g*s-CLOSED SETS IN BIMINIMAL STRUCTURE SPACES

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Abstract: In this paper, we introduce the concept of (i,j) –mg*s-closed sets in biminimal structure spaces and study some of their properties.

Key words: m-structure, (i,j)-mg*s-closed sets, (i,j)-mgs-closed sets, (i,j)-msg-closed sets.

Introduction: Valeriu Popa and Takshi.Noiri [8] introduced the concept of minimal structure on a nonempty set. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-cl and m_X-int operators respectively. They introduced m-continuous functions [7] and studied some of its basic properties.T.Noiri introduced the concept of mg-closed sets in minimal structures which is analogs to g-closed sets in topological space introduced by Levine. Pushpalatha and Anitha [1] introduced mω-closed sets in minimal structures and g*s-closed sets [2] in topological spaces. N.Selvanayaki introduced on-g*s-closed sets [6] in bitopological spaces. S.Saranya and A.Parvathi[10] introduced Gπ-closed sets in biminimal structure spaces. Pushpalatha.A and K.Anitha [9] introduced mg*s-closed sets in minimal structures. C.Boonpok [3] introduced the concept of biminimal structure spaces and studied some fundamental properties of m_X^1, m_X^2-closed sets and m_X^1, m_X^2-open sets in biminimal structure spaces. Moreover,C.Boonpok [4] introduced the notion of M-continuous functions on biminimal structure spaces and studied some characterizations and several properties of such functions.

In this paper we introduce the concept of (i,j)-mg*s-closed sets in biminimal structure spaces and study some of their properties.

Preliminaries

Definition:2.1 [3] Let X be a nonempty set and P(X) be the power set of X. A sub family $m_X$of P(X) is called a minimal structure (briefly m-structure) if $\emptyset \in m_X$ and X$\in m_X$. By (X,m_X).
we denote a nonemptyset X with a m-structure on X and it is called a m-space. Each member of $m_X$is said to be a m_X-open set and the complement of a m_X-open set is said to be m_X-closed.

Definition:2.2 [3] Let X be a nonempty set and m_X be m-structure on X. For a subset A of X the m_X-closure of A and the m_X -interior of A are defined as follows:

(1) m_X-cl(A)=∩{F/A⊆F,X-F∈ m_X}

(2) m_X-Int(A)=∪{U / U ⊆A, U ∈ m_X}

Definition:2.3 [9] Let X be a non-empty set and P(X) the power set of X.A subfamily $m_X$of P(X) is called a minimal structure (m-structure) on X if ∅∈m_X and X ∈m_X. The pair (X, m_X) is called minimal space (or m-space).

Definition:2.4[5]Let X be a nonempty set and m_X-a minimal structure on
X. For subsets A and B of X the following properties hold.
(i) m_X-cl(X-A)= X-m_X-int(A) and m_X–Cl(A).
(ii) If (X-A)∈m_X then m_X-cl(A)= A and if A∈m_X then m_X-int(A)=A.
(iii) m_X -cl(∅)=∅,m_X -cl(X)=X, m_X -int(∅)=∅, and m_X -int(X)=X.
(iv) If A⊂B then m_X -cl(A)⊂m_X -cl(B) and m_X -cl(A) ⊂m_X -cl(A)
(v) A⊂m_X -cl(A) and m_X -int(A) ⊂A
(vi) m_X -cl(m_X -cl(A)) = m_X -cl(A) and m_X -int(m_X -int(A))= m_X -int(A).

Lemma:2.5 Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then x∈m_X -cl(A) iff U ∩ A≠∅ for every U ∈m_X containing X.

Definition:2.6 [9] A minimal structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belong to m_X.

Remark:2.7 A minimal structure m_X with property B coincides with a generalized topology on the sense of Lugojan.

Lemma:2.8 [9] Let X be a non-empty set and m_X a minimal structure on X satisfying property B. For a subset A of X, the following properties hold:
(i)A∈m_X iff m_X -int(A)=A.
(ii) A is m_X -closed iff m_X - Cl(A)=A.
(iii) m_X -int (A)∈m_X and m_X - Cl(A) is m_X -closed.

Definition:2.9 Let (X, m_X) be an m-space, A subset A of X is said to be mg-closed [9] if m_X - Cl(A)⊑G whenever A⊑G and G is m_X –open. The complement of mg-closed set is said to be mg-open set.

Definition:2.10 Let (x, m_X) be an m-space.A subset A of X is called m_X - semi-open if there exists an m_X - open set G such that G⊑A⊑m_X - Cl(G). A subset A of X is said to be m_X -semi-closed [9] if A^c is m_X - semi-open.

Definition:2.11 [9] Let (X, m_X) be an m-space. A subset A of X is said to be minimal generalized semi-closed b(briefly mgs-closed) if m_X - scl(A) ⊑U. Whenever A⊑U and U is m_X -open in X.

Definition:2.12 [9] Let (X, m_X) be an m-space. A subset A of X is said to be minimal semi generalized closed (briefly msg-closed) if A ⊑U and U is m_X - semiopen in X.

Definition:2.13 [9] Let (X, m_X) be an m-space. A subset A of X is said to be mg*s-closed set if m_X -scl(A) ⊆U Whenever A⊆U and U is gs -open.

Definition:2.14 [4] Let X be a nonempty set and Let m_X^1, m_X^2 be minimal structures on X. A triple (X, m_X^1, m_X^2) is called a biminimal structure space (briefly bim-space).

Throughout this paper, (X, m_X^1, m_X^2) denote a biminimal structure space and A is a subset of X. The m_X-closure and m_X-interior of A with respect to m_X^i are denoted by m_X^i-cl(A) and m_X^i-Int(A), respectively, for i=1,2.

Definition:2.15 [4] For a f:$(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$, the following properties are equivalent:

(a) f is (i,j)-M-continuous at a point x∈X.

(b) x∈ m_X^j-Int(f^{-1}(V)) for every V∈ $m_Y i$containing f(X).

(c) x∈ $f^{-1}(m_Y i$-cl(f(A))) for every subset A of X with x ∈ m_X^j-cl(A).

(d) x∈ $f^{-1}(m_Y i$-cl(B)) for every subset B of Y with x∈ m_X^j-cl(f^{-1}(B)).

(e) x∈ m_X^j-int(f^{-1}(B) for every subset B of Y with x∈ $f^{-1}(m_Y i$-Int(B))

(f) x∈ f^{-1} (F) for every $m_Y i$-closed set F of Y such that x∈ m_X^j-cl(f^{-1}(F)).

Definition:2.16 [4] For a function f:$(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$, the following properties are equivalent:

(a)f is (i,j)-M-continuous.

(b)f^{-1}(V)=m_X^j-int(f^{-1}(V)) for every V ∈ m_Y^i

(c)f(m_X^j-cl(A))⊆ $m_Y i$-cl(f(A)) for every subset A of X.

(d)m_X^j-clf^{-1}(B))⊆ $f^{-1}(m_Y i$-cl(B)) for every subset B of Y.

(e)$f^{-1}(m_Y i$-Int(B)) ⊆ $m_X^{j\text{-INt}(}f^{-1}(B))$ for every subset B of Y.

(f)m_X^j-cl(f^{-1}(F) for every m_Y^i-closed set F of Y.

Definition:2.17 A subset A of biminimal structure space (X, m_X^1, m_X^2) is said to be $m_X^{(i,j)}$-closed if m_X^i-cl(m_X^j-cl(A))=A. Where i,j=1,2 and i≠j.

The complement of a $m_X^{(i,j)}$-closed set is said to $m_X^{(i,j)}$-open.

3.g*s-closed sets in Biminimal structure spaces

In this section we introduce the concept of (i,j)-mg*s-closed sets in Biminimal structure spaces and study some of their properties.

Definition :3.1 A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i,j)-msg-closed set if m_X^j-scl(A)⊆U Whenever A⊆U and U is m_X^i –sg open in X where i,j=1,2 and i=j .

The complement of (i,j)-msg closed set is (i,j)-msg open set.

Definition :3.2 A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i,j)-mgs-closed set if m_X^j-scl(A)⊆U Whenever A⊆U and U is m_X^i –open in X where i,j=1,2 and i=j .

The complement of (i,j)-mgs closed set is (i,j)-mgs open set.

Definition :3.3 A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i,j)-mg*s-closed set if m_X^j-scl(A)⊆U Whenever A⊆U and U is m_X^i –gs open in X where i,j=1,2 and i=j .

The complement of (i,j)-mg*s closed set is (i,j)-mg*s open set.

Definition :3.4 A subset A of a biminimal structure space (X, m_X^1, m_X^2) is called pairwise (i,j)-mg*s-closed set if (1,2)- mg*s closed set and (2,1)-mg*s closed set.

The complement of pairwise (i,j)-mg*s closed is (i,j)-mg*s open.

Remark:3.5 The family of all (i,j)-mg*s-closed sets (resp. (i,j)-mg*s-open sets) of (X, m_X^1, m_X^2) is denoted by (i,j)-mg*sC(x) (resp. (i,j)-mg*sO(x)) i,j=1,2 and i= j.

Theorem:3.6 Every m_X^j-closed subset of a biminimal structure space (X, m_X^1, m_X^2) is (i,j)-mg*s-closed set.

Proof: Let A be a m_X^2 –closed set in X. Let U be a gs-open in m_X^1 such that A⊆U . Since A is m_X^2-closed , m_X^2- cl(A)=A, m_X^2- cl(A)⊆U. But m_X^2- scl(A)⊆m_X^2-cl(A)⊆U. Hence m_X^2- scl(A)⊆U. Hence A is (i,j)-mg*s closed set in X.

The converse of the above theorem need not be true in the following example.

Example:3.7 Let X ={a,b,c} consider tow minimal structures m_X^1={∅,X,{a},{b},{a,b}} and m_X^2={∅,X,{b},{ b,c }}.Then the set A={c} is (1,2)- mg*s closed set but not m_X^2-closed in (X, m_X^1, m_X^2)

Theorem:3.8 If A and B are (i,j)-mg*s closed then A∪B is (i,j)-mg*s closed.

Proof: Let U be a gs-open in m_X^1 such that A∪B⊆U. Then A⊆U and B ⊆U. Since A and B are (i,j)-mg*s closed set then m_X^2- scl(A)⊆U and m_X^2- scl(B)⊆U. Hence m_X^2- scl(A∪B)⊆ m_X^2- scl(A)∪m_X^2- scl(B)⊆U. That is m_X^2- scl(A∪B)⊆ U whenever A∪B⊆U. Hence A∪B is (i,j)-mg*s closed.

Theorem:3.9 In a biminimal structure space (X, m_X^1, m_X^2), every (i,j)- mg*s-closed set is (i,j)- mgs-closed set.

Proof: Let A⊆U is U is m_x-open in m_x^1 . Since every m_x^1-open is m_x^1-gs open and A is (i,j)-mg*s-closed, we have m_X^2- scl(A)⊆U. Therefore A is (i,j)- mgs-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example:3.10 Let X={a,b,c}.Consider two minimal structures m_x^1={∅,X,{b},{ b,c }} and m_X^2={∅,X,{a},{ a,c }}. Then A={a} is (1,2)-gs closed but not (1,2) –mg*s closed.

Theorem:3.11 In a biminimal structure space (X, m_X^1, m_X^2), every m_x^2-semiclosed set is (i,j)- mg*s-closed set.

Proof: Let A be a m_x^2-semiclosed. Let U be a m_x^1-gs open such that A⊆U. Since A is m_x^2-semiclosed, we have m_X^2- scl(A)=A. Therefore m_X^2- scl(A)⊆U. Hence A is (i,j)- mg*s-closed set.

The converse of the above theorem need not be true from the following example.

Example:3.12 Let X={a,b,c}.Consider two minimal structures m_x^1={∅,X,{b},{ b,c }} and m_X^2={∅,X,{c}}. Then the subset{a,c} is (1,2)-mg*s closed but not m_x^2-semiclosed.

Theorem:3.13 A subset A of (X,m_X^1, m_X^2) is (i,j)-mg*s-closed set iff m_X^2- scl(A)~A contains no nonempty m_X^1-gs closed set.

Proof: Suppose that F is a nonempty m_X^1-gs closed subset of m_X^2- scl(A)~A. Now F⊆ m_X^2- scl(A)~A. Then F⊆ m_X^2- scl(A)∩A^c. Therefore F⊆ m_X^2- scl(A) and F⊆A^c. Since F^c is m_X^1-gs open set and A is (i,j)-mg*s-closed, m_X^2- scl(A)⊆F^c. That is F⊆(m_X^2- scl(A)$)^c$. Hence F⊆ m_X^2- scl(A)∩ (m_X^2- scl(A)$)^c$=∅. Thatis F=∅. Thus m_X^2- scl(A)~A contains no nonempty m_X^1-gs closed set.

Conversely assume m_X^2- scl(A)~A contains no nonempty m_X^1-gs closed set. Let A⊆U, U is m_X^1-gs open set. Suppose that m_X^2- scl(A) is not contained in U. Then m_X^2- scl(U)∩U^c is a nonempty m_X^1-gs closed set and contained in m_X^2- scl(A)~A which is a contradiction. Therefore m_X^2- scl(A)⊆U and hence A is (i,j)- mg*s-closed set.

Theorem:3.14 For each element X of (X,m_X^1, m_X^2),{x} is m_X^1-gs closed set or $\{x\}^c$ is (i,j)- mg*s-closed set.

Proof: If{x} is not m_X^1-gs closed set, then $\{x\}^c$ is not m_X^1-gs open and a m_X^1-gs open set containing $\{x\}^c$ is x only. Also, m_X^2- scl($\{x\}^c$)⊆X. Therefore $\{x\}^c$ is (i,j)-mg*s-closed set.

Theorem:3.15 If A is a (i,j)- mg*s-closed set of (X,m_X^1, m_X^2) such that A⊆B⊆m_X^j- scl(A) then B is (i,j)- mg*s-closed set when i,j=1,2 and i≠j.

Proof: Let A be a (i,j)- mg*s-closed set and A⊆B⊆m_X^j-scl(A). Let B⊆U and U is m_X^i-gs open. Then A⊆U. Since A is (i,j)- mg*s-closed, we have m_X^j- scl(A)⊆U. Since B⊆ m_X^j- scl(A), then m_X^j- scl(B)⊆ m_X^j-scl(A)⊆U. Hence B is (i,j)- mg*s-closed set.

REMARK:3.16 (1,2)- mg*s-C(X) is generally not equal to (2,1)- mg*s-C(X) as seen from the following example.

Example:3.17 Let X={a,b,c}.Consider two minimal structures m_x^1={∅,X,{b},{ b,c }} and m_X^2={∅,X,{c}}. Then (1,2)- mg*s-C(X)={∅,X,{a},{b},{a,b},{b,c},{a,c}} and (2,1)- mg*s-C(X)= {∅,X,{a},{c},{a,b},{a,c}}. Thus (1,2)- mg*s-C(X) is not equal to (2,1)- mg*s-C(X).

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