EFFECT OF COUPLESTRESS FLUID WITH SURFACE ROUGHNESS COUPLESTRESSES ON JOURNAL SQUEEZEFILM BEARINGS LUBRICATION USING RAPID-NARANG TECHNIQUE

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Abstract: In this paper the effect of couplestress fluid with surface roughness and slip in the lubrication of finite journal bearing using Rapid-Narang technique is studied. The case of long journal bearing and short bearing are analyzed. Thus two combined to get the results of finite journal bearing using Rapid-Narang technique. The effects of long chain molecule on load capacity is studied numerically by plotting graphs.

Introduction: The phenomenon of two lubricated surfaces approaching each other with a normal velocity is known as squeezefilm lubrication. The time required to squeeze out the lubricant depends upon surface configuration, fluid properties and the load applied. In general the behavior of a lubricated bearing system depends upon the nature of surface in contact, lubricant used between them, environmental conditions. A lubricated system with rough bearing surface can be studied by both deterministic and stochastic methods. In the deterministic approach the rough surface is assumed to be represented by a sin or cosine functions. The film thickness which is a function of surface roughness is accordingly modified and usual hydrodynamic Reynolds equation is solved to study the bearing characteristics. In the stochastic approach, the surface roughness is assumed to be as a stochastic function. Then by taking the statistical average or mean of various terms the Reynolds equation is solved to study the characteristics of bearing system.

The equation governing the pressure generated in the lubricant film can be obtained by coupling the equation of motion with the equation of continuity and was derived by Reynolds and is known as Reynolds equation[19]. The generalized Reynolds equation is derived and taking into account the effects of surface roughness and additives in the form of couple stresses is applied to various squeeze film bearings. Now we apply this generalized Reynolds equation to see the effects of couplestress fluid on the squeeze film of finite journal bearing.It may be noted that the effects of velocity-slip at the surface, is important on the flow behavior of gases and liquids particularly when the film thickness is very small. To study the effect of slip, Burgdorfer[6] modified Reynolds equation for gas lubricated hydrodynamic bearings with 'slip flow' under isothermal condition and pointed out that if $0 < h/\lambda < 1$ gas flow may be continuous and analysis can be carried out with modified slip boundary conditions. The effect of slip is also important on the behavior of liquids especially when the bearing surface is very smooth and is operating at higher temperature .The squeeze film characteristics of long journal bearings lubricated with couple stress fluids is studied by Lin[15] in the year 1997. Naduvinamani et al [3] studied the squeeze film lubrication of a short porous journal bearing with couple stress fluis. Jayachandra Reddy et al [1] studied the analysis of load carrying capacity using rapid technique. Raghavendra[14] studied the effects of couple stresses and surface roughness on roller bearings under lightly loaded conditions. Naduvinamani et al [4] studied the combined effects of surface roughness and couple stresses on squeeze film lubrication between porous circular plates.

BasicEquation:

The equation governing to the Fluid flow in the bearing is given by equation

$$\frac{\partial}{\partial x}\left[\frac{h^3}{12\mu}F\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{h^3}{12\mu}F\frac{\partial p}{\partial z}\right]=U\frac{dh}{dt} \quad (1)$$

Where $F=\frac{M-\tanh(M)}{kM3}\,6_{\beta}3+\frac{6_{\beta}tanh2M}{2kM}\left[(1-{}_{\beta})+{}_{\beta}\frac{\tanh(M)}{M}\right]^2+(1-\beta)^3-12\frac{l^2}{h^2}(1-\beta)+24\frac{l^3}{h^3}\tanh\left(\frac{h(1-\beta)}{2l}\right)$

Here h is the film thickness L is the characteristic the couple stress property of the lubricant

The oil film thickness is given by $h = c(1+\varepsilon\cos\theta)$

$$\frac{\partial h}{\partial t}=c\frac{d\varepsilon}{dt}\cos\theta \quad (3)$$

Where r-R is the clearance width and $\varepsilon=\frac{e}{c}$ is the eccentricity ratio as shown fig (1)

The modified Reynolds equation is

$$\frac{\partial}{\partial x}\left[\frac{h^3}{12\mu}F\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{h^3}{12\mu}F\frac{\partial p}{\partial z}\right]=c\frac{d\varepsilon}{dt}\cos\theta \qquad (4)$$

Where F is given by equation (2)

Introducing the non-dimensional variables and parameters

$$\theta=\frac{x}{R},\ \bar{z}=\frac{z}{L},\lambda=\frac{L}{2R},\bar{h}=\frac{h}{c}\ \beta=2\frac{h_s}{h_n} \qquad \bar{\phi}=\frac{\phi}{c^2}\ \ M=h\alpha \qquad (5)$$

The modified Reynolds equation in a non-dimensional form can be return as

$$\frac{\partial}{\partial\theta}\left[\frac{\bar{h}^3}{12\mu}\bar{F}\frac{\partial\bar{p}}{\partial\theta}\right]+\frac{1}{4\lambda^2}\frac{\partial}{\partial\bar{z}}\left[\frac{\bar{h}^3}{12\mu}\bar{F}\frac{\partial\bar{p}}{\partial\bar{z}}\right]=12\frac{\mu R^2U}{c^2}\frac{d\varepsilon}{dt}\cos\theta \qquad (6)$$

Where

$$\bar{F}=\frac{M-\tanh(M)}{kM3}\,6_\beta 3+\frac{6_\beta\tanh(2M)}{2kM}\left[(1-_\beta)+_\beta\frac{\tanh(M)}{M}\right]^2+(1-\beta)^3-12\frac{\bar{l}^2}{\bar{h}^2}(1-\beta)+24\frac{\bar{l}^3}{\bar{h}^3}\tanh\left(\frac{\bar{h}(1-\beta)}{2\bar{l}}\right) \qquad (7)$$

The non-dimensionless pressure is given by $\bar{p}=\dfrac{pc^2}{U\mu R^2\frac{d\varepsilon}{dt}}$

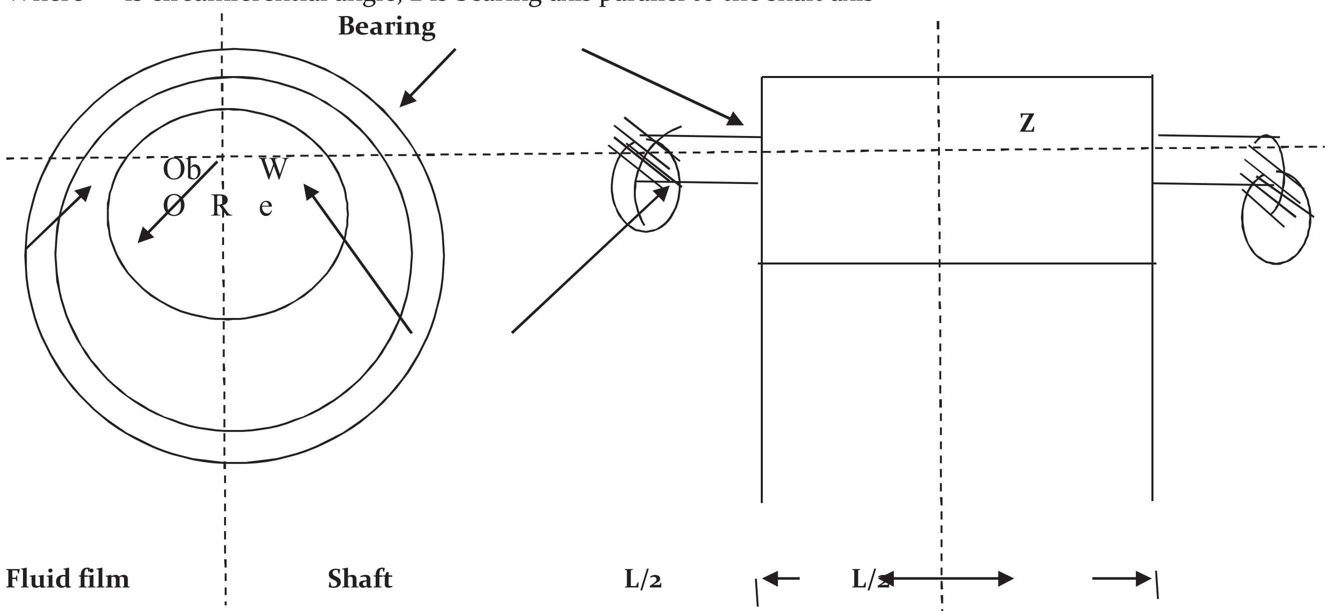
The equation (6) reduces to

$$\frac{\partial}{\partial\theta}\left[\frac{\bar{h}^3}{12\mu}\bar{F}\frac{\partial\bar{p}}{\partial\theta}\right]+\frac{1}{4\lambda^2}\frac{\partial}{\partial\bar{z}}\left[\frac{\bar{h}^3}{12\mu}\bar{F}\frac{\partial\bar{p}}{\partial\bar{z}}\right]=12\cos\theta \qquad (8)$$

Where $\bar{F}$ is given in equation (7) The boundary conditions for the equation (8) are

$$\bar{p}=0,\ at\ \theta=\frac{\Pi}{2},\ \frac{3\Pi}{2} \qquad \bar{p}=0\ \text{at}\ \bar{z}=\pm\frac{1}{2} \qquad (9) \qquad \frac{\partial\bar{p}}{\partial\bar{z}}=0\ at\ \bar{z}=0$$

Where θ is circumferential angle, z is bearing axis parallel to the shaft axis



Short bearing analysis:

If it is called short bearing or narrow bearing. Neglecting pressure variations in the x direction, the modified Reynolds equation reduces to

$$\frac{\partial}{\partial \bar{z}}\left[\overline{h^3}\,\overline{F}\,\frac{\partial \bar{p}}{\partial \bar{z}}\right] = 48\lambda^2 \cos\theta \tag{10}$$

Integrating the above equation (10) twice, the equation becomes,

$$\bar{p} = \frac{48\lambda^2 \cos\theta\, \overline{z^2}}{\overline{2h^3 F}} + \frac{c_1}{Fh^3}\bar{z} + c_2 \tag{11}$$

Where c_1 and c_2 are constants of integration and evaluated using boundary conditions of equation(9)

$$c_1 = 0 \quad c_2 = \frac{48\lambda^2 \cos\theta}{\overline{h^3 F}}\left(\frac{1}{8}\right) \tag{12}$$

Substituting equation(12) in (11), the pressure for a short bearing is

$$\bar{p} = \frac{24\lambda^2 \cos\theta}{\overline{h^3 F}}\left(\bar{z}^2 - \frac{1}{4}\right) \tag{13}$$

Pressure at the centre line of the bearing is, i.e., $\bar{z}$ =0

$$\bar{p} = \frac{-6\lambda^2 \cos\theta}{\overline{h^3 F}} \tag{14}$$

Therefore the non-dimensional pressure for short bearing is

$$\bar{p}_s = \frac{-6\lambda^2 \cos\theta}{\overline{h^3 F}} \tag{15}$$

$\overline{F}$ is given in equation (7)
The load carrying capacity is

$$W_s = -2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}.\int_0^{\frac{L}{2}} PRCos\theta dz \tag{16}$$

$$W_s = -2.\frac{\mu UR^3 \frac{d\varepsilon}{dt} L}{c^2}\int_0^{\frac{3\pi}{2}} \frac{6\lambda^2 Cos^2\theta}{\overline{h^3 F}}\, d\theta \tag{17}$$

And the dimensionless squeeze load is given by

$$\overline{W}_s = \frac{W_s c^2}{\mu UR^{3.} \frac{d\varepsilon}{dt} L} = 6\lambda^2 \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{Cos^2\theta}{\overline{h^3 F}}\, d\theta \tag{18}$$

Where,

$$\bar{\ } = \frac{-\tanh(\ )}{3}\, 6\beta 3 + \frac{6\beta\tanh(2\ )}{2}\left[(1-\beta) + \beta\frac{\tanh(M)}{M}\right]^2 + (1-\beta)^3 - 12\frac{\bar{\ }^2}{\bar{h}^2}(1-\beta) + 24\frac{\bar{\ }^3}{\bar{h}^3}\tanh\left(\frac{\bar{h}(1-\beta)}{2\bar{\ }}\right)$$

Long bearing Analysis:

If it is called long bearing. That is the bearing is infinitely long in axial direction and the pressure is constant in that direction. Thus neglecting the $\frac{\partial p}{\partial z}$, the modified Reynolds equation reduces to

$$\frac{\partial}{\partial \theta}\left[\overline{h^3}\,\overline{F}\,\frac{\partial \bar{p}}{\partial \theta}\right] = 12\cos\theta \tag{19}$$

Integrating with respect to θ

$$.\overline{h^3 F}\frac{\partial \bar{p}}{\partial \theta} = 12\sin\theta + B \qquad (20)$$

Where B is the integral constant

Apply boundary conditions $\frac{\partial \bar{p}}{\partial \theta} = 0 at \theta = \pi$

The constant B=o then

$$\frac{\partial \bar{p}}{\partial \theta} = \frac{12\sin\theta}{\overline{h^3 F}} \qquad (21)$$

Again integrating and applying the boundary conditions

$\bar{p} = 0, at\theta = \frac{\Pi}{2}, 3\frac{\Pi}{2}$ $\quad \bar{p} = 0$ at $\bar{z} = \mp\frac{1}{2}$ $\quad .\frac{\partial \bar{p}}{\partial \bar{z}} = 0 at \ \bar{z} = 0$

And the dimensionless pressure is

$$\overline{p_l} = 12\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\overline{h^3 F}} d\theta \qquad (22)$$

Squeeze load capacity W_l for the long bearing is

$$W_l = 2L\int_0^{\pi} PR\sin\theta d\theta \quad W_l = .\frac{\mu U R^3 \frac{d\varepsilon}{dt} L}{c^2}\int_0^{\pi} \frac{-24 \ \sin^2\theta}{\overline{h^3 F}} d\theta \qquad (23)$$

And the dimensionless load capacity is

$$\overline{W}_l = \frac{W_l c^2}{\mu U R^{3.} \frac{d\varepsilon}{dt} L} = 24\int_0^{\pi} \frac{\sin^2\theta}{\overline{h^3 F}} d\theta \qquad (24)$$

Where

$$\bar{F} = \frac{M - \tanh(M)}{kM3} 6_\beta 3 + \frac{6_\beta \tanh(2M)}{2kM}\left[(1-_\beta) + _\beta \frac{\tanh(M)}{M}\right]^2 + (1-_\beta)^3 - 12\frac{\bar{l}^2}{\bar{h}^2}(1-_\beta) + 24\frac{\bar{l}^3}{\bar{h}^3}\tanh\left(\frac{\bar{h}(1-_\beta)}{2\bar{l}}\right)$$

Finite bearing analysis:

For finite bearings, the two dimensional Reynolds equation is solved using Rapid-Narang technique. If p,ps and pl are the pressure in finite, narrow and long bearings respectively, then the relationship between them is

$$\frac{1}{p} = \frac{1}{p_s} + \frac{1}{p_l} \qquad (25)$$

The finite bearing pressure is

$$p = \frac{p_s p_l}{p_s + p_l} \qquad (26)$$

Load carrying capacity:

As the load is proportional to the pressure, the load carrying capacity for the finite bearing is

$$\frac{1}{W_f} = \frac{1}{W_s} + \frac{1}{W_l}$$

$$W_f = \frac{W_s W_l}{W_s + W_l} \tag{27}$$

Substituting the short and long bearing load equations(18) and (24) in the above equation(27) the finite bearing load carrying capacity in non-dimensional form is

$$\overline{W} = \frac{W_f c^2}{\mu U R^{3.} \dfrac{d\varepsilon}{dt} L} = \frac{W_s W_l}{W_s + W_l} \tag{28}$$

Equation(28) is solved numerically. Graphs have been plotted for for various values of different parameters

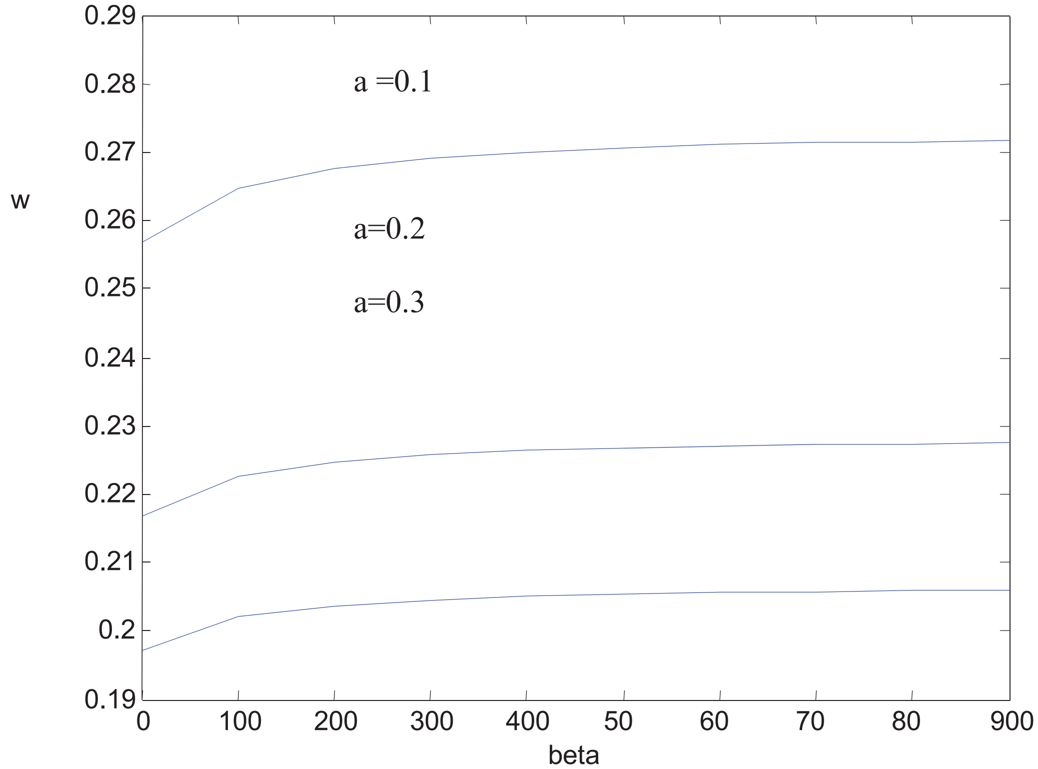


Figure 1

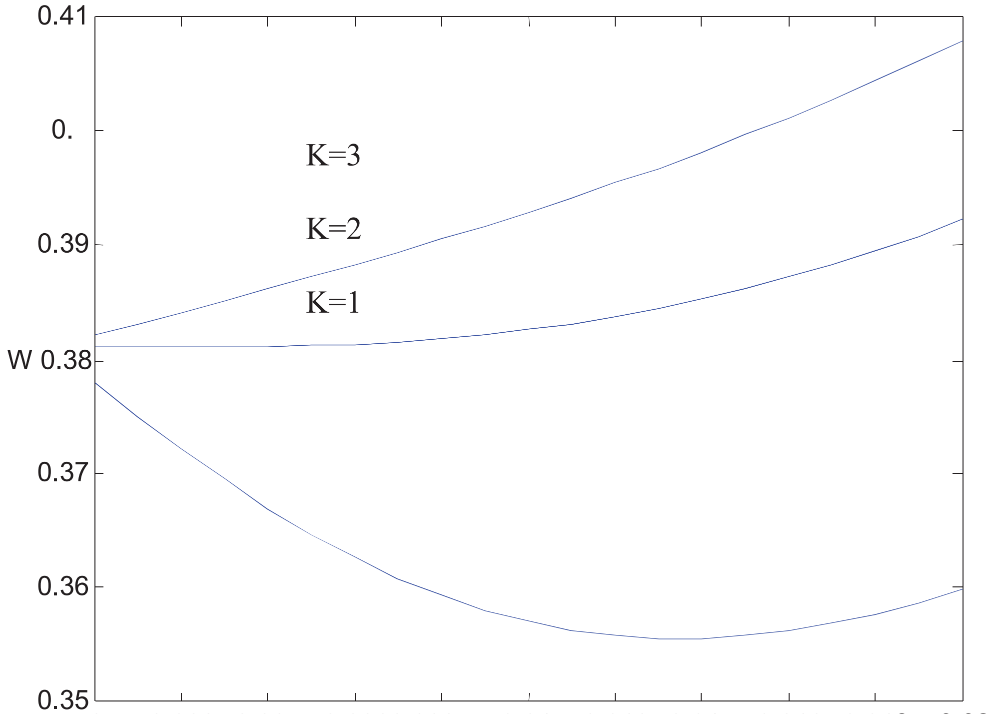


Figure 2

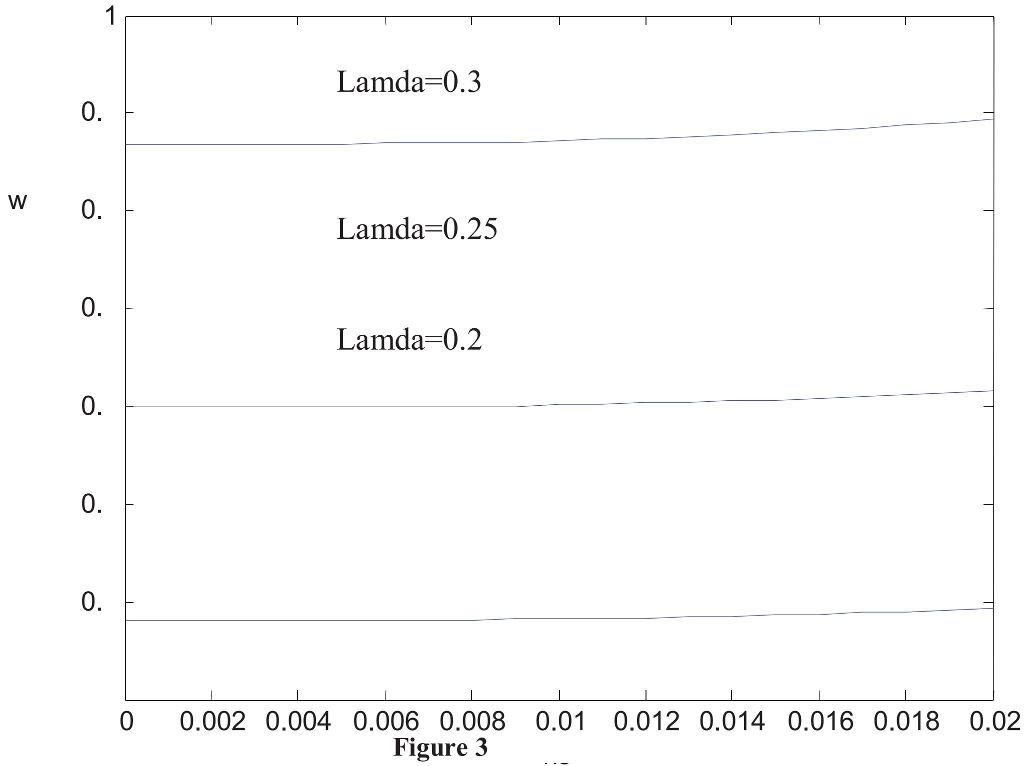


Figure 3

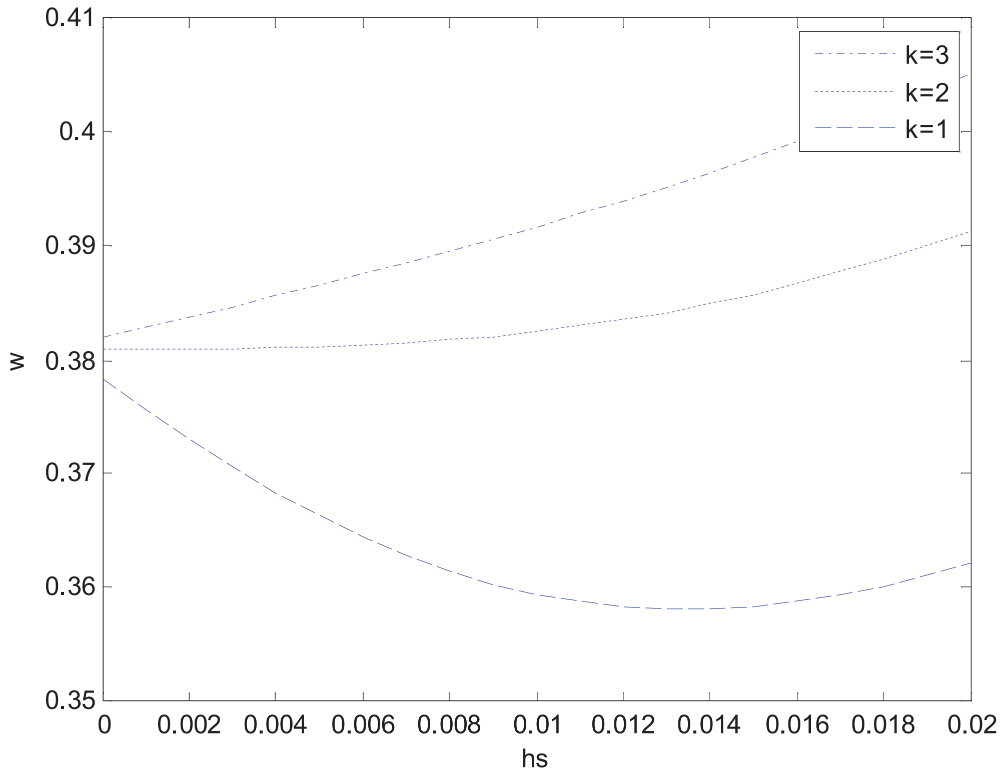


Figure 4

Results and results:

In fig (1) the load carrying capacity are plotted with β ,slip parameter for different values of a . As the parameter β increases the load carrying capacity increases , also increases with the decrease of a .

It is seen that from fig(2) the load carrying capacity is plotted with mean height of roughness asperity hs for different values of ratio of viscosities. As the roughness hs increases the load carrying capacity decreases and increases with the increase of ratio of viscosities. From fig(3) the load carrying capacity is plotted with roughness parameter for different values of lamda. As the roughness increases the load carrying capacity is linear and increases with the increase of lamda. From fig(4) the load carrying capacity is plotted with roughness parameter for different values of k. As the roughness increases the load carrying capacity is linear and increases with the increase of k.

Summary: In this paper the generalized Reynolds equation with the effect of couplestress fluid with roughness and slip is applied to finite journal bearing. The cases of short and long journal bearing is analyzed and is applied to study the finite journal bearing using Rapid-Narang technique.

Nomenclature:

a	The mean height surface asperities in the symmetric roughness case
c	Radial clearance
e	Eccentricity ε Eccentricity ratio
h	total film thickness
H	Dimensionless oil film thickness
R	Radius of Shaft
r	radius of the bearing
U	velocity of the surface
l	couplestress parameter
p	Hydrodynamic pressue
w	load carrying capacity
hn	nominal film thickness
hs	mean height of the roughness asperities

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