A STUDY ON MULTIOBJECTIVE FULLY FUZZY LINEAR PROGRAMMING PROBLEMS

SANGEETHA RAGHU

Abstract: The modeling and solving the optimization problem is one of the most important daily problems. For the data in practice which are imprecise, Fully Fuzzy Linear Programming (FFLP) problem is a powerful tool in modeling the practical optimization problem. A method to solve Multi Objective Fully Fuzzy Linear Programming (MFFLP) is proposed.

Keywords: Fully Fuzzy Linear Programming, Multi objective Fully Fuzzy Linear Programming,

Fuzzy logic – Introduction: Fuzzy logic is a form of many-valued logic derived from fuzzy set theory to deal with reasoning that is fluid or approximate rather than fixed and exact. In contrast with "crisp logic", where binary sets have two-valued logic, fuzzy logic variables may have a truth value that ranges in degree between 0 and 1. In other words, fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth, where the truth value may range between completely true and completely false. Furthermore, when linguistic variables are used, these degrees may be managed by specific functions. The concept of Fuzzy Logic(FL) was first conceived by Lofti Zadeh, a professor at the university of California at Berkley, and presented a control methodology, but as a way of processing data by allowing partial set membership rather than crisp set membership or non membership. Though fuzzy logic has been applied to many fields, from at control theory to artificial intelligence, it still remains controversial among most statisticians, who prefer Bayesian logic, and some control engineers, who prefer traditional two-valued logic. Today, Fuzzy Logic concept used widely in many implementations like automobile engine and automatic gear control systems, air conditioners, automatic focus control, video enhancement in TV sets, washing machines, behavior-based mobile robots, sorting and handling data, Information Systems, traffic control systems and so on.

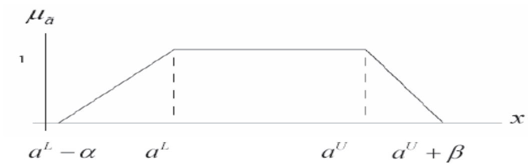
2. Basic definitions: A fuzzy number is an extension of a regular number in the sense that it does not refer to not single value but rather to a connected set of possible values, where each possible value has its own weight 0 and 1. This weight is called the membership function. Fuzziness refers to indistinctness, the quality of being indistinct and without sharp outlines. The membership function of a fuzzy set is a generalization of the indicator function in classical sets. In fuzzy logic, it represents the degree of truth, as an extension of valuation.

For any set X, a membership function on X is any function from X to the real unit interval [0, 1]

The membership function which represents a fuzzy set Ã is usually denoted by μ_A. For an element x of X, the value $\mu_A(x)$ is called the membership degree of x in the fuzzy set Ã. In traditional logic system, an item that strictly does or does not belong to a group is called a set. In a fuzzy set, the transitions between a member or a non member occur in a continuous ray being a membership degree associated between "0" (totally non member) and "1" (totally member)

For example, an animal either is or is not a dog. Fuzzy logic allows an object to belong to a set to a certain degree or with a certain confidence.

Applications of fuzzy logic in contemporary computer systems are too numerous to cite, but they control things like heating mixtures and tooling parts. With fuzzy logic, prepositions can be represented with degree of truthfulness and falsehood. For example, the statement, **today is sunny**, might be 100% true if there are no clouds, 80% true if there are a few clouds, 50% true if its hazy and 0% true if it rains all day. A fuzzy number is a trapezoidal fuzzy number if the membership function of it be in the following form:



1. Trapezoidal Fuzzy number

A convenient method for comparing of the fuzzy numbers is by use of ranking functions. A ranking function is a map from F(R) into the real line. Now, we define orders on F(R) as following:

$$\tilde{a} \underset{\Re}{\geq} \tilde{b} \text{ if and only if } \Re(\tilde{a}) \geq \Re(\tilde{b})$$

$$\tilde{a} \underset{\Re}{\succ} \tilde{b} \text{ if and only if } \Re(\tilde{a}) \succ \Re(\tilde{b})$$

$$\tilde{a} \underset{\Re}{=} \tilde{b} \text{ if and only if } \Re(\tilde{a}) = \Re(\tilde{b})$$

3. Basic arithmetic operators on fuzzy numbers: Let $X_1 = \phi\,(a_1, b_1, c_1, d_1)$, $X_2 = \phi\,(a_2, b_2, c_2, d_2)$ $X_1 + X_2 = \phi\,(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2)$

$X_1 - X_2 = \phi\,(a_1-a_2, b_1-b_2, c_1-c_2, d_1-d_2)$

$X_1 \bullet X_2 = \phi$ {min (a_1a_2, a_1d_2, d_1a_2, d_1d_2), min (b_1b_2, b_1c_2, c_1b_2, c_1c_2), Max (b_1b_2, b_1c_2, c_1b_2, c_1c_2), max (a_1a_2, a_1d_2, d_1a_2, d_1d_2)}

Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x \in R$. Then the results of applying on the trapezoidal fuzzy numbers as shown in the following:

Image of $\tilde{a}$: $-\tilde{a} = (-a^U, -a^L, \beta, \alpha)$

Addition:

$$\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$$

Scalar multiplication:

$$x \succ 0, x\tilde{a} = (xa^L, xa^U, x\alpha, x\beta)$$

$$x \prec 0, x\tilde{a} = (xa^U, xa^L, -x\beta, -x\alpha)$$

Fuzzy linear programming

Decision making is possibly the most important and inevitable aspect of application of mathematical methods in various fields of human activity. Currently fuzzy techniques are very much applied in the field of decision making. Fuzzy methods have been developed in virtually all branches of decision making, including multi objective, multi person and multi stage decision making.

Decision making in the real world problems are not known exactly is our concern. For instance the structure of a crisp linear programming problem is defined by its parameter set, the matrix A, the vectors B and C. There are many cases when A, B and C cannot be precisely given. To deal quantitatively with imprecision one can use the concepts and techniques of fuzzy set theory.

Fuzzy constraints or fuzzy parameters can reflect the capacity to absorb changes encountered in real life.

A general linear programming is said to be fuzzy linear programming (FLP) if either of the below conditions are satisfied. The objective function is fuzzy (i.e. c_i's are fuzzy). The constraint is fuzzy (either a_{ij}'s or b_i's or both are fuzzy). The relationship between objective function and constraint are fuzzy. (x_i's are fuzzy) The general model of linear programming under fuzzy constraints is as follows.

$$\begin{aligned}
&Max / Min \ f = c_1x_1 + c_2x_2 + \ldots + c_nx_n \\
&subject\ to \quad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1 + d_1 \\
&\qquad a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2 + d_2 \\
&\qquad a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n \le b_3 + d_3 \\
&\qquad \vdots \\
&\qquad a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m + d_m \\
&\qquad x_1, x_2, x_3, \ldots, x_n \ge 0 \\
&\qquad d_1, d_2, d_3, \ldots, d_m \ are\ stretch(fuzzy)
\end{aligned}$$

$X = (x_1, x_2, x_3, \ldots, x_n)^T$ is decision variable.

f is objective function.

The linear programming problem with fuzzy parameters in the objective function: It often happens that the unit profit or cost cannot be precisely determined. Therefore, consider the linear programming with imprecise coefficients in the objective function is valuable.

The problem is formulated as follows:

Maximize $\tilde{C}X$

Subject to $(AX)_i \le b_i$, $\forall$ i

$X \ge o$

Fully Fuzzy Linear Programming: In the conventional approach value of the parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. In the real life problems there may exists uncertainty about the parameters of linear programming problems which are represented as fuzzy numbers.

The linear programming problem is said to be fully fuzzy linear programming problem when all of its parameters are fuzzy numbers.

Solution to Fully Fuzzy Multi Objective Linear Programming Problem:

A ranking function is a function $\Re$: F(R) $\rightarrow$ R, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.

Let Ã = (a, b, c) be a triangular fuzzy number. Then $\Re$ (Ã) = $\frac{a_1 + 2a_2 + a_3}{4}$ Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then

$\tilde{A} \le \tilde{B}$ iff $a_1 \le b_1$, $a_2 - a_1 \le b_2 - b_1$, $a_3 - a_2 \le b_3 - b_2$, Ã >= B̃ iff $a_1 \ge b_1, a_2 - a_1 \ge b_2 - b_1, a_3 - a_2 \ge b_3 - b_2$,

$\tilde{A} = \tilde{B}$ iff $a_1 = b_1, a_2 = b_2, a_3 = b_3$

Arithmetic operations: The arithmetic operations between two triangular fuzzy numbers defined on universal set of real numbers R is as follows.

$$\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)$$

$$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$-\tilde{A} = -(a_1, a_2, a_3) = (-a_3, -a_2, -a_1)$$

$$\tilde{A} - \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3)$$

$$= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

Let $\tilde{A} = (a_1, a_2, a_3)$ be any triangular fuzzy number and $\tilde{B} = (b_1, b_2, b_3)$ be a non negative triangular fuzzy number then $\tilde{A} \otimes \tilde{B} =$

$$\begin{cases} (a_1 b_1, a_2 b_2, a_3 b_3), a \geq 0 \\ (a_1 b_3, a_2 b_2, a_3 b_3), a < 0, c \geq 0 \\ (a_1 b_3, a_2 b_2, a_3 b_1), c < 0 \end{cases}$$

Logarithmic Least Square Method: Triangular fuzzy numbers is used in all pair wise comparison matrices. Hence, criteria weights were calculated as the triangular fuzzy numbers, and then these fuzzy criteria weights are inserted to the objective function and the multi objective is converted into single objective. Pair wise Comparison matrices are structured by using triangular fuzzy numbers (l, m, u). The $m \times n$ triangular fuzzy matrix can be given as follows:

$$\tilde{A} = \begin{pmatrix} (a_{11}{}^{l}, a_{11}{}^{m}, a_{11}{}^{u}) & \cdots & (a_{1n}{}^{l}, a_{1n}{}^{m}, a_{1n}{}^{u}) \\ \vdots & \ddots & \vdots \\ (a_{m1}{}^{l}, a_{m1}{}^{m}, a_{m1}{}^{u}) & \cdots & (a_{mn}{}^{l}, a_{mn}{}^{m}, a_{mn}{}^{u}) \end{pmatrix}$$

If $\tilde{A}^*$ is a pair wise comparison matrix, the element a_{mn} represents the comparison of component m (row element) with component n (column element) it is assumed that it is reciprocal, and the reciprocal value, $1/a_{mn}$, is assigned to the element $\tilde{a}_{mn}$

$\tilde{A}^*$ is also a triangular fuzzy pair wise comparison matrix. Logarithmic least square method is reasonable and effective, and it is used here as follows:

$$\begin{pmatrix} (1,1,1) & \cdots & (a_{1n}{}^{l}, a_{1n}{}^{m}, a_{1n}{}^{u}) \\ \vdots & \ddots & \vdots \\ \left(\frac{1}{a_{1n}{}^{u}}, \frac{1}{a_{1n}{}^{m}}, \frac{1}{a_{1n}{}^{l}}\right) & \cdots & (1,1,1) \end{pmatrix}$$

The logarithmic least square method for calculating triangular fuzzy weights can be given as follows:

$$\tilde{w}_k = (w_k^l, w_k^m, w_k^u) \quad k = 1, 2, 3, .., n,$$

$$\text{where } w_k^s = \frac{\left(\prod_{j=1}^{n} a_{kj}^s\right)^{1/n}}{\sum_{i=1}^{n} \left(\prod_{j=1}^{n} a_{ij}^m\right)^{1/n}} \quad s \in \{l, m, u\}$$

Algorithm: Given Multi objective Fully Fuzzy Linear Programming problem

$$\text{Max/Min} \begin{cases} \tilde{f}_1 = \tilde{c}_{11}\tilde{x}_1 + \tilde{c}_{12}\tilde{x}_2 + \cdots + \tilde{c}_{1n}\tilde{x}_n \\ \tilde{f}_2 = \tilde{c}_{21}\tilde{x}_1 + \tilde{c}_{22}\tilde{x}_2 + \cdots + \tilde{c}_{2n}\tilde{x}_n \\ \vdots \\ \tilde{f}_n = \tilde{c}_{n1}\tilde{x}_1 + \tilde{c}_{n2}\tilde{x}_2 + \cdots + \tilde{c}_{nn}\tilde{x}_n \end{cases}$$

$$\text{Subject to} \begin{cases} \tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n \leq \tilde{b}_1 \\ \tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n \leq \tilde{b}_2 \\ \vdots \\ \tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \cdots + \tilde{a}_{nn}\tilde{x}_n \leq \tilde{b}_n \end{cases}$$

(1)

Where $\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n$ and all parameters are non negative triangular fuzzy numbers

Step 1: Convert the Multi objective Fully Fuzzy Linear Programming problem into single objective fully fuzzy linear programming problem using the calculated fuzzy weights as below

Form the Ã matrix using the constraint coefficients of $\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n$

$$\tilde{A} = \begin{pmatrix} (a^l{}_{11}, a^m{}_{11}, a^u{}_{11}) & \cdots & (a^l{}_{1n}, a^m{}_{1n}, a^u{}_{1n}) \\ \vdots & \ddots & \vdots \\ (a^l{}_{m1}, a^m{}_{m1}, a^u{}_{m1}) & \cdots & (a^l{}_{mn}, a^m{}_{mn}, a^u{}_{mn}) \end{pmatrix} \quad (2)$$

From Ã matrix obtain the triangular fuzzy pair wise comparison matrix $\tilde{A}^*$ as follows

$\tilde{A} =$

$$\begin{pmatrix} (1,1,1) & \cdots & (a^l_{1n}, a^m_{1n}, a^u_{1n}) \\ \vdots & \ddots & \vdots \\ \left(\frac{1}{a^l_{sn}}, \frac{1}{a^m_{sn}}, \frac{1}{a^u_{sn}}\right) & \cdots & (1,1,1) \end{pmatrix} \quad (4.42)$$

(3)

Calculate the triangular fuzzy weights using logarithmic least square method as follows $\tilde{w}_k = (\tilde{w}_k^l, \tilde{w}_k^m, \tilde{w}_k^u)$ k=1, 2, …, n

$$\text{Where } w_k^s = \frac{\left(\prod_{j=1}^{n} a_{kj}^{s}\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^{m}\right)^{1/n}}, s \in \{l, m, u\} \quad (4)$$

Now the given linear programming problem is converted as follows

$$\text{Maximize} \quad \tilde{w}_1 \tilde{f}_1 + \tilde{w}_2 \tilde{f}_2 + \cdots + \tilde{w}_n \tilde{f}_n$$

$$\text{Subject to} \quad \begin{cases} \tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n \le \tilde{b}_1 \\ \tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n \le \tilde{b}_2 \\ \vdots \\ \tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \cdots + \tilde{a}_{nn}\tilde{x}_n \le \tilde{b}_n \end{cases} \quad (5)$$

Where $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$ are non negative triangular fuzzy numbers.

Step 2: Convert all the inequalities of the constraints into equations by introducing non negative fuzzy variables $\tilde{x}_j$ to the left side of the constraint $\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \le b_i$ for some i and to the right side of the constraint $\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \ge b_i$ for some i

Now the above linear programming problem is converted as

$$\text{Maximize} \quad \tilde{w}_1 \tilde{f}_1 + \tilde{w}_2 \tilde{f}_2 + \cdots + \tilde{w}_n \tilde{f}_n$$

Subject to

$$\begin{cases} \tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n + \tilde{s}_1 = \tilde{b}_1 \\ \tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n + \tilde{s}_2 = \tilde{b}_2 \\ \vdots \\ \tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \cdots + \tilde{a}_{nn}\tilde{x}_n + \tilde{s}_n = \tilde{b}_n \end{cases} \quad (6)$$

where $\tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_m$ are non negative fuzzy numbers.

$$Max \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$$

Sub to $\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j = b_i \; \forall\, i = 1, 2, \ldots, m$

$\tilde{x}_j$ is a non negative triangular fuzzy number. Here

$$\tilde{C}_j = \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix} \quad (7)$$

$$\tilde{x}_j = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \cdot \\ \vdots \\ \tilde{x}_n \end{pmatrix} \quad \tilde{a}_{ij} = \begin{pmatrix} \tilde{a}_{11} & \cdots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \cdots & \tilde{a}_{mn} \end{pmatrix}$$

Step 3: Let $\tilde{c}_j = (p_j, q_j, r_j)$

$\tilde{x}_j = (x_j, y_j, z_j)$

$\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij})$

$\tilde{b}_j = (b_i, g_i, h_i)$

$\forall$ i = 1,2,3,…,m and j = 1,2,3,..,n.

Here (x_j, y_j, z_j) is a non negative triangular fuzzy number

Now the fully fuzzy linear programming problem obtained in step 2 may be written as Maximize

$$\sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j)$$

Subject to

$$\sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) = (b_i, g_i, h_i)$$

$$\forall i = 1, 2, 3, \ldots, m \quad (8)$$

(x_j, y_j, z_j) is a non negative triangular fuzzy number.

Step 4. Assuming $(a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) = (m_{ij}, n_{ij}, o_{ij})$ the FFLP problem obtained in step 3, may be written as,

Maximize $\sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j)$ Subject to $\sum_{j=1}^{n} (m_{ij}, n_{ij}, o_{ij}) = (b_i, g_i, h_i)$ (9)

(x_j, y_j, z_j) is a non negative triangular fuzzy number.

Now we use the ranking function $\Re$ which maps each fuzzy number into the real line.

$$\text{Maximize}\,\Re\left(\sum_{j=1}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)\right)$$

$$\text{Subject to } \sum_{j=1}^{n}(m_{ij},n_{ij},o_{ij})=(b_i,g_i,h_i)$$

$$\forall i=1,2,3,\ldots,m$$

(x_j,y_j,z_j) *is a non negative triangular fuzzy number*

Step 5: Using arithmetic operations, the FFLP obtained in step 4 is converted into the following crisp linear programming (CLP) problem

$$\text{Maximize}\,\Re\left(\sum_{j=1}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)\right)$$

$$\text{Subject to } \sum_{j=1}^{n}m_{ij}=b_i,\forall i=1,2,3\ldots,m$$

$$\sum_{j=1}^{n}n_{ij}=g_i,\forall i=1,2,3\ldots,m$$

$$\sum_{j=1}^{n}o_{ij}=h_i,\forall i=1,2,3\ldots,m$$

$$y_j-x_j>=0,\; z_j-y_j>=0 \tag{10}$$

$$\forall j=1,2,3,\ldots,n$$

Step 6: Find the optimal solution x_j,y_j,z_j by solving the CLP problem obtained in step 5.

Step 7: Find the fuzzy optimal solution by putting the values of x_j,y_j,z_j in

$$\tilde{x}_j=(x_j,y_j,z_j)$$

Step 8: Find the fuzzy optimal solution by putting $\tilde{x}_j$ in objective function $\sum_{j=1}^{n}\tilde{c}_j\otimes\tilde{x}_j$

10. Numerical Example:

Given FFLPP

$$Max\quad (1,2,3)\tilde{x}_1+(2,3,4)\tilde{x}_2$$

$$sub\,to\quad (2,3,4)\tilde{x}_1+(3,4,5)\tilde{x}_2\le(1,10,27)$$

$$(1,2,3)\tilde{x}_1+(5,6,7)\tilde{x}_2\le(2,11,28)$$

$\tilde{x}_1,\tilde{x}_2$ are non negative triangular fuzzy numbers. (11)

Solution: $$\tilde{A}=\begin{pmatrix}(2,3,4) & (1,2,3)\\ (1,2,3) & (5,6,7)\end{pmatrix} \tag{12}$$

Using logarithmic least squares method, we have,

$$\tilde{A}=\begin{pmatrix}(1,1,1) & (1,2,3)\\ \left(\frac{1}{3},\frac{1}{2},1\right) & (1,1,1)\end{pmatrix} \tag{13}$$

Now using fuzzy weights we convert multi objective into single objective.

By equation (4), we have,

$$\tilde{w}_k=\left(w_k^l,w_k^m,w_k^u\right)\quad k=1,2,3,..,n,$$

$$where\;\; w_k^s=\frac{\left(\prod_{j=1}^{n}a_{kj}^s\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n}a_{ij}^m\right)^{1/n}}\qquad s\in\{l,m,u\}$$

$$w_1^{\ell}=\frac{\prod_{j=1}^{2}\left(a_{1j}^{\ell}\right)^{1/2}}{\sum_{i=1}^{2}\left(\prod_{j=1}^{2}a_{1j}^{m}\right)^{1/2}} \tag{15}$$

$$\left(\prod_{j=1}^{2}a_{1j}^{m}\right)^{1/2}=\sqrt{2}+\frac{1}{\sqrt{2}} \tag{16}$$

$$\prod_{j=1}^{2}\left(a_{1j}^{\ell}\right)^{1/2}=1 \tag{17}$$

Substituting we get, w_1^{ℓ} =0.471

Similarly, w_1^m = 0.667, w_1^u = 0.816

$$\tilde{w}_1=(0.47,0.67,0.82)$$

$$w_2^s=\frac{\prod_{j=1}^{n}\left(a_{2j}^s\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{2}a_{ij}^m\right)^{1/n}} \tag{18}$$

We have $\tilde{w}_2=\left(w_2^{\ell},w_2^m,w_2^u\right)$ Now

$$w_2^{\ell}=\frac{\prod_{j=1}^{2}\left(a_{2j}^{\ell}\right)^{1/2}}{\sum_{i=1}^{2}\left(\prod_{j=1}^{2}a_{ij}^{m}\right)^{1/2}} \tag{19}$$

$$\left(\prod_{j=1}^{2}a_{2j}^{\ell}\right)^{1/2}=\frac{1}{3} \tag{20}$$

Substituting we get, w_2^{ℓ} =0.272.

Similarly, w_2^{m} =0.33, w_2^{u} =0.471

$$\tilde{w}_2 = (0.27, 0.33, 0.47)$$

Substituting 4.24 and 4.28 in 4.17, the given LPP becomes,

$$\begin{aligned} \max \quad & (0.47, 0.67, 0.82)\left[(1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2\right] - \\ & (0.27, 0.33, 0.47)\left[(2,3,4)\tilde{x}_1 + (3,4,5)\tilde{x}_2\right] \\ sub\,to \quad & (2,3,4)\tilde{x}_1 + (3,4,5)\tilde{x}_2 \le (1,10,27) \\ & (1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2 \le (2,11,28) \end{aligned}$$

$$\begin{aligned} \max \quad & (-1.41, 0.35, 1.92)\tilde{x}_1 - (-1.41, 0.69, 2.47)\tilde{x}_2 \\ sub\,to \quad & (2,3,4)\tilde{x}_1 + (3,4,5)\tilde{x}_2 \le (1,10,27) \\ & (1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2 \le (2,11,28) \end{aligned}$$

Now substituting $\tilde{x}_1 = (x_1, y_1, z_1)$, $\tilde{x}_2 = (x_2, y_2, z_2)$ in we get,

$$\begin{aligned} \max \quad & (-1.41, 0.35, 1.92)(x_1, y_1, z_1) - (-1.41, 0.69, 2.47)(x_2, y_2, z_2) \\ subto \quad & (2,3,4)(x_1, y_1, z_1) + (3,4,5)(x_2, y_2, z_2) \le (1,10,27) \\ & (1,2,3)(x_1, y_1, z_1) + (2,3,4)(x_2, y_2, z_2) \le (2,11,28) \end{aligned}$$

$$\begin{aligned} \max \quad & \Re(-1.41x_1 - 1.41x_2, 0.35y_1 + 0.69y_2, 1.92z_1 + 2.47z_2) \\ subto \quad & (2x_1 + x_2 + s_1, 3y_1 + 2y_2 + t_1, 4z_1 + 3z_2 + u_1) = (1,10,27) \\ & (,2,3)(2x_1 + x_2 + s_2, 2y_1 + 6y_2 + t_2, 3z_1 + 7z_2 + u_2) = (2,11,28) \end{aligned}$$

Now using ranking function we get,

$$\begin{aligned} \max \quad & \left[\frac{1}{4}(-1.41x_1 - 1.41x_2 + 0.7y_1 + 1.38y_2 + 1.92z_1 + 2.47z_2)\right] \\ subto \quad & 2x_1 + x_2 + s_1 = 1 \\ & 3y_1 + 2y_2 + t_1 = 10 \\ & 4z_1 + 3z_2 + u_1 = 27 \\ & 2x_1 + x_2 + s_2 = 2 \\ & 2y_1 + 6y_2 + t_2 = 11 \\ & 3z_1 + 7z_2 + u_2 = 28 \\ & y_1 - x_1 \ge 0,\ z_1 - y_1 \ge 0,\ y_2 - x_2 \ge 0,\ z_2 - y_2 \ge 0 \\ & t_1 - s_1 \ge 0,\ u_1 - t_1 \ge 0, t_2 - s_2 \ge 0, u_2 - t_2 \ge 0 \end{aligned}$$

The above LPP is crisp linear programming.

Conclusion: In this work we concentrate on fuzzy linear programming problem with single objective function and multiple objective functions. We considered fuzzy number linear programming problems and introduced the basic feasible solution for these problems. We proposed a new algorithm for solving these problems. We proposed a new algorithm for solving these problems directly, by use of linear ranking functions.

Also we concentrate on a fully fuzzy linear programming problem, and solve this problem byusing a kind of defuzzification method. In our approach, the core of the nearest symmetric triangular fuzzy number is applied for an approximation of fuzzy numbers in objective function and coefficient matrix in the constraints. Finally, a new method is proposed to find the fuzzy optimal solution of multi objective fully fuzzy linear programming problems with inequality constraints by representing all the parameters as triangular fuzzy numbers. By using the proposed method the fuzzy optimal solution of FFLP with inequality constraints occurring in real life situation, can be easily obtained. Here the multi objective fully fuzzy linear programming model is converted into single objective fully fuzzy linear programming using logarithmic least square method. Logarithmic least squares method is reasonable and effective for calculating triangular weights of criteria. Numerical examples are solved to illustrate the proposed method.

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No.6, Ramakrishna Street, Adambakkam, Chennai 600088, Tamilnadu, India/Lecturer in Mathematics/
Theivanai Ammal College For Women(Autonomous), Villupuram, Tamilnadu,
India/raghusangeetha59@gmail.com