PROPAGATION OF ION-ACOUSTIC SOLITARY WAVES IN INHOMOGENEOUS PLASMAS

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Abstract: The propagation of ion-acoustic solitary waves in inhomogeneous plasma are found to be interesting which give rise to some important scientific observation in laboratory plasma as well as in many other astrophysical plasmas. The presence of non-isothermal electrons leads to some addition to the findings on solitary waves in multicomponent plasma. In this present paper the fluid model of plasma is considered and the system of equations for this plasma model has been treated by reductive perturbation method along with appropriate boundary conditions. We have used an appropriate spate-time stretched coordinate in the formulation process. A modified Korteweg-de-Vries (mKdV) equation is derived from the system of equations. From the mKdV equation, the solution of the ion-acoustic solitary waves is obtained from which the effective conditions for soliton propagation in multicomponent inhomogeneous plasma have been analyzed.

Keywords: Inhomogeneous plasma, ion-acoustic solitons, mKdV equation.

Introduction: Ion-acoustic solitary waves in plasma can be governed by the well known Korteweg-de-Vries(KdV) equation which was established first time by Washimi and Taniuti[1]. Since then various investigations both theoretically as well as experimentally have been done in different plasma models[2[-[4]. Most of the theoretical results of the heuristic features on the propagation of solitary waves were limited to homogeneous plasma. In practice, we invariably encounter inhomogeneous plasmas both in the laboratory as well as in space due to the density gradient or that of temperature or it could be due to the magnetic field in space and they are more pronounced closer to the edges and the boundaries of the system. In case of inhomogeneous plasma [5]-[9], the KdV equation gets modified with the variable coefficients or an additional term occurred due to presence of density gradient. The studies related to the propagation of ion acoustic waves were conducted in the inhomogeneous plasma having isothermal electrons that follow the Boltzmann distribution [10-15]. Sakanaka[10] and Tappet[11] studied the propagation of ion acoustic waves in inhomogeneous plasma with warm adiabatic ions and then soliton propagation in weakly inhomogeneous plasma has been studied first by Asano[12], ion acoustic case by Nishikawa and Kaw[13]and Gell and Gomberoff[14]which has an inconsistency due to neglect of the zero order quantities like ion-fluid velocity and electric field which arises due to the presence of inhomogeneity. Later Rao and Verma[15]eliminated these shortcomings by using a right set of 'stretched coordinates' appropriate for the spatially inhomogeneous system. However in space related plasmas, there may exist groups of electrons at different temperatures in which the electron velocity distribution may be represented by the superposition of Maxwellian for the isothermal electrons[16] or vortex-like distributions for the nonisothermal electrons[17]-[18]. This type of plasmas of two temperature electron components are possible in various situations like hot turbulent plasmas of thermonuclear interest, hot cathode discharge plasma, strong electron beam plasma interactions etc. Different studies relating to two temperature nonisothermal electrons which modifies the properties of ion-acoustic solitary waves [17]-[24].

In this present paper, we have derived a modified KdV equation in weakly inhomogeneous plasma having two temperature nonisothermal electrons. To solve the problem we use the well known reductive perturbation technique along with employing a set of 'stretched coordinates' appropriate for fluid model of inhomogeneous plasma.

Basic equations: We have considered unmagnetised weakly inhomogeneous and collisionless plasma having finite temperature ions and two temperature nonisothermal electrons. The continuity, momentum and Poisson equation for this plasma model with density distribution for low and high temperature electrons[18]are as follows:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv)=0 \quad (2.1a)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{\partial \phi}{\partial t}+\frac{2\sigma}{n}\frac{\partial n}{\partial x}=0 \quad (2.1b)$$

$$\frac{\partial^2 \phi}{\partial x^2}+n-n_{el}-n_{eh}=0 \quad (2.1c)$$

Where

$$\sigma=\frac{T_i}{T_{eff}},\ T_{eff}=\frac{(n_{el0}+n_{eh0})T_{el}T_{eh}}{n_{el0}T_{eh}+n_{eh0}T_{el}}$$

(2.2)

T_i = Temperature of ions

T_{eff} = Effective temperature of electrons.

and T_{el}, T_{eh} are the temperatures of low and high temperature electrons together with n_{el0} and n_{eh0} as the unperturbed values of the low and high temperature electron densities. The density distribution of low and high temperature electrons are as follows:-

$$n_{el} = n_{el0}\left[1+\frac{T_{eff}}{T_{el}}\phi - \frac{4}{3}b_l\left(\frac{T_{eff}}{T_{el}}\phi\right)^{\frac{3}{2}} + \frac{1}{2}\left(\frac{T_{eff}}{T_{el}}\phi\right)^2 + ...\right]$$

(2.3a)

$$n_{eh} = n_{eh0}\left[1+\frac{T_{eff}}{T_{eh}}\phi - \frac{4}{3}b_h\left(\frac{T_{eff}}{T_{eh}}\phi\right)^{\frac{3}{2}} + \frac{1}{2}\left(\frac{T_{eff}}{T_{eh}}\phi\right)^2 + ...\right]$$

(2.3b)

where b_l and b_h are nonisothermal parameters such that

$$b_l = \left(1-\frac{T_{el}}{T_{eff}}\right)\pi^{-\frac{1}{2}},\ b_h = \left(1-\frac{T_{eh}}{T_{eff}}\right)\pi^{-\frac{1}{2}} \quad (2.4)$$

Here n, n_{el} and n_{eh} are densities of ions, low temperature electrons and high temperature electrons respectively which are normalized by unperturbed plasma density n_0, v is the ion flow velocity normalized by the ion acoustic speed $\left(T_{eff}/m_i\right)^{1/2}$ where m_i is the ion mass. The space and time coordinates x and t are normalized by the Debye length $\left(\varepsilon_0 T_{eff}/n_0 e^2\right)^{1/2}$ and inverse ion plasma frequency $\left(n_0 e^2/\varepsilon_0 m_i\right)^{-1/2}$. The electric potential ϕ is normalized by T_{eff}/e, where e is the electron charge.

Derivation of the modified KdV equation: We use the following stretched coordinates for the time and space variables as

$$\xi = \varepsilon^{\frac{1}{4}}\left(\frac{x}{\lambda_0} - t\right), \qquad \eta = \varepsilon^{\frac{3}{4}}x \quad (3.1)$$

where ε is the expansion parameter and λ_0 is the phase velocity of the ion-acoustic wave.

Using (3.1) we get the following transformations for the space and time derivatives

$$\frac{\partial}{\partial x} = \frac{\varepsilon^{\frac{1}{4}}}{\lambda_0}\frac{\partial}{\partial\xi} + \varepsilon^{\frac{3}{4}}\frac{\partial}{\partial\eta} \quad (3.2a)$$

$$\frac{\partial}{\partial t} = -\varepsilon^{\frac{1}{4}}\frac{\partial}{\partial\xi} \quad (3.2b)$$

$$\frac{\partial^2}{\partial x^2} = \frac{\varepsilon^{\frac{1}{2}}}{\lambda_0^2}\frac{\partial^2}{\partial\xi^2} + \frac{2\varepsilon}{\lambda_0}\frac{\partial^2}{\partial\xi\partial\eta} + \varepsilon^{\frac{3}{2}}\frac{\partial^2}{\partial\eta^2} - \frac{\varepsilon}{\lambda_0^2}\frac{\partial\lambda_0}{\partial\eta}\frac{\partial}{\partial\xi}$$

(3.2c)

Substituting (3.2a)-(3.2c) in (2.1a) – (2.1c) we get

$$-\frac{\partial n}{\partial\xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial\xi}(nv) + \varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\eta}(nv) = 0 \quad (3.3a)$$

$$-n\frac{\partial v}{\partial\xi} + \frac{nv}{\lambda_0}\frac{\partial v}{\partial\xi} + \varepsilon^{\frac{1}{2}}nv\frac{\partial v}{\partial\eta} + \frac{n}{\lambda_0}\frac{\partial\phi}{\partial\xi} +$$
$$\varepsilon^{\frac{1}{2}}n\frac{\partial\phi}{\partial\eta} + \frac{2\sigma}{\lambda_0}\frac{\partial n}{\partial\xi} + \varepsilon^{\frac{1}{2}}2\sigma\frac{\partial n}{\partial\eta} = 0 \quad (3.3b)$$

$$\frac{\varepsilon^{\frac{1}{2}}}{\lambda_0^2}\frac{\partial^2\phi}{\partial\xi^2} + \frac{2\varepsilon}{\lambda_0}\frac{\partial^2\phi}{\partial\xi\partial\eta} + \varepsilon^{\frac{3}{2}}\frac{\partial^2\phi}{\partial\eta^2} -$$
$$\frac{\varepsilon}{\lambda_0^2}\frac{\partial\lambda_0}{\partial\eta}\frac{\partial\phi}{\partial\xi} + n - n_{el} - n_{eh} = 0 \quad (3.3c)$$

Using the well-known reductive perturbation technique, which requires the expansion of dependent quantities like densities, velocities, potentials etc. about their equilibrium position in the form of smallness parameter ε, the smallness of which determines the order/strength of the perturbation. The lower power of ε imply that physical quantity varies at a faster rate in comparison with the one that carries higher power of ε. So, it is clear from (3.1) that the physical quantities very slowly with space in comparison with their variation with time and hence the phase velocity λ_0 can be treated as slowly varying function. Due to the very large scale length of the density inhomogeneity in our present plasma model, the phase velocity λ_0 is treated to be constant and hence the inhomogeneity contributes through the density gradient only. Therefore we expand the densities, fluid velocity and electric potential in terms of ε as follows:

$$\left.\begin{aligned} n &= n_0 + \varepsilon n_1 + \varepsilon^{\frac{3}{2}}n_2 + \varepsilon^2 n_3 + ... \\ n_{el} &= n_{el0} + \varepsilon n_{el1} + \varepsilon^{\frac{3}{2}}n_{el2} + \varepsilon^2 n_{el3} + ... \\ n_{eh} &= n_{eh0} + \varepsilon n_{eh1} + \varepsilon^{\frac{3}{2}}n_{eh2} + \varepsilon^2 n_{eh3} + ... \\ v &= v_0 + \varepsilon v_1 + \varepsilon^{\frac{3}{2}}v_2 + \varepsilon^2 v_3 + ... \\ \phi &= \varepsilon\phi_1 + \varepsilon^{\frac{3}{2}}\phi_2 + \varepsilon^2\phi_3 + ... \end{aligned}\right\} \quad (3.4)$$

In view of the inhomogeneity of the plasma in space, the zeroth order quantities are taken to be the functions of η only and hence

$$\frac{\partial n_0}{\partial\xi} = \frac{\partial n_{el0}}{\partial\xi} = \frac{\partial n_{eh0}}{\partial\xi} = \frac{\partial v_0}{\partial\xi} = \frac{\partial\lambda_0}{\partial\xi} = 0 \quad (3.5)$$

By putting (3.4) and using (3.5) in (3.3a)-(3.3c) we obtain equations of orders of ε^0, $\varepsilon^{\frac{1}{2}}$, ε and $\varepsilon^{\frac{3}{2}}$.

From O (ε^0), we get

$$n_0 = n_{el0} + n_{eh0} \quad (3.6)$$

From O ($\varepsilon^{\frac{1}{2}}$), we get

$$n_0 \frac{\partial v_0}{\partial \eta} + v_0 \frac{\partial n_0}{\partial \eta} = 0 \quad (3.7a)$$

$$n_0 \frac{\partial v_0}{\partial \eta} + \frac{2\sigma}{v_0} \frac{\partial n_0}{\partial \eta} = 0 \quad (3.7b)$$

Equating the coefficients, we get

$$\frac{2\sigma}{v_0} = v_0 \quad \Rightarrow v_0 = \sqrt{2\sigma} \quad (3.8)$$

From O (ε), we get

$$-\frac{\lambda_0 - v_0}{\lambda_0} \frac{\partial n_1}{\partial \xi} + \frac{n_0}{\lambda_0} \frac{\partial v_1}{\partial \xi} = 0 \quad \Rightarrow v_1 = \frac{\lambda_0 - v_0}{n_0} n_1$$

(3.9)

$$\left[-\lambda_0 + 2v_0\right] \frac{\partial n_1}{\partial \xi} + \frac{n_0}{\lambda_0} \frac{\partial \phi_1}{\partial \xi} = 0 \quad (3.10)$$

$$n_1 = \left(n_{el0} \frac{T_{eff}}{T_{el}} + n_{eh0} \frac{T_{eff}}{T_{eh}} \right) \phi_1 = n_0 \phi_1 \quad (3.11)$$

From (3.9) - (3.11) we get

$$\lambda_0 - v_0 = \sqrt{(1 + 2\sigma)} \quad (3.12)$$

For the next higher order of ε i.e. O ($\varepsilon^{\frac{3}{2}}$), we get

$$-\frac{\lambda_0 - v_0}{\lambda_0} \frac{\partial n_2}{\partial \xi} + \frac{n_0}{\lambda_0} \frac{\partial v_2}{\partial \xi} + n_0 \lambda_0 \frac{\partial \phi_1}{\partial \eta} + \lambda_0 \phi_1 \frac{\partial n_0}{\partial \eta} = 0$$

(3.13)

$$-\frac{n_0(\lambda_0 - v_0)}{\lambda_0} \frac{\partial v_2}{\partial \xi} + n_0 v_0 (\lambda_0 - v_0) \frac{\partial \phi_1}{\partial \eta} + n_0 (\lambda_0 - v_0) \phi_1 \frac{\partial v_0}{\partial \eta} +$$
$$n_0 v_0 \phi_1 \frac{\partial v_0}{\partial \eta} + \frac{n_0}{\lambda_0} \frac{\partial \phi_2}{\partial \xi} + n_0 \frac{\partial \phi_1}{\partial \eta} + \frac{2\sigma}{\lambda_0} \frac{\partial n_2}{\partial \xi} + 2\sigma n_0 \frac{\partial \phi_1}{\partial \eta} +$$
$$\frac{2\sigma}{n_0} \phi_1 \frac{\partial n_0}{\partial \eta} = 0 \quad (3.14)$$

$$\frac{1}{\lambda_0^2} \frac{\partial^2 \phi}{\partial \xi^2} + n_2 - n_0 \phi_2 + \frac{4}{3} \left[n_{el0} b_l \left(\frac{T_{eff}}{T_{el}} \right)^{3/2} + n_{ch0} b_h \left(\frac{T_{eff}}{T_{eh}} \right)^{3/2} \right] = 0$$

Differentiating this equation w.r.t. ξ, we get

$$\frac{1}{\lambda_0^3} \frac{\partial^3 \phi}{\partial \xi^3} + \frac{1}{\lambda_0} \frac{\partial n_2}{\partial \xi} - \frac{n_0}{\lambda_0} \frac{\partial \phi_2}{\partial \xi} + \frac{2}{\lambda_0} \left[n_{el0} b_l \left(\frac{T_{eff}}{T_{el}} \right)^{3/2} + n_{ch0} b_h \left(\frac{T_{eff}}{T_{eh}} \right)^{3/2} \right] \phi_1^{1/2} \frac{\partial \phi_1}{\partial \xi} = 0$$

(3.15)

Eliminating second order quantities from (3.13)-(3.15), finally we get the following modified KdV equation

$$\frac{\partial \phi_1}{\partial \eta} + \alpha \phi_1^{1/2} \frac{\partial \phi_1}{\partial \xi} + \beta \frac{\partial^3 \phi}{\partial \xi^3} + \gamma \phi_1 \frac{\partial n_0}{\partial \eta} = 0$$

(3.16)where

$$\left.\begin{aligned} \alpha &= \frac{1}{n_0 \lambda_0^2 (\lambda_0 - v_0)} \left[n_{el0} b_l \left(\frac{T_{eff}}{T_{el}} \right)^{3/2} + n_{ch0} b_h \left(\frac{T_{eff}}{T_{eh}} \right)^{3/2} \right] \\ \beta &= \frac{1}{2 n_0 \lambda_0^4 (\lambda_0 - v_0)} \\ \gamma &= \frac{n_0 \lambda_0 (\lambda_0 - 2v_0) + 2\sigma}{2 n_0^2 \lambda_0 (\lambda_0 - v_0)} \end{aligned}\right\} \quad (3.17)$$

Solution of the modified KdV equation: To find the solution of modified KdV equation (3.16) we use the following transformations

$$\phi_1 = b(\eta) \overline{\psi}, \qquad b(\eta) = \exp\left[-\int_\eta \gamma \left(\frac{\partial n_0}{\partial \eta} \right) d\eta \right] \quad (4.1)$$

Using these transformations, (3.16) becomes

$$\frac{\partial \overline{\psi}}{\partial \eta} + \alpha b^{1/2} \overline{\psi}^{1/2} \frac{\partial \overline{\psi}}{\partial \xi} + \beta \frac{\partial^3 \overline{\psi}}{\partial \xi^3} = 0 \quad (4.2)$$

Putting $\overline{\psi}^{1/2} = \psi$ in this equation, we get

$$\psi \frac{\partial \psi}{\partial \eta} + \alpha b^{1/2} \psi^2 \frac{\partial \psi}{\partial \xi} + \beta \left[3 \frac{\partial \psi}{\partial \xi} \frac{\partial^2 \psi}{\partial \xi^2} + \psi \frac{\partial^3 \psi}{\partial \xi^3} \right] = 0 \quad (4.3)$$

The solution of (4.3) cannot be determined by ordinary methods due to the presence of variable coefficients. So we use a more appropriate method known as sine-cosine method [25]

To solve (4.3), we use another variable transformation as

$$\zeta = g(\xi - U\eta), \ \psi(\xi, \eta) = \Phi(\xi) \quad (4.4)$$

The equation (4.3) becomes

$$-U\Phi \frac{d\Phi}{d\zeta} + \alpha b^{1/2} \Phi^2 \frac{d\Phi}{d\zeta} + \beta k^2 \left[3 \frac{d\Phi}{d\zeta} \frac{d^2\Phi}{d\zeta^2} + \Phi \frac{d^3\Phi}{d\zeta^3} \right] = 0$$

(4.5)

Using Sine-Cosine method, the solution of (4.5) can be written as

$$\Phi(\omega) = A_0 + \sum_{i=1}^{p} (B_i \sin\omega + A_i \cos\omega) \cos^{i-1}\omega$$

(4.6)

Where $\frac{d\omega}{d\zeta} = \sin\omega$ and A_i, B_i are functions of ζ and ω but they will not appear explicitly as functions of $\sin\omega$ and $\cos\omega$. Also, p is determined by the balance of the leading order of nonlinear to linear terms. As we have a nonlinearity of lower order, so we take p=2 in our present case. With this, the solution in the form of intermediate variable ω can be written as

$$\Phi(\omega) = A_0 + A_1 \cos\omega + B_1 \sin\omega + A_2 \cos^2\omega + B_2 \cos\omega \sin\omega \quad (4.7)$$

To determine the coefficients A_i, B_i, U and k putting the values of $\Phi(\omega)$ from (4.7) in (4.5) and then the coefficients of the various trigonometric identities are put equal to zero. As the odd functions $\sin\omega$, $\cos\omega \sin\omega$ etc. do not play any rule in solution yielding $A_1 = B_1 = B_2 = 0$.

Finally we get

$$A_0 = -A_2 = \frac{13}{6\alpha b^{1/2} U}, \qquad k = \sqrt{\frac{1}{12 U^3 \beta}}$$

(4.8)

and consequently, the solution of modified

KdV equation (3.16) is obtained as

$$\Phi(\omega) = \frac{13}{6\alpha b^{1/2} U} \sec h^2 \left[\sqrt{\frac{1}{12U^3 \beta}} (\xi - U\eta) \right] \quad (4.9)$$

In terms of ϕ_1, this solution becomes

$$\phi_1 = \frac{169}{36\alpha^2 U^2} \sec h^4 \left[\sqrt{\frac{1}{12U^3 \beta}} (\xi - U\eta) \right] = \phi_m \sec h^4 \left[\sqrt{\frac{(\xi - U\eta)}{W}} \right] \quad (4.10)$$

where the peak amplitude ϕ_m and width w of the soliton are

$$\phi_m = \frac{169}{36\alpha^2 U^2} = \frac{169 n_0^2 \lambda_0^4 (\lambda_0 - v_0)^2}{36 \left[n_{el0} b_l \left(\frac{T_{eff}}{T_{el}} \right)^{3/2} + n_{ch0} b_h \left(\frac{T_{eff}}{T_{eh}} \right)^{3/2} \right]^2 U^2} \quad (4.11)$$

$$w = \sqrt{\frac{1}{12U^3}} = \sqrt{\frac{n_0 \lambda_0^4 (\lambda_0 - v_0)}{6U^3}} \quad (4.12)$$

Conclusion: From the three relations (4.10) – (4.12), we can conclude that the peak amplitude depends upon the densities, temperature of different components as well as on the nonisothermal parameters of electrons. As the peak amplitude is always positive, so only compressive solitons are possible. Peak amplitude of the solitons increases as the phase velocity increases and decreases as shift velocity increases. In case of soliton width w, it is depended upon densities, phase velocity as well as shift velocity but independent of nonisothermal parameters and temperature of ions, electrons. Soliton width increases as density and phase velocity increases and decreases as shift velocity increases.

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