INTUITIONISTIC FUZZY REGULAR WEAKLY GENERALIZED CLOSED MAPPINGS

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Abstract: In this paper we introduce intuitionistic fuzzy regular weakly generalized closed mappings and intuitionistic fuzzy regular weakly generalized open mappings. We investigate some of their properties. We also introduce intuitionistic fuzzy R-weakly generalized closed mappings as well as intuitionistic fuzzy R-weakly generalized open mappings. We provide the relation between intuitionistic fuzzy R-weakly generalized closed mappings and intuitionistic fuzzy regular weakly generalized closed mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy regular weakly generalized closed mappings and intuitionistic fuzzy regular weakly generalized open mappings.

Introduction: After the introduction of Fuzzy set (FS) by Zadeh [15] in 1965 and fuzzy topology by Chang [2] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology.The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] in 1983 as a generalization of fuzzy sets. In 1997 Coker [3] introduced the concept of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy regular weakly generalized closed mappings and intuitionistic fuzzy regular weakly generalized open mappings and study some of their properties. We also introduce intuitionistic fuzzy R-weakly generalized closed mappings as well as intuitionistic fuzzy R-weakly generalized open mappings. We provide the relation between intuitionistic fuzzy R-weakly generalized closed mappings and intuitionistic fuzzy weakly generalized closed mappings.

Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then (a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are the empty set and the whole set of X, respectively.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i\in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by $int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$ [14].

Definition 2.5: An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be

(a) [4] intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$

(b) [4] intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$

(c) [4] intuitionistic fuzzy pre-closed set (IFPCS in short) if $cl(int(A)) \subseteq A$

(d) [4] intuitionistic fuzzy regular closed set (IFRCS in short) if $cl(int(A)) = A$

(e) [13] intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS

(f) [10] intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS

(g) [8] intuitionistic fuzzy α generalized closed set (IFαGCS in short) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$

and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IαFGOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6: [5] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS in short) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X . The family of all IFRWGCSs of an IFTS (X, τ) is denoted by IFRWGC(X).

Definition 2.7: [5] An IFS A is said to be an intuitionistic fuzzy regular weakly generalized open set (IFRWGOS in short) in (X, τ) if the complement A^c is an IFRWGCS in X.

The family of all IFRWGOSs of an IFTS (X, τ) is denoted by IFRWGO(X).

Result 2.8: [5] Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFRWGCS but the converses need not be true in general.

Definition 2.9: [6] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by

$rwgint(A) = \cup \{G / G$ is an IFRWGOS in X and $G \subseteq A\}$

$rwgcl(A) = \cap \{K / K$ is an IFRWGCS in X and $A \subseteq K\}$.

Definition 2.10: [3] Let f be a mapping from an IFTS (X, τ) to an IFTS (Y, σ). If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle / y \in Y\}$ is an IFS in Y, then the pre-image of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle / x \in X\}$. If $A = \{\langle x, \lambda_A(x), \nu_A(x)\rangle / x \in X\}$ is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by $f(A) = \{\langle y, f(\lambda_A(y)), f\_(\nu_A(y))\rangle / y \in Y\}$ where $f\_(\nu_A) = 1-f(1-\nu_A)$.

Definition 2.11: [7] A mapping f: (X, τ)$\to$ (Y, σ) from an IFTS (X, τ) into an IFTS (Y, σ) is called an intuitionistic fuzzy regular weakly generalized continuous (IFRWG continuous in short) if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFCS B of (Y,σ).

Definition 2.12: [6] A mapping f: (X, τ) $\to$ (Y, σ) from an IFTS (X, τ) into an IFTS (Y, σ) is called an intuitionistic fuzzy regular weakly generalized irresolute (IFRWG irresolute in short) if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFRWGCS B of (Y, σ).

Definition 2.13: A mapping f: (X, τ) $\to$ (Y, σ) from an IFTS (X, τ) into an IFTS (Y, σ) is said to be

(a) [11] intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X.

(b) [4] intuitionistic fuzzy semi closed mapping (IFSCM for short) if f(A) is an IFSCS in Y for every IFCS A in X.

(c) [4] intuitionistic fuzzy pre-closed mapping (IFPCM for short) if f(A) is an IFPCS in Y for every IFCS A in X.

(d) [4] intuitionistic fuzzy α-closed mapping (IFαCM for short) if f(A) is an IFαCS in Y for every IFCS A in X.

(e) [9] intuitionistic fuzzy α-generalized closed mapping (IFαGCM for short) if f(A) is an IFαGCS in Y for every IFCS A in X.

(f) [14] intuitionistic fuzzy pre regular closed mapping (IFPRCM for short) if f(A) is an IFPCS in Y for every IFRCS A in X.

Definition 2.14: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $rwT_{1/2}$ (IF $rwT_{1/2}$ in short) space if every IFRWGCS in X is an IFCS in X.

Definition 2.15: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $rwgT_{1/2}$ (IF $rwgT_{1/2}$ in short) space if every IFRWGCS in X is an IFPCS in X.

Intuitionistic Fuzzy Regular Weakly Generalized Closed Mappings and Intuitionistic Fuzzy Regular Weakly Generalized Open Mappings

In this section we introduce intuitionistic fuzzy regular weakly generalized closed mappings and intuitionistic fuzzy regular weakly generalized open mappings. We investigate some of their properties.

Definition 3.1: A mapping f: (X, τ) $\to$ (Y, σ) from an IFTS (X, τ) into an IFTS (Y, σ) is called an intuitionistic fuzzy regular weakly generalized closed mapping (IFRWGCM in short) if f(A) is an IFRWGCS in Y for every IFCS A in X.

Example 3.2: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.8, 0.7), (0.2, 0.2)\rangle$, $T_2 = \langle y, (0.2, 0.3), (0.8, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y,σ) by f (a) = u and f (b) = v. Then f is an IFRWGCM.

Theorem 3.3: Every IFCM is an IFRWGCM but not conversely.

Proof: Let f: (X, τ) $\to$ (Y, σ) be an IFCM. Let A be an IFCS in X. Then f (A) is an IFCS in Y. Since every IFCS is an IFRWGCS, f(A) is an IFRWGCS in Y. Hence f is an IFRWGCM.

Example 3.4: Let X = {a, b}, Y = {u, v} and $T_1 = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$, $T_2 = \langle y, (0.4, 0.5), (0.6, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y,σ) by f (a) = u and f (b) = v. Then f is IFRWGCM but not an IFCM since IFS $A = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$ is an IFCS in X but $f(A) = \langle y, (0.2, 0.3), (0.8, 0.7)\rangle$ is not an IFCS in Y, since $cl(f(A)) = T_2^C \neq f(A)$.

Theorem 3.5: Every IFαCM is an IFRWGCM but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFαCM. Let A be an IFCS in X. Then f (A) is an IFαCS in Y. Since every IFαCS is an IFRWGCS, f(A) is an IFWGCS in Y. Hence f is an IFRWGCM.

Example 3.6: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.6, 0.8), (0.4, 0.2)⟩, T_2 = ⟨ y, (0.5, 0.3), (0.5, 0.7)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is IFRWGCM but not an IFαCM since IFS A = ⟨ x,(0.4, 0.2), (0.6, 0.8) ⟩ is an IFCS in X but f(A) = ⟨ y, (0.4, 0.2), (0.6, 0.8) ⟩ is not an IFαCS in Y, since cl(int(cl(f(A)))) = $T_2^C \not\subseteq$ f(A).

Theorem 3.7: Every IFPCM is an IFRWGCM but not conversely.

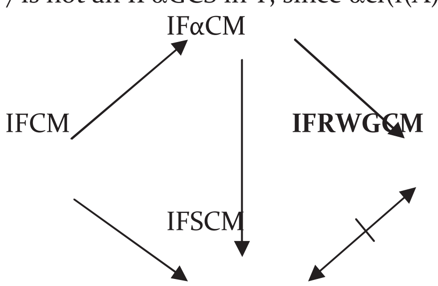
Proof: Let f: (X, τ) → (Y, σ) be an IFPCM. Let A be an IFCS in X. Then f (A) is an IFPCS in Y. Since every IFPCS is an IFRWGCS, f(A) is an IFRWGCS in Y. Hence f is an IFRWGCM.

Example 3.8: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.1, 0.6), (0.9, 0.3)⟩, T_2 = ⟨ y, (0.5, 0.3), (0.5, 0.7)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is IFRWGCM but not an IFPCM since IFS A = ⟨ x, (0.9, 0.3), (0.1, 0.6) ⟩ is an IFCS in X but f(A) = ⟨ y, (0.9, 0.3), (0.1, 0.6) ⟩ is not an IFPCS in Y, since cl(int(f(A))) = $1_\sim \not\subseteq$ f(A).

Theorem 3.9: Every IFαGCM is an IFRWGCM but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFαGCM. Let A be an IFCS in X. Then f (A) is an IFαGCS in Y. Since every IFαGCS is an IFRWGCS, f(A) is an IFRWGCS in Y. Hence f is an IFRWGCM.

Example 3.10: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.6, 0.5), (0.3, 0.5)⟩, T_2 = ⟨ y, (0.5, 0.6), (0.5, 0.4)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is IFRWGCM but not an IFαGCM since IFS A = ⟨ x, (0.3, 0.5), (0.6, 0.5) ⟩ is an IFCS in X but f(A) = ⟨ y, (0.3, 0.5), (0.6, 0.5) ⟩ is not an IFαGCS in Y, since αcl(f(A)) = $1_\sim \not\subseteq T_2$.



In this diagram by "A → B" we mean A implies B but not conversely and "A $\nleftrightarrow$ B" means A and B are independent of each other.

Theorem 3.17: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) are both IFrWGCM, then their composition gof: (X, τ) →(Z, δ) is an IFrWGCM.

Remark 3.11: An IFSCM and IFRWGCM are independent to each other as seen from the following examples.

Example 3.12: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.6, 0.7), (0.4, 0.3)⟩, T_2 = ⟨ y, (0.4, 0.3), (0.6, 0.7)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is IFSCM but not an IFRWGCM since IFS A = ⟨ x, (0.4, 0.3), (0.6, 0.7) ⟩ is an IFCS in X but f(A) = ⟨ y, (0.4, 0.3), (0.6, 0.7) ⟩ is not an IFRWGCS in Y, since cl(int(f(A))) = $T_2^C \not\subseteq$ T.

Example 3.13: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.3, 0.4), (0.7, 0.6)⟩, T_2 = ⟨ y, (0.9, 0.7), (0.1, 0.2)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is IFRWGCM but not an IFSCM since IFS A = ⟨ x, (0.7, 0.6), (0.3, 0.4) ⟩ is an IFCS in X but f(A) = ⟨ y, (0.7, 0.6), (0.3, 0.4) ⟩ is not an IFSCS in Y, since int(cl(f(A))) = $1_\sim \not\subseteq$ f(A).

Definition 3.14: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy r-weakly generalized closed mapping (IFrWGCM in short) if f(A) is an IFRWGCS in Y for every IFRWGCS A in X.

Theorem 3.15: Every IFrWGCM is an IFRWGCM but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFrWGCM. Let A be an IFCS in X. Then A is an IFRWGCS in X. By hypothesis f (A) is an IFRWGCS in Y. Hence f is an IFRWGCM.

Example 3.16: Let X = {a, b}, Y = {u, v} and T_1 = ⟨x, (0.4, 0.3), (0.6, 0.7)⟩, T_2 = ⟨ y, (0.2, 0.4), (0.7, 0.4)⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is IFRWGCM but not an IFrWGCM since IFS A = ⟨x, (0.2, 0.3), (0.7, 0.7) ⟩ is an IFRWGCS in X but f(A) = ⟨ y, (0.2, 0.3), (0.7, 0.7) ⟩ is not an IFRWGCS in Y.

The relations between various types of intuitionistic fuzzy closedness are given in the following diagram.

Proof: Let A be an IFRWGCS in X. Then f(A) is an IFRWGCS in Y, by hypothesis. Since g is an IFrWGCM, g(f(A)) = gof(A) is an IFRWGCS in Z. Hence gof is an IFrWGCM.

Theorem 3.18: If f: (X, τ) → (Y, σ) is an IFCM and g: (Y, σ) → (Z, δ) is IFrWGCM, then their composition gof: (X, τ) →(Z, δ) is an IFRWGCM.

Proof: Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since every IFCS is an IFRWGCS, f(A) is an IFRWGCS in Y. Since g is IFrWGCM, g(f(A)) = gof(A) is an IFRWGCS in Z. Hence gof is an IFRWGCM.

Theorem 3.19: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the following statements are equivalent.

(a) f is an IFRWGCM.(b) rwgcl(f(A)) ⊆ f(cl(A)) for each IFS A of X.

Proof: (a) ⇒ (b): Let A be an IFS in X. Then cl(A) is an IFCS in X. (a) implies f(cl(A)) is an IFRWGCS in Y. Therefore rwgcl(f(cl(A))) = f(cl(A)). Now f(A) ⊆ f(cl(A)).That is rwgcl(f(A)) ⊆ rwgcl(f(cl(A))) = f(cl(A)). Hence rwgcl(f(A)) ⊆ f(cl(A)) for each IFS A of X.

(b) ⇒ (a): Let A be any IFCS in X. Then cl(A) = A. (b) implies that rwgcl(f(A)) ⊆ f(cl(A)) ⊆ A. This implies f(A) is an IFRWGCS in Y. Hence f is an IFRWGCM.

Theorem 3.20: Let f: (X, τ) → (Y, σ) be a bijection from an IFTS (X, τ) into an IFTS (Y, σ). Then the following statements are equivalent.

(a) f is an IFRWGCM

(b) rwgcl (f(A)) ⊆ f(cl(A)) for each IFS A of X

(c) f^{-1}(rwgcl(B)) ⊆ cl(f^{-1}(B)) for every IFS B of Y.

Proof: (a) ⇔ (b): Is obviously true from the Theorem 3.19. **(b)⇒(c):** Let B be an IFS in Y. Then f^{-1}((B)) is an IFS in X. Since f is onto, rwgcl(B) = rwgcl(f(f^{-1}(B))) and (b) implies rwgcl(f(f^{-1}(B))) ⊆ f(cl(f^{-1}(B))). Therefore rwgcl(B) ⊆ f(cl(f^{-1}(B))). Now f^{-1}(rwgcl(B)) ⊆ f^{-1} (f(cl(f^{-1}(B)))). Since f is one to one, f^{-1}(rwgcl(B)) ⊆ cl(f^{-1}(B)).

(c) ⇒ (b): Let A be any IFS of X. Then f(A) is an IFS of Y. Since f is one to one, (c) implies that f^{-1}(rwgcl(f(A))) ⊆ cl(f^{-1}(f(A))) = cl(A). Therefore f(f^{-1}(rwgcl(f(A)))) ⊆ f(cl(A)). Since f is onto, rwgcl(f(A)) = f(f^{-1}(rwgcl(f(A)))) ⊆ f(cl(A)).

Theorem 3.21: Let f: (X, τ) → (Y, σ) be an IFRWGCM. Then for every IFS A of X, f(cl(A)) is an IFRWGCS in Y.

Proof: Let A be any IFS in X. Then cl(A) is an IFCS in X. By hypothesis, f(cl(A)) is an IFRWGCS in Y.

Theorem 3.22: Let f: (X, τ) → (Y, σ) be an IFRWGCM where Y is an IF $rwT_{1/2}$ space. Then f is an IFCM.

Proof: Let f be an IFRWGCM. Then for every IFCS A of X, f(A) is an IFRWGCS in Y. Since Y is an IF $rwT_{1/2}$ space, f(A) is an IFCS in Y. Hence f is an IFCM.

Theorem 3.23: Let f: (X, τ) → (Y, σ) be an IFRWGCM where Y is an IF $rwgT_{1/2}$ space. Then f is an IFPRCM if every IFPCS is an IFRCS in Y.

Proof: Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. By hypothesis, f(A) is an IFRWGCS in Y. Since Y is an IF IF $rwgT_{1/2}$ space, f(A) is an IFPCS in Y and hence an IFRCS in Y, by hypothesis. This implies f(A) is an IFPRCM.

Theorem 3.24: If every IFS is an IFCS, then an IFRWGCM is an IFRWG continuous mapping.

Proof: Let A be any IFS in Y. Then f^{-1}(A) is an IFS in X. Therefore f^{-1}(A) is an IFCS in X. Since every IFCS is an IFRWGCS, f^{-1}(A) is an IFRWGCS in X. This implies that f is an IFRWG continuous mapping.

Theorem 3.25: Let A be an IFGCS in X. An onto mapping f: (X, τ) → (Y, σ) is both IF continuous mapping and IFRWGCM, then f(A) is an IFRWGCS in Y.

Proof: Let f(A) ⊆ U where U is an IFOS in Y, then A ⊆ f^{-1}(U) where f^{-1}(U) is an IFOS in X, by hypothesis. Since A is an IFGCS, cl(A) ⊆ f^{-1}(U) in X. Hence, f(cl(A)) ⊆ f(f^{-1}(U)) = U. But f(cl(A)) is an IFRWGCS in Y, since cl(A) is an IFCS in X and f is an IFRWGCM. We have rwgcl(f(cl(A))) ⊆ U. Now rwgcl(f(A)) ⊆ rwgcl(f(cl(A))) ⊆ U. Hence f(A) is an IFRWGCS in Y.

Theorem 3.26: If f: (X, τ) → (Y, σ) is an IFCM and g: (Y, σ) → (Z, δ) is IFRWGCM, then gof: (X, τ) →(Z, δ) is an IFRWGCM.

Proof: Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is IFRWGCM, g(f(A)) = gof(A) is an IFRWGCS in Z. Hence gof is an IFRWGCM.

Definition 3.27: A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy regular weakly generalized open mapping (IFRWGOM in short) if f(A) is an IFRWGOS in Y for every IFOS A in X.

Definition 3.28: A mapping f: (X, τ) → (Y,σ) is said to be an intuitionistic fuzzy M-regular weakly generalized open mapping (IFMRWGOM in short) if f(A) is an IFRWGOS in Y for every IFRWGOS A in X.

Theorem 3.29: If f: (X, τ) → (Y, σ) be a mapping. Then the following are equivalent.

(a) f is an IFRWGOM

(b) f(int(A)) ⊆ rwgint(f(A)) for each IFS A of X

(c) int(f^{-1}(B)) ⊆ f^{-1}(rwgint(B)) for every IFS B of Y.

Proof:

(a) ⇒ (b): Let f be an IFRWGOM. Let A be any IFS in X. Then int(A) is an IFOS in X. (a) implies that f(int(A)) is an IFRWGOS in Y. Therefore rwgint(f(int(A))) = f(int(A)) ⊆ f(A). Now f(int(A))=rwgint(f(int(A))) ⊆ rwgint(f(A)).

(b) ⇒ (c): Let B be any IFS in Y. Then f^{-1}(B) is an IFS in X. (b) implies that f(int(f^{-1}(B))) ⊆ rwgint((f(f^{-1}(B))) = rwgint(B). Now int (f^{-1}(B)) = f^{-1}(f (int (f^{-1}(B)))) ⊆ f^{-1} (rwgint(B)). Hence int(f^{-1}(B)) ⊆ f^{-1}(rwgint(B)).

(c) ⇒ (a): Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. (c) implies that int(f^{-1}(f(A))) ⊆ f^{-1}(rwgint(f(A))). Now A = int(A) ⊆ int (f^{-1}(f(A))) ⊆ f^{-1}(rwgint(f(A))). Therefore f(A) ⊆ f(f^{-1}(rwgint(f(A)))) =

rwgint(f(A)) ⊆ f(A). This implies rwgint(f(A)) = f(A). Hence f(A) is an IFRWGOS in Y. Thus f is an IFRWGOM.

Theorem 3.30: A mapping f: (X, τ) → (Y,σ) is an IFRWGOM if f(rwgint(A)) ⊆ rwgint(f(A)) for every A ⊆ X.

Proof: Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) ⊆ f(rwgint(A)) ⊆ rwgint(f(A)), by hypothesis. But rwgint(f(A)) ⊆ f(A). Therefore f(A) is an IFRWGOS in X. Hence f is an IFRWGOM.

Theorem 3.31: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) then the following statements are equivalent.

(a) f is an IFMRWGCM

(b) f(A) is an IFRWGCS in Y for every IFRWGCS A in X

(c) f(A) is an IFRWGOS in Y for every IFRWGOS A in X.

(d) f(rwgint(B)) ⊆ rwgint(f(B)) for every IFS B in X

(e) rwgcl(f(B)) ⊆ f(rwgcl(B)) for every IFS B in X.

Proof:

(a) ⇒ (b): is obvious from the Definition 3.14.

(b) ⇒ (c): Let A be an IFRWGOS in X. Then A^C is an IFRWGCS in X. By hypothesis, $f(A^C)$ is an IFRWGCS in Y. That is $f(A)^C$ is an IFRWGCS in Y and hence f(A) is an IFRWGOS in Y.

(b) ⇒ (c): Let B be any IFS in X. Since rwgint(B) is an IFRWGOS in X, f(rwgint(B)) is an IFRWGOS in Y, by hypothesis. Therefore f(rwgint(B)) = rwgint(f(rwgint(B))) ⊆ rwgint(f(B)).

(c) ⇒ (d): Let B be any IFS in X. Since rwgint(B) is an IFRWGOS in X, f(rwgint(B)) is an IFRWGOS in Y, by hypothesis. Therefore f(rwgint(B)) = rwgint(f(rwgint(B))) ⊆ rwgint(f(B)).

(d) ⇒ (e): is obvious by taking complement in (c).

(e) ⇒ (a): Let A be an IFRWGCS in X. By hypothesis, rwgcl(f(A)) ⊆ f(rwgcl(A)). Therefore rwgcl(f(A)) ⊆ f(rwgcl(A)) = f(A) ⊆ rwgcl(f(A)). Hence f(A) is an IFRWGCS in Y. Hence f is an IFMRWGCM.

Theorem 3.32: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) are both IFRWGCM and Y is an $IFr_wT_{1/2}$ space, then their composition gof: (X, τ) →(Z, δ) is an IFRWGCM.

Proof: Let A be an IFCS in X. Then f(A) is an IFRWGCS in Y, by hypothesis. Since Y is an $IFr_wT_{1/2}$ space, f(A) is an IFCS in (Y,σ). Since g is an IFRWGCM, g(f(A)) = gof(A) is an IFRWGCS in Z. Hence gof is an IFRWGCM.

Theorem 3.33: If f: (X, τ) → (Y, σ) is IFRWGCM and g: (Y, σ) → (Z, δ) is IFPCM [respectively IFSCM, IFαCM and IFαGCM and (Y,σ) is an $IFr_wT_{1/2}$ space, then their composition gof: (X, τ) →(Z, δ) is an IFPCM [respectively IFSCM, IFαCM and IFαGCM.

Proof: Let A be an IFCS in X. Therefore f(A) is an IFRWGCS in Y, by hypothesis. Since Y is a $IFr_wT_{1/2}$ space, f(A) is an IFCS in Y. Since g is a IFPCM, g(f(A)) = gof(A) is a IFPCS in Z. Hence gof is an IFPCM [respectively IFSCM, IFαCM and IFαGCM].

Theorem 3.34: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be any two maps. If gof: (X, τ) → (Z, δ) is a IFMRWGOM and f is surjective IFRWG continuous map, then g is IFRWGOM.

Proof: Let V be any IFOS in (Y,σ). Since f is IFRWG continuous, $f^{-1}(V)$ is IFRWGOS in (X, τ). Since gof is IFMRWGOM and f is surjective, $(gof)(f^{-1}(V))$ = g (V) is IFRWGOS in (Z,δ). Hence g is IFRWGOM.

Conclusion : In this paper we have introduced Intuitionistic Fuzzy Regular Weakly Generalized Closed Mappings and Intuitionistic Fuzzy Regular Weakly Generalized Open Mappings and studied some of its basic properties. Also we have studied the relationship between Fuzzy Regular Weakly Generalized Closed Mappings and some of the intuitionistic fuzzy mappings already exist.

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