi continuous mapping, $f^{-1}(A^c)$ is an intuitionistic fuzzy π - generalized semi open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi closed set in X.

generalized semi open set, $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π-generalized semi continuous mapping.

Theorem 3.23 : Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy contra π-generalized semi continuous mapping, then f is an intuitionistic fuzzy contra continuous mapping if X is an intuitionistic fuzzy $\pi T_{1/2}$ space.

Proof : Let A be an intuitionistic fuzzy closed set in Y. Then $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X, since f is an intuitionistic fuzzy contra π-generalized semi continuous. Since X is an intuitionistic fuzzy $\pi T_{1/2}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy open set in X. Hence f is an intuitionistic fuzzy contra continuous mapping.

Theorem 3.24 : Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy contra π - generalized semi continuous mapping, then f is an intuitionistic fuzzy contra generalized continuous mapping if X is an intuitionistic fuzzy π generalized $T_{1/2}$ space (IF$\pi gT_{1/2}$ space).

Proof : Let A be an intuitionisitic fuzzy closed set in Y. Then $f^{-1}(A)$ is an intuitionistic fuzzy π -

Sufficiency: Let A be an intuitionistic fuzzy closed set in Y. This implies A^c is an intuitionistic fuzzy open set in Y. By hypothesis,$f^{-1}(A^c)$ is an intuitionistic fuzzy π - generalized semi closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence f is an intuitionistic fuzzy contra π-generalized semi continuous mapping.

Theorem 3.22 : Let f : (X, τ) → (Y,σ) be a mapping and let $f^{-1}(A)$ be an intuitionistic fuzzy regular open set in X for every intuitionistic fuzzy closed set A in Y. Then f is an intuitionistic fuzzy contra π-generalized semi continuous mapping.

Proof : Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular open set in X. Since every intuitionistic fuzzy regular open set is an intuitionistic fuzzy π - generalized semi open set in X, by hypothesis. Since X is an IF$\pi gT_{1/2}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy contra generalized closed set in X. Hence f is an intuitionistic fuzzy contra generalized continuous mapping.

Theorem 3.25 : Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy π - generalized semi continuous mapping and g : (Y, σ) → (Z, δ) is intuitionistic fuzzy contra continuous mapping, then g o f : (X, τ) → (Z, δ) is an intuitionistic fuzzy contra π - generalized semi continuous mapping.

Proof : Let A be an intuitionistic fuzzy closed set in Z. Then $g^{-1}(A)$ is an intuitionistic fuzzy open set in Y, since g is an intuitionistic fuzzy contra continuous mapping. By hypothesis, Since f is an an intuitionistic fuzzy π - generalized semi continuous mapping, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy π - generalized semi open set in X. That is $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence g o f is an intuitionistic fuzzy contra π - generalized semi continuous mapping .

Theorem 3.26 : If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy contra π - generalized semi continuous mapping and g : (Y, σ) → (Z, η) is an intuitionistic fuzzy continuous mapping, then

$g \circ f : (X, \tau) \to (Z, \eta)$ is an intuitionistic fuzzy contra π - generalized semi continuous mapping.

Proof : Let A be an intuitionistic fuzzy closed set in Z. Then $g^{-1}(A)$ is an intuitionistic fuzzy closed set in Y, by hypothesis. Since f is an intuitionistic fuzzy contra π - generalized semi continuous mapping, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy π - generalized semi open set in X. That is $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy π - generalized semi open set in X. Hence $g \circ f$ is an intuitionistic fuzzy contra π - generalized semi continuous mapping.

Conclusion : In this paper we have introduced intuitionistic fuzzy contra π-generalized semi continuous mappings and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy contra generalized semi continuous mappings and some of the intuitionistic fuzzy contra continuous mappings already exist

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