DEDUCTION OF CONSTANTS OF LOG-LOGISTIC CURVE USING THREE SELECTED POINTS METHOD

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Abstract: The time series Growth models are fitted for analyze and forecast the data. According to data we have to fit different growth models. Simple and popular growth models are linear Growth model, Second degree parabolic growth model, n^{th} degree polynomial growth model, exponential growth model, Second degree polynomial growth model and the most popular growth models are modified exponential , Gompertz curve and logistic growth models. In this paper we present a new growth model of name log-logistic growth model and the constants are estimated by three selected points method.

Introduction: Stock market persons, business persons, industrial persons, atmospheric scientists wants to forecaste the values using some Growth curves. In this paper we present a log-logistic growth model for univariate data. There are so many methods such as linear growth modal, parabolic growth model, n^{th} degree polynomial growth model, Gompertz growth model, logistic growth model and Modified exponential growth model for estimation of constants. The most popularly used methods for estimation of constants is Method of least squares, three selected points method and partial sum of squares. Philip Hans Franses a published an article "Fitting a Gompertz Curve". In this paper we proposed a simple Gompertz curve procedure. An application to forecasting the stock of lans in Netherlands is examined using Gompertz curve. The data taken for smoothing is from 1956 to 1989. An applications in telecommunications is forecasted by using logistic growth model by R. Bewley and D.C. Fiebig (1988) in his research paper "A flexible logistic growth model with applications in telecommunications". "Forecasting using Growth curves – An adaptive approach" was given by N. Meade, in 1985.

Deciding About Type Of Trend Fitted To Data:

Straight Line: When time series is found to be increasing by equal absolute amounts, then the straight line trend is used. If change of first degree is constant i.e., Δu_t = constant then we use linear growth model.

$u_t = a + bt$

Second degree parabolic growth model: The second degree parabolic growth model is used when double difference of time series value is constant i.e., $\Delta^2 u_t$ is constant.

$u_t = a + bt + ct^2$

n^{th} degree polynomial growth model: nth degree polynomial growth model is used when n^{th} difference of time series value is constant i.e., $\Delta^n u_t$ = constant.

$u_t = b_o + b_1 t + b_2 t^2 + b_3 t^3 + \text{------} + b_n t^n$.

Exponential trend curve:

When $\Delta \log u_t$ is constant then we use exponential trend curve.

$u_t = a\, b^t$

Second degree curve fitted to logarithms: When $\Delta^2 (\log u_t)$ is constant then we use second degree curve fitted to logarithms

$u_t = a\, b^t\, c^{t^2}$

Modified exponential curve: If $\frac{\Delta u_t}{\Delta u_{(t-1)}}$ is constant, we use modified exponential curve.

Gompertz curve: If $\frac{\Delta(\log u_t)}{\Delta(\log u_{t-1})}$ is constant we use Gompertz curve.

Logistic curve: If $\frac{\Delta(\frac{1}{u_t})}{\Delta(\frac{1}{u_{t-1}})}$ is constant, we use logistic curve.

Method of estimation of constants for models: There are many methods for estimation of constants in growth models. The most popular methods for estimation of constant is

1. Method of Least squares: This method is popularly used method for polynomial equation. The steps involved in this method are
   (a) Estimate error sum of squares to the equation.
   (b) Partially differentiate these error sum of squares with respect to constants and equate it to zero. We call there equations as normal equations.
   (c) Upon solving these normal equations we obtain constants.
2. Method of three selected points: Steps involved in this method are
   (a) we have to select three equidistant values of t, (say) t_1, t_2, t_3 such that $t_2 - t_1 = t_3 - t_2$.
   (b) We get three time series ordinates for these three time points.
   (c) Upon solving by using these time series ordinates we obtain constants.
3. Method of partial Sums: The steps involved this method are
   (a) The given time series data are split up into three equal parts each containing n consecutive value of u_t curve.
   (b) We have to calculate partial sums for each part.

$$S_1 = \sum_{t=1}^{n} u_t, \quad S_2 = \sum_{t=n+1}^{2n} u_t, \quad S_3 = \sum_{t=2n+1}^{3n} u_t$$

Upon solving these S_1, S_2 and S_3 equations, we obtain constant.

Log-Logistic curve:

For time 't', time series value u_t is

$$u_t = \frac{k}{1+\beta_0 e^{-\beta_1 \ln t}}$$

Let $\beta_0 = e^{b_0}$, $\beta_1 = b_1$, therefore

$$u_t = \frac{k}{1+e^{b_0-b_1 \ln t}} \quad \text{............ (1)}$$

The above log- logistic curve is solved by the method of three selected points. Three selected points T_1, T_2, T_3 such that ln t_1, ln t_2, ln t_3 are at equidistant such that ln(t_1/t_2) is equal to ln(t_2/t_3).

$$u_1 = \frac{k}{1+e^{b_0-b_1 \ln t_1}} \Rightarrow \frac{k}{u_1} = 1 + e^{b_0-b_1 \ln t_1}$$

$$\frac{k}{u_1} - 1 = e^{b_0-b_1 T_1}, \quad \text{where } \ln t_1 = T_1$$

$$\ln\left(\frac{k}{u_1} - 1\right) = b_0 - b_1 T_1 \quad \text{............ (2)}$$

$$u_2 = \frac{k}{1+e^{b_0-b_1 \ln t_2}} \Rightarrow \frac{k}{u_2} = 1 + e^{b_0-b_1 \ln t_2}$$

$$\frac{k}{u_2} - 1 = e^{b_0-b_1 T_2},$$

where $\ln t_2 = T_2$

$$\ln\left(\frac{k}{u_2} - 1\right) = b_0 - b_1 T_2 \quad \text{............ (3)}$$

$$u_3 = \frac{k}{1+e^{b_0-b_1 \ln t_3}} \Rightarrow \frac{k}{u_3} = 1 + e^{b_0-b_1 \ln t_3}$$

$$\frac{k}{u_3} - 1 = e^{b_0-b_1 T_3},$$

where $\ln t_3 = T_3$

$$\ln\left(\frac{k}{u_3} - 1\right) = b_0 - b_1 T_3 \quad \text{............ (4)}$$

Subtracting equation (2) from the equation (3), we get

$$b_0 - b_1 T_2 - b_0 + b_1 T_1 \Rightarrow b_1(T_1 - T_2) = b_1(\ln t_1 - \ln t_2) = b_1 \ln\left(\frac{t_1}{t_2}\right)$$

$$\ln\left(\frac{k}{u_2} - 1\right) - \ln\left(\frac{k}{u_1} - 1\right) = \ln\left(\frac{\frac{k}{u_2} - 1}{\frac{k}{u_1} - 1}\right)$$

$$\ln\left(\frac{\frac{k}{u_2} - 1}{\frac{k}{u_1} - 1}\right) = b_1 \ln\left(\frac{t_1}{t_2}\right) \quad \text{............ (5)}$$

Subtracting equation (3) from the equation (4), we get

$$b_0 - b_1 T_3 - b_0 + b_1 T_2 = b_1(T_2 - T_3) = b_1 \ln\left(\frac{t_2}{t_3}\right)$$

$$\ln\left(\frac{k}{u_3} - 1\right) - \ln\left(\frac{k}{u_2} - 1\right) = \ln\left(\frac{\frac{k}{u_3} - 1}{\frac{k}{u_2} - 1}\right)$$

$$\ln\left(\frac{\frac{k}{u_3} - 1}{\frac{k}{u_2} - 1}\right) = b1 \ln\left(\frac{t_2}{t_3}\right) \quad \text{............... (6)}$$

From (5) and (6), if R.H.S. of equation then

$$\frac{\frac{k}{u_2} - 1}{\frac{k}{u_1} - 1} = \frac{\frac{k}{u_3} - 1}{\frac{k}{u_2} - 1}$$

$\Rightarrow \left(\frac{k}{u_2} - 1\right)^2 = \left(\frac{k}{u_3} - 1\right)\left(\frac{k}{u_1} - 1\right)$ is equal, because $\ln\left(\frac{t_1}{t_2}\right) = \ln\left(\frac{t_2}{t_3}\right)$

$$\frac{(k - u_2)^2}{u_2^2} = \frac{(k - u_3)}{u_3} \cdot \frac{(k - u_1)}{u_1}$$

$$(k - u_2)^2 \, u_3 u_1 = (k - u_3)(k - u_1) u_2^2$$

$$(k^2 + u_2^2 - 2k\, u_2) u_1 u_3 = (k^2 - k u_1 - k u_3 + u_1 u_3) u_2^2$$

$$k^2 u_1 u_3 + u_2^2 u_1 u_3 - 2k\, u_2 u_1 u_3 = k^2 u_2^2 - k u_1 u_2^2 - k u_3 u_2^2 + u_1 u_3 u_2^2$$

$$k^2 (u_2^2 - u_1 u_3) = u_2^2 k(u_1 + u_3) - 2k u_1 u_2 u_3$$

$$k (u_2^2 - u_1 u_3) = u_2^2 (u_1 + u_3) - 2 u_1 u_2 u_3$$

$$k = \frac{u_2^2(u_1 + u_3) - 2u_1 u_2 u_3}{(u_2^2 - u_1 u_3)}$$

.... (7)

From (5) , we have

$$b_1 \ln\left(\frac{t_1}{t_2}\right) = \ln\left(\frac{\frac{k}{u_2} - 1}{\frac{k}{u_1} - 1}\right)$$

$$\Rightarrow b_1 = \ln\left(\frac{\frac{k}{u_2} - 1}{\frac{k}{u_1} - 1}\right) \Bigg/ \ln\left(\frac{t_1}{t_2}\right) \quad \text{.......... (8)}$$

From (2), we have

$$b_0 - b_1 T_1 = \ln\left(\frac{k}{u_1} - 1\right)$$

$$b_0 = b_1 \ln t_1 + \ln\left(\frac{k}{u_1} - 1\right)$$

$$= \left(\ln\left(\frac{\frac{k}{u_2} - 1}{\frac{k}{u_1} - 1}\right) \Bigg/ \ln\left(\frac{t_1}{t_2}\right)\right) \ln t_1 + \ln\left(\frac{k}{u_1} - 1\right)$$

.... (9)

Equations (7), (8) and (9) gives estimates of the log-logistic curve.

Conclusions: Growth curves are modified according to data, because if you fit good model to data may explain approximate forecasted value. In this paper we are estimated constants of log-logistic model by using well known popular method, three selected points method upon assuming the $\ln t_1$ - $\ln t_2$ is equal to $\text{L}nt_2$ - $\text{L}nt_3$ I.E., The Natural Logarithmic Values Of Time Point T_1, T_2, T_3 Are At Equidistant.

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