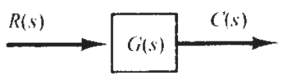
DESIGN AND ANALYSIS OF SIGNAL FLOW GRAPHS IN CONTROL SYSTEMS

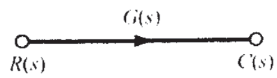
V. MANJULA

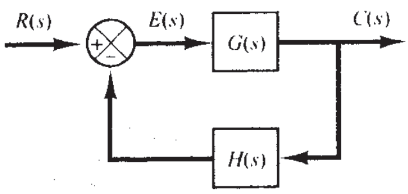
Abstract : This paper describes the application of Graph Theory in control systems. Because of its inherent simplicity Graph Theory has a wide range of applications in Engineering, physical, social and biological sciences. This paper is an attempt to show that Graph Theory can be used to represent so many problems involving discrete arrangement of objects. It is not with the concern of internal properties of control system but the relationship with Graph Theory in Engineering sciences. A signal Flow Graph is a graphical representation of control Theory .It is a diagram that represents a set of simultaneous equations. The advantage of signal flow graph is Mason's gain formula so that the overall gain of the system can be computed easily. This method is simpler than block diagram techniques that are tedious. In the analysis of linear system problems can be reduced to set of simultaneous linear algebraic equations usually solved by matrix method can also be solved by Graph Theory .The graph theoretic approach is often faster and displays cause –effect relationships between the variable. Each signal flow graph consists of a network in which nodes are connected by directed branches and each node represents a system variable, each branch acts as a signal multiplier. The signal flows in the direction can be indicated by the arrows.

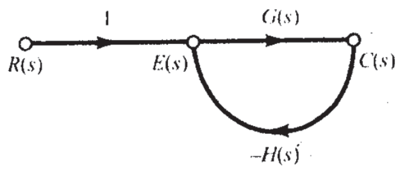
Key words: Introduction, Interpretation, construction of Signal Flow Graphs

Introduction: In the early study of control systems the analysis was based upon the solution of the differential equations and it becomes tedious and does not indicate readily what changes should be made to improve the system performance. Usage of Laplace transforms simplifies this analysis to some extent. Graph theory is one of the important areas in mathematics for the study of control systems. In analyzing and designing control systems we must have a basis of comparison of performance of various systems.Asignalflow graphis adiagram that represents a set of simultaneous linear algebraic equations and eachgraph consists ofa networkin which nodes are connected bydirected branches. Eachnode represents a system variable,andeach branch acts asasignal multiplier.The signal flows will beindicated by the arrows.









In the analysis of control system it is very convenient to obtain block diagram of different components and their interconnecting. It is also possible to determine the transfer function of a complicated block diagram directly using masons gain formula.

Ma son's Rule

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

Where M is the total gain of the system, represented as the ratio of the output gain (y_{out}) to the input gain (y_{in}) of the system. M_k is the gain of the k^{th} forward path, and Δ_k is the loop gain of the k^{th} loop.

Fundamentals of control systems: A control system is an interconnection of components forming a systemconfiguration that will provide a desired system response. The basis for analysisof a system is the foundation provided by linear system, which assumes a causeeffect relationship for the components of a system.The study of control systems involves how components are integrated together to control a parameter such as velocity. Torque and pressure to force an output parameter to remain at a specific value. Control systems use different types of components: electrical. Mechanical, fluid values for developing a strategy for control of a plant the dynamic, static mathematical model for the system must be developed.

*Temperature is one of the most often measured parameters. The maintenance of constant temperature and the detection of over or under temperature conditions is extremely important.

*Fluid systems occur often in control systems. They are used for things such as maintaining the fluid level in a tank so the tank remains full under changing conditions of outflow from tank.

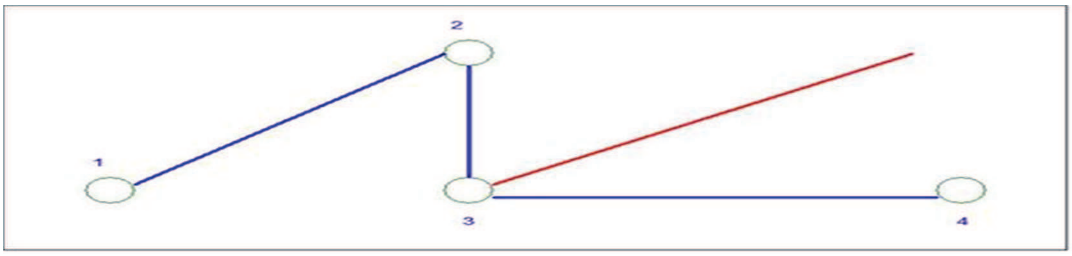
* The voltage developed across a resistor is a function of the resistance and current represents the electrical equivalent of friction

Why study signal flow graphs: For the analysis and design of control systems a mathematical description of the system known as 'model' need to be formulated .The basic models are differential equations obtained by application of appropriate laws of nature. These are inconvenient for the analysis and design manipulations . So the Laplace transforms which converts differential equations to algebraic equations. These equations will be in transfer function form and the system modeled graphically a transfer function of block diagram. Alternatively signal flow graph may be used.

Interpretation of Graph Theory in control systems

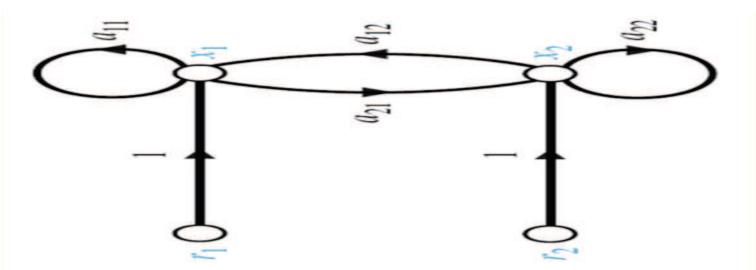
signal flow graphs are used to describe a control system transfer function. Generally graphs are simpler than a block diagram since only loops,nodes and branches are used to show the configuration of the system.

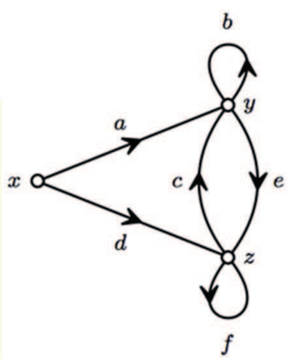
A graph is a set of nodes (vertices) and a set of branches (lines, edges) that connect two nodes



In signal flow graphs the outer circles indicate nodes which are variables and the lines which are connecting braches indicate transfer relationships. The central node is a summing junction. A path that starts on a node and ends on the same node passing through no. of nodes more than once is called a loop. A self branch that passes through nodes in the same direction when the path passes through the nodes only once is called a forward path. Block diagram is the most extensively used graphical representation of feedback control system. But for complicated systems the block diagram reduction becomes tedious signal flow diagram technique is alternative approach developed by J.J.Manson

Signal flow graph is a pictorial representation of simultaneous equations describing a system. It graphically represents the transmission of signals through system.A feedback loop is a path that forms a closed cycle along which each node is encountered once per cycle . A path gain is the product of the branch gains along that path. The loop gain of a feedback loop is the product of the gains of the branches forming that loop. The gain of a flow graph is the signal appearing at the sink per unit signal applied at the source. Only one source and one sink need be considered,since sources are superposable and sinks are independent of each other.

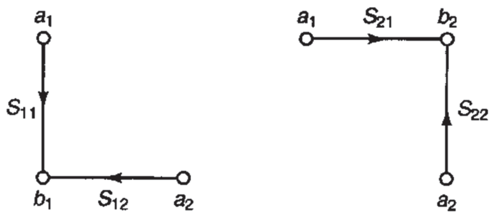




Some Elementary properties of a signal flow graph

- Signal flow applies only to linear systems.
- The equations based on which a signal flow graph is drawn must be algebraic
  equations in the form of effects as a function of causes.
  Nodes are used to represent variables. Normally the nodes are arranged left to
  right, following a succession of causes and effects through the system.
- Signals travel along the branches only in the direction described by the arrows ofthe branches.
- The branch directing from node Xk to Xj represents dependence of the variable Xj on Xk but not the reverse.
- The signal traveling along the branch Xk and Xj is multiplied by branch gain akj
  and signal akjXk is delivered at node Xj.
  A signal flow graph is a network of directed branches which connect at nodes. Branch jk originates at node jand terminates upon node k, the direction from j to kbeing indicated by an arrowhead on the branch. Each

Branchjk has associated with it a quantity called the branch gain gik and each node j has an associated quantity called the node signal **xj.** The various node signals are related by the associated equations L xjgik = xk, k = 1, 2,



$b_1 = S_{11}a_1 + S_{12}a_2$ $\quad$ $b_2 = S_{21}a_1 + S_{22}a_2$

Construction of Signal Flow Graphs

The signal flow graph of linear feedback control system can be constructed by direct reference to the block diagram of the system. Each variable of block diagram becomes a node and each block becomes branch.

1) Arrange the input to output nodes from left to right.

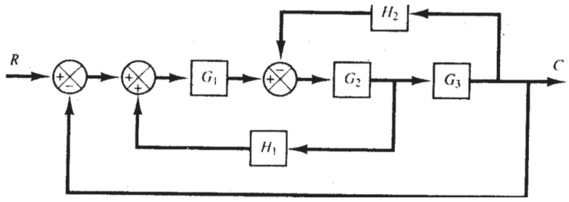
2) Connect the nodes by appropriate branches.

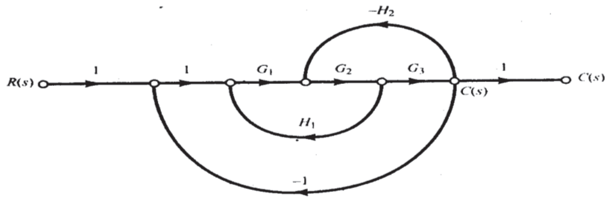
3) If the desired output node has outgoing branches, add a dummy node and a unity
Gain branch.

4) Rearrange the nodes and/or loops in the graph to achieve pictorial clarity

Choose the nodes to represent the variables say as shown in the block diagram..

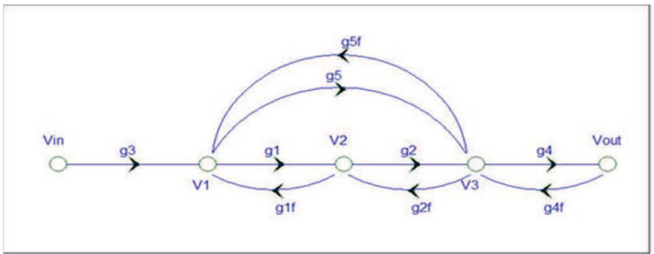
Connect the nodes with appropriate gain along the branch. The signal flow graph is shown in Fig.





Analysis

A circuit component can create different values of signal flow for different directions. With a directed graph, we can have different values for each direction.



Each branch has a Source node and a Destination node. Vin as a source node and node V1 as a destination node. A Source node is the input for the branch gain, and the Destination node is the output for the branch gain. The branch receives a signal from the Source node and sends it to the Destination node. A path is a sequence of branches that does not go through any node more than once. Any node can be included in a path as a source node only once. A path must also contain a node as a destination node only once. In the above figure g3 – g1 – g2 is a valid path. The path g3 – g1 – g1f –g1 – g2 is not a valid path since it goes through node V2 twice. Any branch can be contained in a path only once..

. The path gain is the product of the branch gains that make the path.

In the SFG above one path from Vin to Vout has a path gain of g3*g1*g2*g4. The other path gas the gain g3*g5*g4

A path that has the same node as the source node and the final destination node is a Loop. In the figure g1 – g1f is a loop. The path g5 – g2f – g1f is also a loop. There are additional loops. A loop that has only a single branch is called a Self Loop.

A node that has only branches leaving the node is an Input node. In the SFG Vin is an input node. A node that has at least one incoming branch is an Output node. Nodes V1, V2, V3, and Vout are output nodes.

Concept: Block diagrams and signal flow graphs are used to convey information about a system and its parameters whether the system is continuous or discrete a transfer function will be obtained from the type of the diagram used. A transfer function is the ratio of output to input of a system. Using transfer function the order of the system can be determined. In some cases it may be desirable to increase the order of the system to achieve zero order and to improve steady state performance. It is important to know how the overall system reacts when a change occurs in a specific parameter value and how to design for minimizing the change of output of the system . The good in all the systems is that when a reference input is applied the output will have the exact value of the system input in steady state. The output should have no error with regard to input.

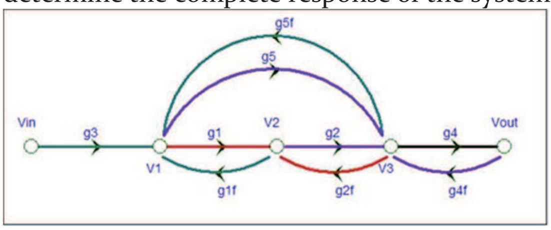
Three common signal types used to test control system are

1. Step input
2. Ramp input
3. Parabolic input

Each of the signals requires a specific type a system for input signal with either zero error or some specified error.

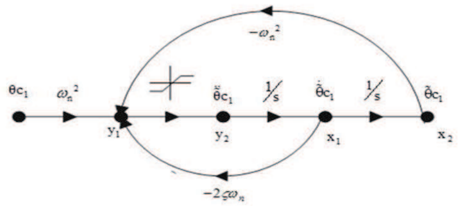
Importance

Signal Flow Diagrams visually representing a system. These are especially useful, because they allow for particular methods of analysis, such as **Mason's Gain Formula**.Signal flow diagrams typically use curved lines to represent wires and systems, instead of using lines at right-angles, and boxes, respectively. Every curved line is considered to have a multiplier value, which can be a constant gain value, or an entire transfer function. Signals travel from one end of a line to the other, and lines that are placed in series with one another have their total multiplier values multiplied together..Signal flow diagrams help us to identify structures called "loops" in a system, which can be analyzed individually to determine the complete response of the system

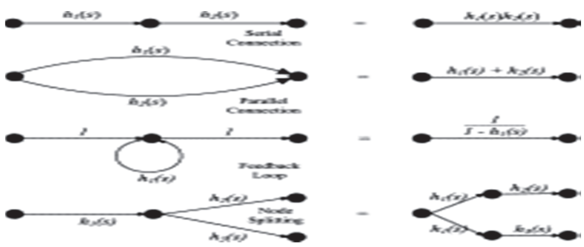


An output node sums the incoming branch signals to form the node value

V1 = (Vin * g3) + (V2 * g1f) + (V3 * g5f)
V2 = (V1 * g1) + (V3 * g2f)
V3 = (V1 * g5) + (V2 * g2) + (Vout * g4f)
Vout = (V3 * g4)



$$\frac{\ddot{\theta}_1(s)}{\theta c_1(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$



Conclusion : A crucial problem in engineering design and analysis is the determination of a mathematical model must be in a quantativemanner, the input and output of the system which may be regarded as the 'cause' and 'effect'. The transfer function represents a linear model of the system and is usually shown in the form of a block diagram and it is unidirectional. In many practical situations the model obtained by using block diagrams complicated .in such cases the alternate approach for system identification is signal flow graphs. It can be regarded as a type of curve fitting since the transfer function is selected to fit the overall lable data in optimum manner. Using signal flow graphs the overall transfer function of the system can be determined. 'Block diagram' pictorial representation of control systems.

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Basic Engineering Department,
DVR& Dr. HS MIC College of Technology, Kanchikacherla
manju_adiraju@yahoo.co.in