A DIFFERENT APPROACH OF INTRODUCING BETA AND GAMMA FUNCTION TO FIRST YEAR UNDERGRADUATE ENGINEERING STUDENTS

HIREMATH SURESH SHIVARUDRAPPA , BIRAJDAR GAJANAN MALLESHI

Abstract: In this paper we have tried to introduce the general types of definite integrals which are to be solved by gamma and beta functions.

Keywords: Beta Function, Definite integrals, Engineering student,Gamma Function.

Introduction: While reading through many engineering mathematics books, we have observed that the definite integrals which are to be solved by using beta and gamma functions are classified into different types of integrals. Here in this paper we have tried to reduce the types of integrals to simple minimum number of types. The types which we have introduced here cover all definite integrals which are solvable by beta and gamma functions. Our attempt is to introduce beta and gamma functions in such a way that the First year engineering undergraduate student can easily memorize the minimum number of types so that they can easily solve the examples of definite integrals using beta and gamma functions.

Gamma Function

Definition:

Let n be a positive integer. The gamma function of n is denoted by Γn and is defined by the following integral

$$\Gamma n = \int_0^\infty e^{-x}\, x^{n-1}\, dx$$

That is $$\Gamma(n+1) = \int_0^\infty e^{-x}\, x^{n}\, dx$$

The definite integrals which can be solved by gamma function are classified into the following five types

Type I) Integrals of the type $\int_0^\infty e^{-ax^n}\, dx$

Type II) Integrals of the type $\int_0^\infty x^m e^{-ax^n}\, dx$

Type III) Integrals of the type $\int_0^1 x^m (\log x)^n\, dx$

Type IV) Integrals of the type $\int_0^\infty \frac{x^a}{a^x}\, dx$

Type V) Integrals of the type $\int_0^\infty a^{f(x)}\, dx$

The above integrals can be solved by using the following substitutions

1) $ax^n = t$
2) $ax^n = t$
3) $\log x = -t$
4) $a^x = e^t$
5) $a^{f(x)} = e^{-t}$ respectively.

Here we have reduced above types into the following three simple types

Type I) Integrals of the type $\int_0^\infty x^m e^{f(x)}\, dx$

Type II) Integrals of the type $\int_0^1 x^m (\log x)^n\, dx$

Type III) Integrals of the type $\int_0^\infty x^a a^{f(x)}\, dx$

Which are solved by using substitutions

1) $f(x) = -t$
2) $\log x = -t$
3) $a^{f(x)} = e^{-t}$ respectively.

We have combined first two types into a single type I. Also fourth and fifth types are combined into a single type III.

With the above three types it becomes very simple and better understanding of solving definite integrals using gamma function.

Beta Function

Definition:

Let m, n be two positive integers. The beta function of m and n is denoted by B(m, n) and is defined by the following definite integral

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$$

That is $$B(m+1,n+1) = \int_0^1 x^m (1-x)^n\, dx$$

That is $$B(m+1,n+1) = \int_0^1 (x-0)^m (1-x)^n\, dx$$

The definite integrals which can be solved by using beta functions are classified into the following types

Type I) Integrals of the type $\int_0^a x^m (a-x)^n\, dx$

Type II) Integrals of the type $\int_0^1 x^m \left(1 - x^n\right)^p \, dx$

Type III) Integrals of the type $\int_0^1 \left(1 - \sqrt[n]{x}\right)^m dx$

Type IV) Integrals of the type $\int_a^b (x-a)^m (b-x)^n \, dx$

Type V) Integrals of the type $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx$

The above types of integrals can be solved by using the following substitutions

1) $x = at$

2) $x^n = t$

3) $x^{\frac{1}{n}} = t$

4) $x = a + (b-a)t$

5) $bx = \frac{at}{1-t}$ respectively.

Here we have reduced above types into the following three simple types

Type I) Integrals of the type $\int_0^a x^m \left(a^n - x^n\right)^p \, dx$

Type II) Integrals of the type $\int_a^b (x-a)^m (b-x)^n \, dx$

Type III) Integrals of the type $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx$

Which are solved by using substitutions

1) $x^n = a^n t$

2) $x = a + (b-a)t$

3) $bx = \frac{at}{1-t}$ respectively

We have combined first three previous types of integrals into a single type.

With the above three types it becomes very simple and better understanding of solving definite integrals using beta function.

Conclusion:

For engineering students finding an efficient way of learning mathematics is very important. To make their learning better mathematics should be introduced to them in a general way. As they do not have sufficient intention and time to study mathematics rigorously. In this paper we have presented introduction of beta and gamma function in a simple and general way. Engineering students will find this very easy and better to understand the concepts of beta and gamma function

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sur.hiremath@gmail.com; ganesh61987@gmail.com

Assistant Professor, Brahmdevdada Mane Institute of Technology, Solapur.
Solapur University, Solapur Maharashtra (INDIA)