IMPROVING SEARCH AND RETRIEVAL OF REUSABLE LEARNING RESOURCES IN A UNIVERSITY CONSORTIUM ENVIRONMENT BY T-CONCEPT LATTICE AND FUZZY GRAPH

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Abstract: In a University Consortium environment, different content providers contribute learning content for the courses offered by the University Consortium. Large networked repositories of learning material may be accessed by a University Consortium education system.In consortium environment, learning objectsare developed according to the standards like the ones established by IMS, ADL (SCORM) or IEEE (LOM) in order to obtain its complete reusability.But, tools are needed to narrow down the available resources for a particular group of learners based on the learning context; in other words, take into account the learning objectives, pedagogical approaches, user profiles, etc.In this paper an approach for search and retrieving existing learning resources is presented with the help of*t*-concept lattice and fuzzy graph exploiting thelearning object metadata (LOM).

Keywords: University Consortium; Web Services; Learning Object (LO); Pedagogical Principles; SCO; SCORM; fuzzy sets; fuzzy graph; formal concept analysis.

Introduction : For the last decade, E-Learning has become an active research area. One of the major developments in e-learning this decade has been the notion of reuse of digital resources i.e., resources created for one particular learning context, made available for reuse in another context.

Many Universities and institutional organizations are now providing large amounts of online learning resources. These learning resources have covered most common education and learning areas and subjects and are always available.There are numerous benefits that accrue when learning objects created by others are reused and re-purposed in new instructional situations. Presumably, this would lead to a considerable reduction in the time and effort taken to produce new content, compared to developing the content from scratch. The reused content may even be of a higher quality than if developed from scratch. Learning Object Metadata (LOM) specifies standardized *metadata* on each learning object and is intended to facilitate the discovery, management, and exchange of learning objects by learners, instructors, and automated systems over the Web.

One of the key issues in using learning objects is their identification by search engines or content management systems. This is usually facilitated by assigning descriptive learning object metadata. The most important pieces of metadata typically associated with a learning object include:

- **Objective:** The educational objective the learning object is instructing
- **Prerequisites:** The list of skills (typically represented as objectives) which the learner must know before viewing the learning object
- **Topic:** Typically represented in a taxonomy, the topic the learning object is instructing
- **Interactivity:**The Interaction Model of the learning object.
- **Technology requirements:** The required system requirements to view the learning object.

Learning objects in repositories are usually described by metadata. Therefore, access to learning objects can take advantage of queries upon metadata for selecting the objects that are most suited to the needs of learners or teachers.

The use of metadata, data about data, can help with this process. Content packaging metadata can be used to hold an aggregation of learning objects together with associated information such as digital rights, descriptive metadata and evaluations.

In a University Consortium environment, different content providers contribute content for courses offered by the consortium[Zaman 2007]. Content provider develops contents following the guidelines of University Consortium including the guideline ofSCORM-compliance for LOs.

But before accepting any content from any provider, it is thoroughly checked by a series of experts (content, instructional expert, instructional designer of sequencing and profiles). The Consortium environment has provided with an automatic process of stamping modification requirements and effective modification of the same [Zaman 2011]. After approval, all remote contents will be available during search operation for content aggregation.

Web Services technology remains a possible technology in academic business that is incorporated in the management of resources and supports communication between member institutions and University Consortium.

The basis for web services incorporates standards for describing, publishing, discovering and binding application interfaces. The set of standards includes a standard specifications for public registries known as

Universal Description Discovery and Integration (UDDI), a description language namely Web Services Description Language (WSDL), a distributed object communication protocol called Simple Object Access Protocol (SOAP) and a dynamic self-defining information specification language with semantic support known as eXtendable Markup Language (XML).

It is assumed that whenever the system wants to develop and deliver a new learning resource (module), it aggregates all the required SCOs using SCORM Aggregation Model. The system first searches its own repository and then it searches the UDDI registry using standard metadata to collect the SCOs. The system also checks the UDDI registry for the approved SCOs for this module.

Details of the learning objects can be investigated by examining number of concepts in any learning object, navigational map etc., using learning object metadata (LOM).

If University Consortium wants to offer a new course, it should have the capability to reuse the existing e-learning content. In this scenario improvement of Search and Retrieval of Reusable Learning Resources in University Consortium environment should be taken into consideration.

In present times, as an effective tool for data analysis, FCA (Formal Concept Analysis) has been extensively applied to fields such as decision making, information retrieval, data mining, knowledge discovery etc. Conceptual clusteringis a machine learning paradigm for unsupervised classification developed mainly during the 1980s. Conceptual clustering is closely related to formal concept analysis (FCA). It is distinguished from ordinary data clustering by generating a concept descriptionfor each generated class. Most conceptual clustering methods are capable of generating hierarchical category structures. FCA (Ganter & Wille, 1999; Wille, 1982) is a formal technique for data analysis and knowledge presentation. It defines formal contexts to represent relationships between objects and attributes in a domain. From the formal contexts, FCA can then generate formal concepts and interpret the corresponding concept lattice so that information can be browsed or retrieved effectively. The basic setting of this technique is based on bivalent logic. However, bivalent logic cannot be of much use as the bulk of information that confronts us is usually fuzzy and imprecise. Zadeh [Zadeh 1965] first introduced the mathematical model of vagueness by using the notion of partial degree of membership, in connection with automatic representation and manipulation of human knowledge. Since then, there have been documented instances of theoretical developments as well as successful applications of fuzzy set theory. As far as our knowledge is concerned, Burusco and Fuentes-Ganzález [Burusco & Fuentes-Ganzález 1994, Burusco & Fuentes-Ganzález 1998] first introduced the theory of FCA in fuzzy setting. Later, Pollandt [Pollandt 1997], Bělohlávek [Bělohlávek 1998], proposed the **L**-context to combine fuzzy logic with FCA. Nevertheless, the downside of FCA has been the existence of a large number of clusters [Bělohlávek 2002] combined with the fact that many of the existing approaches require computation of a whole fuzzy concept lattice which, often, is too large. Handling such large amount of clusters become an unwieldy task and usually impossible. Often it may be required to consider some particular concepts.Given such a situation, it would be of use if one could find a subset containing those required conceptsof the **L**-context without generating a whole set of fuzzy concepts from **L**-context.

In this paper, we present a fuzzy graph theoretic approach to determine a sublattice of the *t*-concept lattice from **L**-context with threshold *t* without generating whole set of *t*-concepts. Beginning by defining a fuzzy graph for a given *L*-context, we show a one-to-one correspondence between the *t*-concepts and cliques of *t*-level graph. Then, corresponding to each query concept we describe a process to cluster all related concepts of query concepts as a sub-lattice of a *t*-concept lattice.

The paper is structured as follows: In section 2,we briefly discuss about fuzzy sets, fuzzy logic, fuzzy graph, formal concept analysis, fuzzy contexts and*t*-concept lattice. The section 3 includes related work. An approach to improve the search Mathematical Model for Retrieval of Reusable Learning Resourcesis presented in the section 4. Section 5 focuses on the demonstration of approach using a sample example. Section 6 provides on conclusion and future work.

Preliminaries

Basics of fuzzy sets, fuzzy logic and Fuzzy Graph

In this section, we recall the basics of fuzzy sets, fuzzy logic and fuzzy graph [Dubois& Prade 1980, Bhattacharya 1987, John&Nair2000].

Since fuzzy logic are developed using general structure of truth degree, in this paper we use a complete residuated lattice as a basic structure of truthdegree. A complete residuated lattice is an algebra $\mathbf{L} = \langle L, \wedge, \vee, \otimes, \to, 0, 1 \rangle$ such that (1) $\langle L, \wedge, \vee, 0, 1 \rangle$ is a complete lattice with 0 and 1 being the leastand greatest element of *L*, respectively; (2) $\langle L, \otimes, 1 \rangle$ is a monoid; (3) $\otimes$ and $\to$ satisfy so called adjointness property, i.e., $a \otimes b \le c$ if and only if $a \le b \to c$, for each $a, b, c \in L$. All elements a of *L* are called truth degrees.Usually, the common choice of **L** is a structure with *L* = [0, 1], or a suitable subset of [0, 1] containing 0 and 1 . Now consideringcomplete residuated lattice **L** as a structure of truthdegree, we present the notions of **L**-set (fuzzy set) and **L**-relation (fuzzy relation). An **L**-set μ in a universe set *X* is a mapping $\mu : X \to L$. $\mu(x)$ is called the truth value (or membership value) of *x* in μ which maps *X* to the membership space *L*.Similarly, an **L**-relation I is a mapping $I : X \times Y \to L$ assigning to any $x \in X$ and $y \in Y$ a truth value *I*(*x*, *y*) to which *x* and *y*is related under *I*.For every $t \in L$, $\mu^t = \{x \in X \mid \mu(x) \geq t\}$ are called level sets or *t*-cut of μ. We let *supp* $(\mu) = \{x \in X \mid \mu(x) > 0\}$. We call *supp* (μ) the *support* of μ. A fuzzy set μ is nontrivial if *supp* $(\mu) \neq \phi$. In this paper, we use the notation $\vee$ for supremum and $\wedge$ for infimum. The set of all fuzzy subsets of *X* is denoted by $\wp(X)$ and is called the fuzzy power set of *X*.Let *h* be the function of $\wp(X)$ into [0, 1] defined by $h(\mu) = \vee\{\mu(x) \mid x \in X\}$ for all $\mu \in \wp(X)$. Then $h(\mu)$ is called the height of μ. Let μ and ν be any two fuzzy subsets of *X* then $\mu \subseteq \nu$ if $\mu(x) \leq \nu(x)$ for all $x \in X$. The union $\mu \cup \nu$ of μ, ν is a subset of X defined by $(\mu \cup \nu)(x) = \mu(x) \vee \nu(x)$ for all $x \in X$ and intersection $\mu \cap \nu$ of μ, ν is also a subset of X defined by $(\mu \cap \nu)(x) = \mu(x) \wedge \nu(x)$ for all $x \in X$ .The fuzzy graphs used in this work are finite and undirected. A fuzzy graph $G = (V, \mu, \rho)$ is a non empty set V together with a pair of functions $\mu : V \to [0,1]$ and $\rho : V \times V \to [0,1]$ such that for all x, y in V, $\rho(x, y) \leq \mu(x) \wedge \mu(y)$. μ is said to be the fuzzy vertex set of *G* and ρ the fuzzy edge set of G, respectively. For $P \subseteq V$, $H = (P, \nu, \tau)$ is called a fuzzy subgraph of $G = (V, \mu, \rho)$ induced by *P* if $\mu(x) = \nu(x)$ for all $x, y \in P$ and $\tau(x, y) = \rho(x, y)$ for all $x, y \in P$. For the sake of simplicity, we sometimes call H a fuzzy subgraph of *G*. Similarly, $H = (P, \nu, \tau)$ is said to be partial fuzzy subgraph of $G = (V, \mu, \rho)$ if $\nu \subseteq \mu$ and $\tau \subseteq \rho$. Let $G = (V, \mu, \rho)$ be a fuzzy graph. For any threshold $t \in [0,1]$, $\mu^t = \{x \in V \mid \mu(x) \geq t\}$ and $\rho^t = \{(x, y) \in V \times V \mid \rho(x, y) \geq t\}$. If $\mu^t \neq \phi$, then the crisp graph $G^t = (\mu^t, \rho^t)$ is said to be *t*-level graph of $G = (V, \mu, \rho)$.

t-concept lattice of data with fuzzy attributes

In this sub-section, we gives the basic notions of *t*-concept lattice of data with fuzzy attributes. A data table with fuzzy attributes, which represents the input table, is a triplet $\langle X, Y, I \rangle$ (**L**-context, in terms of foprmal concept analysis), where *X* is a setof objects,*Y*is a set of attributes, and*I* is a fuzzy relation between *X* and *Y*. $I(x, y) \in L$ (the set of truth values of complete residuated lattice **L**) is interpreted as the truth value of the fact, "the object $x \in X$ has the property $y \in Y$ ". $\langle X, Y, I \rangle$ can be thought of as a table with rows and columns corresponding to object $x \in X$ and attribute $y \in Y$, respectively, and table entries containing the degrees $I(x, y) \in L$.

Let $\langle X, Y, I \rangle$ be an **L**-context, where X and Y are set of objects and set of properties, respectively and I is a fuzzy relation between X and Y. Since the value $I(x, y)$ express the degree to which the object x carries the attribute y. If we set a threshold value $t \in L$ to eliminate the lower degree membership value from fuzzy relation, then the resulting relation is called t-cut of L-context which is basically a binary relation between X and Y and is denoted by I_t. For every confidence threshold $t \in L$, consider two sets: $A' = \{y \in Y \mid \text{for all } x \in A : I(x, y) \geq t\}$, i.e., the set of all attributes from Y shared by all objects of A at least with the degree t and $B' = \{x \in X \mid \text{for all } y \in B : I(x, y) \geq t\}$, i.e., the set of all objects from X sharing attributes from B at least in the degree t. The pair $\langle A, B \rangle \in 2^X \times 2^Y$ is called t-concept iff $A' = B$ and $B' = A$.

If $B\langle X, Y, I \rangle$ denotes the set of all concepts, i.e., $B\langle X, Y, I \rangle = \{\langle A, B \rangle \mid A^{\uparrow} = B, B^{\downarrow} = A\}$ and $\leq$ is a partial order relation on $B\langle X, Y, I \rangle$, defined by $\langle A_1, B_1 \rangle \leq \langle A_2, B_2 \rangle$ if and only if $A_1 \subseteq A_2$ (or, equivalently $B_1 \supseteq B_2$), then the $(B\langle X, Y, I \rangle, \leq)$ is an ordered set. It has some important properties: $(B\langle X, Y, I \rangle, \leq)$ is a complete lattice, called the concept lattice of $\langle X, Y, I \rangle$ [32].

Background and Related Works

In general, browsing and searchingfor learning objects is based on the typical authoritative metadata used for describing content, such as author, title or publicationdate, among others. A social layer can be built on top of a learning object repository, for providing final users with additional services fordescribing, rating and curating learning objects from a teaching perspective. All these interactions among users, services and resources can becaptured and further analyzed, so both browsing and searching can bepersonalized according to user profile and the educational context, helping users to find the most valuable resources for their learning process [Minguillón 2011].Reusable learning objects (RLO) are used in different contexts and are accompanied by their associated metadata for searching, managing, etc. Rahman (2006) proposed an application framework in order to share, search and visualize LOs in a peer-to-peer network. He also provides the user with a 3D interface for creating simple and advanced searches. The information mapped in the 3D metaphor allows an intuitive way to perceive the information and it assists in finding related resources. New users will find it easy to navigate the virtual roads of the exotic landscape using the car metaphor.

As there are more and more collections of such objects, more sophisticated search and reuse techniques are needed. Search and composition of learning objects (resources) that are available in the Web, an iterative search paradigm where the user navigates over the structure of the learning objects and chooses the interesting ones to compose his/herown objects. To achieve this goal, the user has access to intermediate results of the navigational search. The proposal is based on a query language [Gançarski 2007].

An approach that allows estimating the costs needed to adapt existing learning resources. First evaluations of the prototype are promising as the tool helps users in finding the existing learning resource that fits best their needs. The tool also tells users which adaptations have to be performed to adapt the existing resource to the user's requirements [Zimmermann 2007].

Learning objects can also bere-used in multiple contexts and pedagogic settings and can be grouped into coherent collections of digital learning content. In this paper, a fuzzy graph based approach is presented for improving the search capability of LOs in a University Consortium environment.

Finally a cluster of LOs is generated having the presence of topics (p_1,p_2, p_3, p_4, p_5 ....) and particular type of LO. From this cluster educators can choose the best LO that suits the requirement or servers the purpose.

Mathematical Model for Retrieval of Reusable Learning Resources

In this section, first we defining a fuzzy graph corresponding to the *L*-context we show that *t*-concepts can be found corresponding to each maximal cliques of *t*-level graph of the defined fuzzy graph. Then corresponding to each query concept, we cluster all neighbor concepts of query concepts as a sub-lattice of a *t*-concept lattice without generating whole set of *t*-concept.

Fuzzy Graph defined for a Fuzzy context

In this section, we construct an underlying fuzzy graph from a given fuzzy context which is used as a tool to generate all conceptual clusters.

Definition. Let $K = \langle X, Y, I \rangle$ be a fuzzy context where *X* is the set of objects and *Y* the set of properties. Let $\{\alpha_{O_i} \mid o_i \in X\}$ and $\{\beta_{p_j} \mid p_j \in Y\}$ be two family of fuzzy subsets of *Y* and *X* respectively, where for each $o_i \in X$, $\alpha_{o_i}(p_j) = I(o_i, p_j)$ for all $p_j \in Y$ and for each $p_j \in Y$, $\beta_{p_j}(o_i) = I(o_i, p_j)$ for all $o_i \in X$. We construct the fuzzy graph $G_I = (\mu, \rho)$, where $\mu : X \cup Y \to [0,1]$ is defined by

$\mu(o_i) = h(\alpha_{o_i})$ for all $o_i \in X$ and $\mu(p_j) = h(\beta_{p_j})$ for all $p_j \in Y$

and $\rho : (X \cup Y) \times (X \cup Y) \to [0,1]$ is defined by

$$\rho(o_i, o_j) = \begin{cases} h(\alpha_{o_i} \cap \alpha_{o_j}) \; if \; o_i, o_j \in X, i \neq j \\ 0 \; if \; i = j \end{cases}$$

$$\rho(p_i, p_j) = \begin{cases} h(\beta_{p_i} \cap \beta_{p_j}) \; if \; p_i, p_j \in Y, i \neq j \\ 0 \; if \; i = j \end{cases}$$

$\rho(o_i, p_j) = I(o_i, p_j)$ if $o_i \in X$ and $p_j \in Y$

From the obvious definition of the fuzzy graph corresponding to L-context we can easily check thatcorresponding to each clique *C* of *t*-level graph, $G_I^{\,t}$ there always is a unique *t*-concept of the **L**-context $K = \langle X, Y, I \rangle$, where $t \in [0,1]$. Because for $t \in [0,1]$, if *C* be any maximal clique of the *t*-level graph $G_I^{\,t}$, thenthe vertex set *C* of any clique of $G_I^{\,t}$ can be partitioned into two sets $A = \{o \mid o \in X\}$ and $B = \{p \mid p \in Y\}$. Since *C* is a maximal clique of $G_I^{\,t}$, therefore no other vertices in $G_I^{\,t} - C$ could be connected to all vertices in *C*, and also, $A = \{o \in X \mid$ for all $p \in B : I(x, y) \geq t\} = B'$ and $B = \{p \in Y \mid$ for all $o \in A : I(x, y) \geq t\} = A'$ .Therefore, if $C = A \cup B$ be a clique of *t*-level graph $G_I^{\,t}$, then $\langle A, B \rangle$ be the *t*-concept with extent *A* and intent *B*.

Procedure of finding a sublattice of the t-concept lattice

The following steps are involved to find a sublattice of the *t*-concept lattice:

***Step* 1:** Construction of the fuzzy graph corresponding to **L**-context using above definition.

Step 2: Query concept, $\langle A, B \rangle$ (t-concept) determination, i.e., maximal clique $C = A \cup B$ determination from t-level graph, where the threshold value $t \in L$ automatically chosen by user criterion. In this paper, we determine the query concept by the following way: if user query requirements are searching the documents containing the attributes, say, $\{p_i, p_j, p_k\}$, then query concept will corresponds any maximal clique of t-level graph containing the attributes-vertices $\{p_i, p_j, p_k\}$

Step 3: Elimination of the object-vertices of G_I^t which are not adjacent with any of the vertices in *B*. Since elimination of the object-vertex o_i which is not adjacent with any of the vertices contain in *B*, causes the elimination of the cliques containing the object-vertex o_i, as well as elimination of the concept containing the object o_i which are not comparable with $\langle A, B \rangle$.

Step 4: Elimination of the attribute-vertices of G_I^t which are not adjacent with any of the vertices in *A*. Since elimination of the object-vertex p_i which is not adjacent with any of the vertices contain in *A*, causes the elimination of the cliques containing the attribute-vertex p_i, as well as elimination of the concept containing the attribute p_i which are not comparable with $\langle A, B \rangle$.

Step 5: Elimination of the edges (o_r, p_s), where $o_r \notin A$ and $p_s \notin B$. Obviously, elimination of the edge (o_r, p_s), where $o_r \notin A$ and $p_s \notin B$, results the elimination of the cliques containing the edge (o_r, p_s), as well as elimination of the concept containing the object-vertex o_r and the attribute-vertex p_s which are not comparable with $\langle A, B \rangle$.

We can easily observe that Step 3, Step 4, Step 5of the above procedure causes to disappear all non-comparable concepts of query concept from the t-level graph. Therefore without generating the whole t-concept set, all related concepts of query concepts as a sub-lattice of t-concept lattice is generated as per user criterion.

Example: Let us consider the fuzzy **L**-context $K = \langle X, Y, I \rangle$ with $L = \{0, 0.3, 0.6, 1\}$, where $X = \{o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8\}$ is a set of objects consist of three types of learning materials, namely, basic types, intermediate types and advance types, $Y = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ is a set of properties consist of different topics included in the learning materials and *I* is a fuzzy relation between *X* and *Y* defined as, $I(o_i, p_j) = 0$ if the learning object $o_i \in X$ does not include the topic $p_j \in Y$, $I(o_i, p_j) = 0.3$ if the learning object $o_i \in X$ includes the topic $p_j \in Y$ and of basic type, $I(o_i, p_j) = 0.6$ if the learning object $o_i \in X$ includes the topic $p_j \in Y$ and of intermediate type, and $I(o_i, p_j) = 1$ if the learning object $o_i \in X$ includes the topic $p_j \in Y$ and of advance type. The**L**-context $K = \langle X, Y, I \rangle$ is shown in Table 1.

Table 1 Fuzzy L-context of the given example

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
o_1	0.3	0.0	0.6	0.6	0.3	0.6	0.0	0.3
o_2	0.3	1.0	0.0	0.3	0.6	0.0	0.6	0.3
o_3	0.6	0.3	0.6	0.0	0.0	0.6	0.0	0.3
o_4	0.0	0.3	1.0	0.0	0.3	0.6	0.3	0.0
o_5	0.0	0.6	0.3	0.6	0.0	0.0	1.0	1.0
o_6	1.0	1.0	0.0	0.3	0.3	0.6	0.0	0.0
o_7	0.6	0.0	0.3	0.3	1.0	0.0	1.0	0.0
o_8	0.0	0.0	0.3	1.0	0.0	0.3	0.0	1.0

Now searching of specific concept from fuzzy concept lattice has usually a big disadvantage (similarly as other cluster method) – a large size of fuzzy concept lattice is found. Our example is this case - the fuzzy concept lattice obtained from our data has more than 80 fuzzy concepts. Thus if user query set are intermediate type materials on the topic p_4 and advance type on the topic p_8, then to locate and identify a query concept and its related concepts from the given fuzzy concept lattice is not an good quality job. Therefore to generateonly related concepts ofthe query concept from the given fuzzy context, without generating the all *t*-concepts of the fuzzy **L**-context, we use the above technique given in section 4.

First, we construct the fuzzy graph $G_I = (\mu, \rho)$ of the above fuzzy context corresponding to the desired set *Y* as defined in sub-section 3.2. The fuzzy graph G_I is given by the incidence matrix G_I below, where for $i = 1,2,....8,\ \mu(o_i)$ are 0.6, 1.0, 0.6, 1.0, 1.0, 1.0, 1.0, 1.0, respectively, and for $j = 1,2,....8,\ \mu(p_j)$ are 1.0, 1.0, 1.0, 1.0, 1.0, 0.60, 1.0, 1.0, respectively. Also, the 0.6-level graph of the fuzzy graph G_I of the above given fuzzy context is shown by the incidence matrix $G_I^{0.6}$ below. Sinceuser queryset are intermediate type materials on the topic p_4 and advance type on the topic p_8. Also, in the fuzzy context "the learning object $'o_i'$ includes the intermediate type materials of any topic p_j" is presented by the truth value 0.6, and "the learning object $'o_i'$ includes the intermediate type materials of any topic p_j" is presented by the truth value 1.0. Therefore, the query concept will be 0.6-concept which corresponds any one of the maximal cliques of 0.6-level graph containing the vertices p_4 and p_8 which is $\{o_5, o_8, p_4, p_8\}$. Again, the number of 0.6-concept is 18. Therefore without generating whole set of 0.6-concept and applying Step 3, Step 4, Step 5of the proposed procedure given in section 4, we obtained the sublattice by the following way: eliminating the objects-vertices o_2, o_3, o_4, o_6, o_7 which are not adjacent with p_4, p_8 and eliminating the attribute-vertices p_1, p_3, p_5, p_6 which are not adjacent with o_5, o_8 of $G_I^{0.6}$ gives a sub-graph $G_{I'}^{0.6}$ of $G_I^{0.6}$ from which we obtain all related concepts of query concept only. The 0.6-level graph, resulting of sub-graph and corresponding reduced lattice, i.e., sub-lattice of 0.6-concept lattice are shown by incidence matrix $G_I^{0.6}$, $G_{I'}^{0.6}$, and Figure 1, respectively.

$$G_I = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \\ o_6 \\ o_7 \\ o_8 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{array} \begin{array}{cccccccccccccccc} o_1 & o_2 & o_3 & o_4 & o_5 & o_6 & o_7 & o_8 & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\ 0.0 & 0.3 & 0.6 & 0.6 & 0.6 & 0.6 & 0.3 & 0.6 & 0.3 & 0.0 & 0.6 & 0.6 & 0.3 & 0.6 & 0.0 & 0.3 \\ 0.3 & 0.0 & 0.3 & 0.3 & 0.6 & 1.0 & 0.6 & 0.3 & 0.3 & 1.0 & 0.0 & 0.3 & 0.6 & 0.0 & 0.6 & 0.3 \\ 0.6 & 0.3 & 0.0 & 0.6 & 0.3 & 0.6 & 0.6 & 0.3 & 0.6 & 0.3 & 0.6 & 0.0 & 0.0 & 0.6 & 0.0 & 0.3 \\ 0.6 & 0.3 & 0.6 & 0.0 & 0.3 & 0.6 & 0.3 & 0.3 & 0.0 & 0.3 & 1.0 & 0.0 & 0.3 & 0.6 & 0.3 & 0.0 \\ 0.6 & 0.6 & 0.3 & 0.3 & 0.0 & 0.6 & 1.0 & 1.0 & 0.0 & 0.6 & 0.3 & 0.6 & 0.0 & 0.0 & 1.0 & 1.0 \\ 0.6 & 1.0 & 0.6 & 0.6 & 0.6 & 0.0 & 0.6 & 0.3 & 1.0 & 1.0 & 0.0 & 0.3 & 0.3 & 0.6 & 0.0 & 0.0 \\ 0.3 & 0.6 & 0.6 & 0.3 & 1.0 & 0.6 & 0.0 & 0.3 & 0.6 & 0.0 & 0.3 & 0.3 & 1.0 & 0.0 & 1.0 & 0.0 \\ 0.6 & 0.3 & 0.3 & 0.3 & 1.0 & 0.3 & 0.3 & 0.0 & 0.0 & 0.0 & 0.3 & 1.0 & 0.0 & 0.3 & 0.0 & 1.0 \\ 0.3 & 0.3 & 0.6 & 0.0 & 0.0 & 1.0 & 0.6 & 0.0 & 0.0 & 1.0 & 0.6 & 0.3 & 0.6 & 0.6 & 0.6 & 0.3 \\ 0.0 & 1.0 & 0.3 & 0.3 & 0.6 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.3 & 0.6 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.0 & 0.6 & 1.0 & 0.3 & 0.0 & 0.3 & 0.3 & 0.6 & 0.3 & 0.0 & 0.6 & 0.3 & 0.6 & 0.3 & 0.3 \\ 0.6 & 0.3 & 0.0 & 0.0 & 0.6 & 0.3 & 0.3 & 1.0 & 0.3 & 0.6 & 0.6 & 0.0 & 0.3 & 0.6 & 0.6 & 1.0 \\ 0.3 & 0.6 & 0.0 & 0.3 & 0.0 & 0.3 & 1.0 & 0.0 & 0.6 & 0.6 & 0.3 & 0.3 & 0.0 & 0.3 & 1.0 & 0.3 \\ 0.6 & 0.0 & 0.6 & 0.6 & 0.0 & 0.6 & 0.0 & 0.3 & 0.6 & 0.6 & 0.6 & 0.6 & 0.3 & 0.0 & 0.3 & 0.3 \\ 0.0 & 0.6 & 0.0 & 0.3 & 1.0 & 0.0 & 1.0 & 0.0 & 0.6 & 0.6 & 0.3 & 0.6 & 1.0 & 0.3 & 0.0 & 1.0 \\ 0.3 & 0.3 & 0.3 & 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & 0.3 & 0.6 & 0.3 & 1.0 & 0.3 & 0.3 & 1.0 & 0.0 \end{array}$$

$$G_I^{0.6} = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \\ o_6 \\ o_7 \\ o_8 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{array} \begin{array}{c} \begin{array}{cccccccccccccccc} o_1 & o_2 & o_3 & o_4 & o_5 & o_6 & o_7 & o_8 & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0. & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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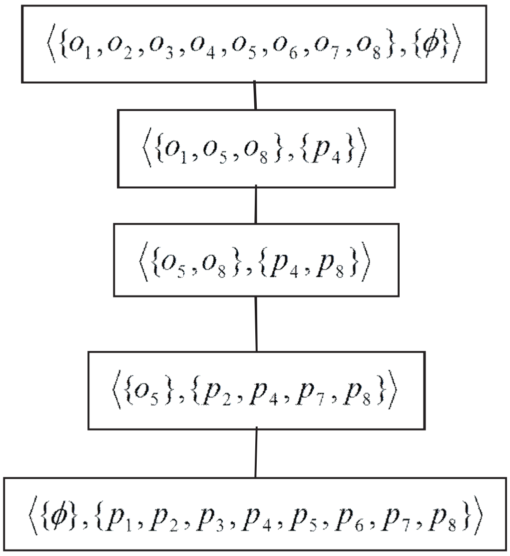

Figure 1. Sublattice corresponding to the query concept of 0.6-concept lattice

$$G_{I'}^{0.6} = \begin{array}{c} \\ o_1 \\ o_5 \\ o_8 \\ p_2 \\ p_4 \\ p_7 \\ p_8 \end{array} \begin{array}{c} \begin{array}{ccccccc} o_1 & o_5 & o_8 & p_2 & p_4 & p_7 & p_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Similarly, if user queryset is intermediate type materials on p_6 then, query concept will be 0.6-concept which corresponds any one of the maximal cliques of 0.6-level graph containing the vertex p_6. Here, $\{o_1, o_3, o_4, o_6, p_6\}$ is a maximal clique of $G_I^{0.6}$ containing the vertex p_6.Now, elimination of the vertices $o_2, o_5, o_7, o_8, p_5, p_7, p_8$ of $G_I^{0.6}$ gives a sub-graph of $G_I^{0.6}$ from which we obtain all related concepts of query concept only. The resulting of sub-graph and corresponding reduced lattice, i.e., sub-lattice of 0.6-concept lattice are shown by incidence matrix $G_{I'}^{0.6}$ and Figure 2, respectively.

$$G_{I'}^{0.6} = \begin{array}{c} \\ o_1 \\ o_3 \\ o_4 \\ o_6 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_6 \end{array} \begin{array}{c} \begin{array}{ccccccccc} o_1 & o_3 & o_4 & o_6 & p_1 & p_2 & p_3 & p_4 & p_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

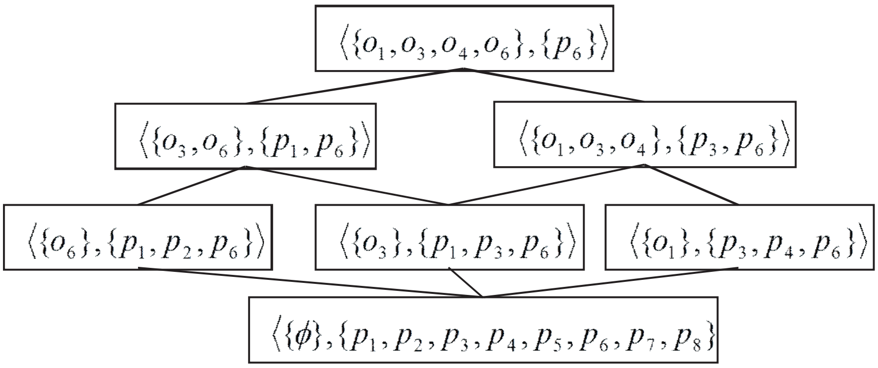

Figure 2. Sublattice corresponding to the query concept of 0.6-concept lattice

Conclusion : In a University Consortium environment, different content providers contribute content for courses offered by the consortium. Content provider develops contents following the guidelines of University Consortium including the guideline ofSCORM-compliance for LOs. Many benefits are expected from the reuse approach, including improved economic and time efficiencies for resource development, and more effective learning and teaching practices.

Learning objects can also bere-used in multiple contexts and pedagogic settings and can be grouped into coherent collections of digital learning content. In this paper, a *t*-concept lattice and fuzzy graph based approach is presented for improving the search capability of LOs in a University Consortium environment. Again, application of fuzzy graph in real world problems now has been well established. Therefore, generation of *t*-concepts using fuzzy graph somehow relates the fuzzy graph theory to the concept lattice theory.

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