WEARER FORM OF CONTRA SUPER CONTINUITY

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Abstract: In this paper, a new class of functions known as contra $\hat{\Omega}$-continuity by utilizing $\hat{\Omega}$-closed sets is introduced and investigated. Some of its characterizations and properties are derived. Also it is compared with the existing mappings.

Keywords: $\hat{\Omega}$ -closed sets, $\hat{\Omega}$-closure, $\hat{\Omega}$-continuous and Kernel.,

Introduction: In 1996, Dontchev [1] introduced and studied a new notion of continuity called contra continuity and studied their properties via compactness, S -closedness and strong S -closedness. In 1999, Dontchev and T.Noiri [2] introduced and investigated RC-continuous functions. S.Jafari and T.Noiri [4] introduced and analyzed the concept of contra-super-continuous functions. In the past ten years, the development of generalized closed sets has been grown rapidly. Among them, Thivagar et.al introduced a new class of generalized closed sets which forms a topology and independent of closed sets known as $\hat{\Omega}$-closed sets. In this paper, a new class of mappings, contra $\hat{\Omega}$-continuity functions is introduced by utilizing $\hat{\Omega}$-closed sets and find some of their characterizations and properties. Moreover, nature of this continuity in subspaces is discussed and one of the significant Lemmas, Pasting Lemma is derived in terms of contra $\hat{\Omega}$-continuity.

Preliminaries: Throughout this paper, (X, τ) represent a topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of X , cl(A) , int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. The family of all $\hat{\Omega}$-open subsets of (X, τ) is denoted by $\hat{\Omega}O(X)$ and

$\hat{\Omega}O(X, x)=\{ U\in \hat{\Omega}O(X):\ x\in X$ such that $x\in U$.

Definition 2.1. A subset A of a topological space (X, τ) is said to be a

(i) semi open [5] if $A \subseteq cl(int(A))$

(ii) $\hat{\Omega}$-closed [6] if $\delta cl(A) \subseteq U$ if for every semi open subset U such that $A\subseteq U$.

Definition 2.2. A function f : (X, τ) → (Y, σ) is called a (i) contra-continuous [1] if $f^{-1}(V)$ is closed in (X, τ) for every open set V of (Y, σ).

(ii) contra pre continuous functions [3] if $f^{-1}(V)$ is pre closed set of (X, τ) for every open set V of (Y, σ).

(iii) contra δ-pre continuous [3] if $f^{-1}(V)$ is δ-pre closed in (X, τ) for every open set V in (Y, σ).

(iv) completely continuous [2] if $f^{-1}(V)$ is regular closed in (X, τ) for every closed set V in (Y, σ).

(v) RC-continuous [2] if $f^{-1}(V)$ is regular closed in (X, τ) for every open set V in Y.

(vi) $\hat{\Omega}$-continuous map [7] if $f^{-1}(V)$ is $\hat{\Omega}$-closed in (X, τ) for every closed set V in (Y, σ).

Definition 2.3.[4] A function f : (X, τ) → (Y, σ) is said to be contra super continuous if for all $x \in X$ and each $F \in C(Y, f(x))$, there exists a regular open set U in (X, τ) containing x such that $f(U)\subseteq F$.

Definition 2.7. Let A be a subset of a space (X, τ). Then kernel of A is given by [4] $\bigcap_{U\subseteq A\in\tau} U$ and denoted by ker(A).

Lemma 2.8.[4] For any subsets A, B of a topological space (X, τ) ,the following properties hold.

(i) $x \in ker(A)$ if and only if $A \cap F \neq \Phi$ for any $F \in C(X, x)$.

(ii) $A\subseteq ker(A)$ and A = ker(B) if A is open in (X, τ).

(iii) If $A\subseteq B$, then $ker(A)\subseteq ker(B)$.

This section starts with the introduction of new class of mapping namely contra $\hat{\Omega}$-continuous function.

Definition 3.1. A function f : (X, τ) → (Y, σ) is said to be a contra $\hat{\Omega}$-continuous function if $f^{-1}(V)$ is $\hat{\Omega}$-closed subset of (X, τ) for every open subset V of (Y, σ).

Example 3.2. Let X = {a, b, c, d} , τ = { Φ , {a}, {b, c, d},X} and Y = {p, q, r}, σ = { Φ ,{p}, {q}, {p, q},Y }. Define f : (X, τ) → (Y, σ) as f(a) = r, f(b) = r, f(c) = p and f(d) = q .Then f is contra $\hat{\Omega}$-continuous function.

Theorem 3.3. A function f : (X, τ) → (Y, σ) is contra $\hat{\Omega}$-continuous function if and only if for every closed subset F of (Y, σ) , $f^{-1}(F) \in \hat{\Omega}O(X)$.

Proof. Necessity-Let f : (X, τ) → (Y, σ) be a contra $\hat{\Omega}$-continuous function and let F be any closed subset of (Y, σ). Then Y \F is open subset of (Y, σ) and by hypothesis, $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is $\hat{\Omega}$-closed subset of (X, τ). Since complement of $\hat{\Omega}$-closed subset is $\hat{\Omega}$-open, $f^{-1}(F)$ is $\hat{\Omega}$-open subset of (X, τ).

Sufficiency-Assume that inverse image of a closed subset of (Y, σ) is $\hat{\Omega}$-open subset of (X, τ). Let V be any open subset of (Y, σ). Then Y \ V is closed subset of (Y, σ). By hypothesis, $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is $\hat{\Omega}$-open subset of (X, τ) and so $f^{-1}(V)$ is $\hat{\Omega}$-closed subset of (X, τ). Thus, f is a contra $\hat{\Omega}$-continuous function.

Theorem 3.4. Every RC (resp. contra super) continuous function is contra $\hat{\Omega}$-continuous function.

Proof. Let f : (X, τ) → (Y, σ) be a RC (resp. contra super) continuous function and V be any open subset of (Y, σ). By hypothesis, $f^{-1}(V)$ is regular closed (resp. δ-closed) subset of (X, τ). Since every regular closed subset is δ-closed, by [6] Theorem 3.2, $f^{-1}(V)$ is $\hat{\Omega}$-closed subset of (X, τ).

Remark 3.5. Reversible implication is not always true for Theorem 3.4 from the example 3.2, because for a open subset {p} of (Y, σ) , $f^{-1}(\{p\})$ = {c} is neither δ-closed nor regular closed subset of (X, τ).

Theorem 3.6. If f : (X, τ) → (Y, σ) is a contra $\hat{\Omega}$-continuous function, then it is contra Pre (resp. contra δ-pre) continuous function.

Proof. By [6], every $\hat{\Omega}$-closed subset is pre closed set and by [3] every pre closed subset is δ-pre closed subset of (X, τ).Therefore, by the definition of contra $\hat{\Omega}$-continuous function, it follows.

Remark 3.7. Reversible implications are not always true for theorem 3.8 from the following example.

Example 3.8. Let X = {a, b, c, d}, τ = { Φ , {a}, {b}, {a, b}, X},Y = {p, q, r, s}, σ = { Φ , {p}, {q}, {p, q}, Y }.Define the function f : (X, τ) → (Y, σ) by f(a) = r, f(b) = s, f(c) = p, f(d) = q. Then f is contra pre (resp.contra δ-pre) continuous function but not a contra $\hat{\Omega}$-continuous function.

Remark 3.9. The following examples show that contra $\hat{\Omega}$-continuous function is independent of contra-continuity and $\hat{\Omega}$-continuous functions.

Example 3.10. Let X = {a, b, c, d}, τ = { Φ , {a}, {a, b}, X},Y = {p, q, r, s}, σ = { Φ , {p}, {p, q}, Y }. Define the function f : (X, τ) → (Y, σ) by f(a) = r, f(b) = q, f(c) = p, f(d) = p. Then f is contra continuous function but not a contra $\hat{\Omega}$-continuous function.

Example 3.11. Let X = {a, b, c, d} , τ = { Φ , {c}, {a, d}, {a, c, d}, X} and Y = {p, q, r, s} ,σ = { Φ , {p}, Y }. Define f: (X, τ) → (Y, σ) as f(a) = p= f(b), f(c) = rand f(d) = s. Then f is contra $\hat{\Omega}$-continuous function but neither a contra continuous function nor a $\hat{\Omega}$-continuous function. If g: (X, τ) → (Y, σ) is defined by g(a) = g(c) = p, f(b) = rand f(d) = s. Then g is $\hat{\Omega}$-continuous but not contra $\hat{\Omega}$-continuous function.

Remark 3.12. The following example shows that composition of two contra $\hat{\Omega}$-continuous function is not always a contra $\hat{\Omega}$-continuous function.

Example 3.13. Let X = {a, b, c, d}, Y = {p, q, r, s}, Z = {a, b, c, d}, , τ = { Φ , {a}, {b}, {a, b}, X}, σ = { Φ , {p, q}, Y } and η = { Φ , {a},Z}. Define f : (X,τ) → (Y, σ) as f(a) = r, f(b) = s, f(c) = p and f(d) = q , g : (Y, σ) → (Z, η) as g(p) = b, g(q) = c, g(r) = a and g(s) = a. Then f and g are contra $\hat{\Omega}$-continuous functions. Define (g o f)(x) = g(f(x)) for all x ∈ X .Then g o f is not contra $\hat{\Omega}$-continuous function.

Theorems on compositions-

Theorem 3.14. If a function f : (X, τ) → (Y, σ) is contra $\hat{\Omega}$-continuous function, then the following statements hold.

(i) If g : (Y, σ) → (Z, η) is a continuous function, then g o f : (X, τ) → (Z, η) is a contra $\hat{\Omega}$-continuous function.

(ii) If g : (Y, σ) → (Z, η) is RC function, then g o f : (X, τ) → (Z, η) is a $\hat{\Omega}$-continuous function.

(iii) If g : (Y, σ) → (Z, η) is a completely continuous function, then g o f : (X, τ) → (Z, η) is a contra $\hat{\Omega}$-continuous function.

Proof. (i) Let F be any closed subset of (Z, η). Since g is continuous function, $g^{-1}(F)$ is closed subset of (Y, σ). Since f is contra $\hat{\Omega}$-continuous function, $f^{-1}(g^{-1}(F)$ = $(g \circ f)^{-1}(F)$ is $\hat{\Omega}$-open subset of (X, τ). Hence g o f is contra $\hat{\Omega}$-continuous function.

(ii) Let F be any closed subset of (Z, η). Since g is RC-continuous map, $g^{-1}(F)$ is regular open subset of (Y, σ) and hence open subset of (Y, σ). Since f is contra $\hat{\Omega}$-continuous function, $f^{-1}(g^{-1}(F))$ = $(g \circ f)^{-1}(F)$ is $\hat{\Omega}$-closed subset of (X, τ). Hence g o f is $\hat{\Omega}$ -continuous function.

(iii) Let F be any closed subset of (Z, η). Since g is completely continuous function, $g^{-1}(F)$ is regular closed and hence closed subset of (Y, σ). Since f is contra $\hat{\Omega}$-continuous function, $f^{-1}(g^{-1}(F)$ = $(g \circ f)^{-1}(F)$ is $\hat{\Omega}$-open subset of (X, τ). Hence g o f is contra $\hat{\Omega}$-continuous function.

Characterizations of contra $\hat{\Omega}$-continuous function-

Theorem 3.15. The following statements are equivalent for the function f : (X, τ) → (Y, σ) .

(i)f is contra $\hat{\Omega}$-continuous function.

(ii) For each point x in X and each closed subset F of Y containing f(x), there exists $\hat{\Omega}$-open subset U of (X, τ) containing x such that f(U) ⊆ F.

(iii) For each point x in X and each subset A of (Y, σ) containing f(x), there exists $\hat{\Omega}$-open subset U of (X, τ) containing x such that f(U) ⊆ cl(A).

(iv) For every subset A of X, f($\hat{\Omega}$ cl(A)) ⊆ kerf(A).

(v) For every subset B of Y, $\hat{\Omega}$ cl($f^{-1}(B)$) ⊆ $f^{-1}(ker(B))$.

Proof. (i) ⇒ (ii) Let x ∈ X and F be any closed subset of Y containing f(x). By hypothesis, $f^{-1}(F)$ is $\hat{\Omega}$ -open subset of (X, τ) containing x. If U = $f^{-1}(F)$, then U is a

$\hat{\Omega}$-open subset of (X, τ) containing x such that f(U) ⊆ F.

(ii) ⇒ (i) Let x ∈X and F be any closed subset of (Y, σ) containing f(x). By hypothesis, there exists U_x∈ $\hat{\Omega}$O(X, x) such that $f(U_x)$ ⊆ F. Therefore, $f^{-1}(F)$ = ∪ {U_x: x ∈$f^{-1}(F)$}. By [6] Theorem 4.16, $f^{-1}(F)$ ∈ $\hat{\Omega}$O(X, x) . Thus, f is contra $\hat{\Omega}$-continuous function.

(ii) ⇒ (iii) Let x ∈X and A be any subset of Y containing f(x). Then cl(A) containing f(x). By hypothesis, there exists U_x∈ $\hat{\Omega}$O(X, x) such that $f(U_x)$ ⊆ cl(A).

(iii) ⇒ (ii) Let x ∈X and F be any closed subset of (Y, σ) containing f(x). By hypothesis, there exists U_x∈ $\hat{\Omega}$ O(X, x) such that $f(U_x)$ ⊆ cl(F) = F.

(ii) ⇒ (iv) Let A be any subset of (X, τ) and suppose y ∉ker(f(A)). By lemma 2.8, there exists a closed set F in (X, τ) containing y such that f(A) ∩ F = Φ. For any x ∈$f^{-1}(F)$, f(x) ∈F. By hypothesis, there exists U_x ∈ $\hat{\Omega}$O(X, x) such that $f(U_x)$ ⊆and hence f(A ∩ U_x) ⊆ f(A) ∩ $f(U_x)$ ⊆ f(A) ∩ F = Φ. Then A ∩ U_x= Φ. By [6] Theorem 5.11,x ∉ $\hat{\Omega}$cl(A) for any x ∈$f^{-1}(F)$. Thus, $f^{-1}(F)$∩ $\hat{\Omega}$cl(A) = Φ

and soF ∩f($\hat{\Omega}$cl(A)) = Φ. Then y ∈F implies that y ∉f($\hat{\Omega}$cl(A).

Hence f($\hat{\Omega}$cl(A) ⊆ ker(f(A)).

(iv) ⇒ (v) Let B be any subset of (Y, σ). By hypothesis, f($\hat{\Omega}$cl($f^{-1}(B)$)) ⊆ ker(f($f^{-1}(B)$)) ⊆ ker(B). Moreover, $\hat{\Omega}$cl($f^{-1}(B)$) ⊆ f^{-1}(f($\hat{\Omega}$cl($f^{-1}(B)$))) ⊆ f^{-1}(ker(B)).

(v) ⇒ (i) Let V be any open subset of (Y, σ). By hypothesis, $\hat{\Omega}$cl($f^{-1}(V)$) ⊆ f^{-1}(ker(V)). By lemma 2.8 (ii), $\hat{\Omega}$cl($f^{-1}(V)$) ⊆ $f^{-1}(V)$) and hence $\hat{\Omega}$cl($f^{-1}(V)$) = $f^{-1}(V)$. By [6] lemma 3.1, $f^{-1}(V)$ is $\hat{\Omega}$-closed subset of (X, τ). Therefore, f is contra $\hat{\Omega}$-continuous function.

Applications

In this section, the first result gives the condition under which contra $\hat{\Omega}$-continuous function and $\hat{\Omega}$-continuous function are related. Next, contra $\hat{\Omega}$-continuous function is studied in subspaces and derive a significant lemma known as Pasting Lemma.

Theorem 4.1. If a function f : (X, τ) → (Y, σ) is contra $\hat{\Omega}$-continuous function and Y is regular, then f is $\hat{\Omega}$-continuous function.

Proof. Let x be an arbitrary point of X and V be any open subset of (Y, σ) containing f(x). Since (Y, σ) is regular, there exists an open subset W of (Y, σ) containing f(x) such that cl(W) ⊆ V. Since f is contra $\hat{\Omega}$-continuous by Theorem 3.15 (ii), there exists U ∈ $\hat{\Omega}$O(X, x) such that f(U) ⊆ cl(W) ⊆ V. By [7] Theorem 4.16 (ii), f is $\hat{\Omega}$-continuous function.

Theorem 4.2. If f : (X, τ) → (Y, σ) is contra $\hat{\Omega}$-continuous function and A is both open and pre closed subset of (X, τ), then the restriction f|A is contra $\hat{\Omega}$-continuous function.

Proof. Let F be any open subset of (Y, σ). By hypothesis, $f^{-1}(F)$ is $\hat{\Omega}$-closed subset of (X, τ). By [6] 6.10, $f^{-1}(F)$ ∩A is $\hat{\Omega}$-closed in (A, τ |A) .Therefore, $(f|A)^{-1}(F)$ is $\hat{\Omega}$-closed in (A, τ |A).Thus, f|A is contra $\hat{\Omega}$-continuous function.

Theorem 4.3. Let {A_α: α ∈Λ} be a cover of X by both δ-open and pre closed subsets of (X, τ). If f|A_α: (A_α, τ |A_α) → (Y, σ) is contra $\hat{\Omega}$-continuous function for each α ∈Λ, then f : (X, τ) → (Y, σ) is contra $\hat{\Omega}$-continuous function.

Proof. Let V be any closed subset of (Y, σ). Since f|$A_\alpha$is contra $\hat{\Omega}$-continuous function, $(f|A_\alpha)^{-1}(V)$ is $\hat{\Omega}$-open in (A_α, τ |A_α). By [6] Theorem 6.9, for each α ∈Λ, $(f|A\alpha)^{-1}(V)$ is $\hat{\Omega}$-open subset of (X, τ). Since $f^{-1}(V)$ = ∪{$(f|A_\alpha)^{-1}(V)$: α ∈Λ} and by [6] Theorem 4.16, $f^{-1}(V)$ is $\hat{\Omega}$-open subset of (X, τ). Thus, f is contra $\hat{\Omega}$-continuous function.

Theorem 4.4. (**Pasting Lemma** For Contra $\hat{\Omega}$-continuous Functions)

Let A and B be any two both δ-open and pre closed subsets of (X, τ) such that X = A ∪ B. Let f : (A, τ |A) → (Y, σ) and g : (B, τ |B) → (Y, σ) be contra $\hat{\Omega}$-continuous functions such that f(x) = g(x) for every x ∈A∩B. Then the combination (f o g)(x) = f(x) for every x ∈A and (f o g)(y) = g(y) for every y ∈B is contra $\hat{\Omega}$-continuous function.

Proof. Let V be any open subset of (Y, σ). Then $(f \circ g)^{-1}(V)$ = (($(f \circ g)^{-1}(V)$ ∩ A) ∪ (($(f \circ g)^{-1}(V)$ ∩ B) = $f^{-1}(V)$ ∪$g^{-1}(V)$ = C ∪D, where C = $f^{-1}(V)$ and D = $g^{-1}(V)$. Since f and g are contra $\hat{\Omega}$-continuous, C and D are $\hat{\Omega}$-closed subsets of (A, τ |A) and (B, τ |B) respectively. By [6] theorem 6.9, C and D are $\hat{\Omega}$-closed subsets of

(X, τ). By [6], theorem 4.12, $(f \circ g)^{-1}(V)$ is $\hat{\Omega}$-closed subsets of (X, τ). Thus, the combination (f o g) is contra $\hat{\Omega}$-continuous function.

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