# ANALYSIS OF CHAIN SAMPLING PLAN FOR EXPONENTIAL FAMILY

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**Abstract :** This paper presents chain sampling plan using various distributions like generalized exponential distribution, beta generalized exponential distribution and Gumbel distribution. The probability of acceptance are compared based on the sample size and acceptance number. The design parameter such as minimum sample size is obtained by satisfying the producer's and consumer's risk. Tables and graphs are shown to evaluate the use of distributions functions in acceptance sampling.

**Introduction :** The important field in statistical quality control is acceptance sampling plan introduced by Dodge and Romig which forms a bridge between no inspection and 100% inspection. Acceptance sampling can be employed if testing is destructive or cost of 100% inspection takes long time. Samples may be single ,double multiple or sequential .The ChSP-1 overcomes the disadvantages of Single Sampling Plan inorder to reject a lot on basis of single nonconforming unit. It makes use of cumulative results of several samples . The use of cumulative results of samples proposed for application to cases where there is repetitive production under same condition and lots to be inspected are submitted in order of production. certain conditions. Complete review and bibliography on chain sampling plans can be seen in Stephens (1982). Soundararajan (1978a, b) constructed tables for the selection of chain sampling plan-1 (ChSP-1) plans. Soundararajan and Govindaraju (1982) made contribution in the designing of ChSP-1 plans.

**Operating procedure for ChSP-1**

The operating procedure for ChSP-1 is as follows: Select a random sample of size 'n' units from a lot with size 'N'. Inspect all the items included in the sample. Let'd' be the number of non-conformities in the sample. If d = 0, accept the lot. If d >1, reject the lot. Accept the lot if d =1 and if no non-conforming units are found in the immediately preceding 'i' samples of size 'n'.

Conditions for application

(i) The production is steady so that results of past, present and future lots are broadly indicative of a continuing process.

(ii) Lots are submitted sequentially in the order of their production.

(iii) Inspection is by attributes, with the lot quality defined as the proportion defective.

(iv) Human involvement should be less in the manufacturing process.

(v) The companies are to have sufficient experience in adopting Six Sigma initiatives in their process to ensure the system has the potentiality to produce nearly zero defectives.

Generally in acceptance sampling only few distributions are considered like Binomial, Poisson , Hypergeometric, Marshall – Olkin extended exponential distributions etc., along with chain sampling plan. Sudamani Ramaswamy and Jayasri (2013) worked on time truncated chain sampling plans for Marshell-olkin extended exponential distributions.An extension of the above work is done here with respect to three distributions like Beta Generalized Exponential Distribution (BGED),Generalized Exponential Distribution and Gumbel Distribution.

**The Beta Generalized Exponential Distribution**The cumulative distribution function (cdf) of the generalized exponential (GE) distribution is given by is $F(x) = (1-e^{-\lambda x})^{\alpha}$ for $x > 0, \lambda > 0$ . The two parameters of the GE distribution represent the shape ($\alpha$ >0) and the scale parameter ($\lambda$ > 0) like the gamma and Weibull distributions. Exponential distribution is a particular case of the GE distribution when $\alpha$ = 1. Nadarajah and Kotz introduced four more exponentiated type distributions: the exponentiated gamma,exponentiated Weibull, exponentiated Gumbel and expon ntiated Fr′echet distributions by generalizing the gamma, Weibull, Gumbel and Fr′echet dis tributions. They also provide some mathematical properties for each exponentiated distribution. The beta GE density function varies significantly depending on the shape parameter $\alpha$ and is given by

$$f(x) = \frac{\alpha\lambda}{B(a,b)} e^{-\lambda x}(1-e^{-\lambda x})^{\alpha a-1}\{1-(1-e^{-\lambda x})^{\alpha}\}^{b-1}, \quad x > 0,$$

The GE distribution is a special case of beta GE for the choice a = b = 1. If in addition $\alpha$ = 1, we obtain the exponential distribution with parameter $\lambda$.

**Generalized Exponential Distribution**

The probability density function of an GE distribution is given by

$f(x) = \alpha\beta(1-e^{-\beta x})^{\alpha-1}e^{-\beta x}, x > 0$ . The GE density function varies significantly depending on the shape parameter $\alpha$. The GE distribution has lots of properties which are quite similar in nature to those of the gamma distribution but it has explicit expressions for the distribution and survival functions like a Weibull distribution. GE can be used as an alternative to Weibull and Gamma distributions. The four parameter beta generalized exponential (BG ) distribution by taking

$$f(x)=\frac{\alpha\lambda}{B(a,b)}e^{-\lambda x}(a-e^{-\lambda x})^{\alpha a-1}\left\{1-(1-e^{-\lambda x})^{\alpha}\right\}^{b-1},x$$

Beta generalized exponential distribution can be considered as a special case of beta exponential and generalized exponential distributions .

**Gumbel Distribution** The extreme value type I distribution has two forms. One is based on the smallest extreme and the other is based on the largest extreme. We call these the minimum and maximum cases, respectively. The extreme value type I distribution is also referred to as the Gumbel distribution. The general formula for the pdf of the Gumbel (minimum) distribution is

$$f(x)=\frac{1}{\beta}e^{\frac{x-\mu}{\beta}}e^{-e^{\frac{x-\mu}{\beta}}},x>0$$

where $\mu$ is the location parameter and $\beta$ is the scale parameter. The case where $\mu$ = 0 and $\beta$ = 1 is called the **standard Gumbel distribution**. The equation for the standard Gumbel distribution (minimum) reduces to $f(x)=e^{x}e^{-e^{x}}$

The general formula for the pdf of the Gumbel (maximum) distribution is

$$f(x)=\frac{1}{\beta}e^{\frac{x-\mu}{\beta}}e^{-e^{-\frac{x-\mu}{\beta}}},x>0$$

where $\mu$ is the location parameter and $\beta$ is the scale parameter. The case where $\mu$ = 0 and $\beta$ = 1 is called the **standard Gumbel distribution**. The equation for the standard Gumbel distribution (maximum) reduces to $f(x)=e^{x}e^{-e^{-x}}$ The formula for the cdf of the Gumbel distribution (minimum) is $F(x)=1-e^{-e^{x}}$ The formula for the cdf of the Gumbel distribution (maximum) is $F(x)=e^{-e^{x}}$

The chain sampling plan is characterized by two parameters n and i. The work done here is to determine the sample size required for beta generalized exponential distribution, generalized exponential distribution and Gumbel distribution for various values of acceptance number i.The probability of acceptance can be considered as a function of the deviation of specified average from the true average. This function is called operating characteristic function of the sampling plan. The minimum sample size is obtained so that one may known the probability of acceptance of a lot. The lot can be accepted or rejected based on the probability of acceptance. The probability of acceptance of chain sampling plan is given by

$$P_a(p)=(1-p)^{n}+np(1-p)^{n-1}(1-p)^{ni}$$

a.

where $p=\frac{1}{\beta}e^{\frac{x-\mu}{\beta}}e^{-e^{\frac{x-\mu}{\beta}}},x>0$ for Gumbel distribution

$p=\alpha\beta(1-e^{-\beta x})^{\alpha-1}e^{-\beta x},x>0$ for generalized exponential distribution

$$p=\frac{\alpha\lambda}{B(a,b)}e^{-\lambda x}(a-e^{-\lambda x})^{\alpha a-1}\left\{1-(1-e^{-\lambda x})^{\alpha}\right\}^{b-1},x>0$$

for beta generalized exponential distribution

The table below shows the probability of acceptance for ChSP-1 using BEGD,GED and GD . For example , let us assume that a experimenter is interested in testing n=10 items and if not more than 1 item fail and if no defective items are found in the immediately preceeding i=1 samples the probability of acceptance are 0.89694, 0.82302 and 0.996003 in case of BGED,GED and GD.

Table 1 : $P_a(p)$ for BGED given n and i

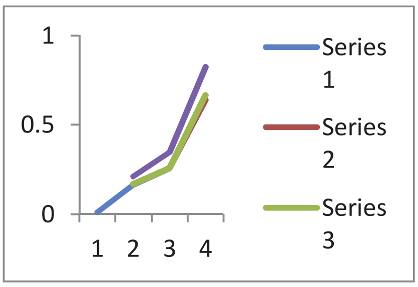
| | | BGED | | |
|---|---|---|---|---|
| i | n=100 | n=40 | n=30 | n=10 |
| 20 | 0.04024 | 0.27766 | 0.381427 | 0.7256 |
| 10 | 0.04024 | 0.27662 | 0.38141 | 0.73476 |
| 5 | | 0.2772 | 0.38444 | 0.77272 |
| 1 | | 0.37655 | 0.52392 | 0.89694 |

Table 2 : $P_a(p)$ for GED given n and i

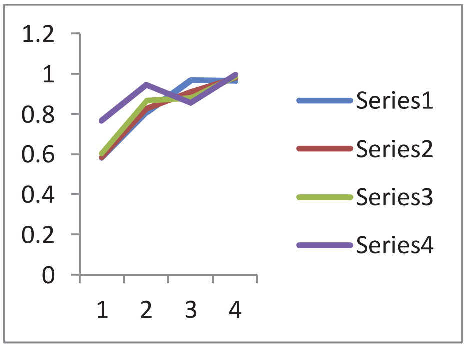
| | | | GED | |
|---|---|---|---|---|
| I | n=100 | n=40 | n=30 | n=10 |
| 20 | 0.010783 | 0.16334 | 0.25694 | 0.63577 |
| 10 | | 0.166336 | 0.25694 | 0.63891 |
| 5 | | 0.16634 | 0.25734 | 0.66632 |
| 1 | | 0.2128 | 0.34871 | 0.82302 |

Table 3 : $P_a(p)$ for GD given n and i

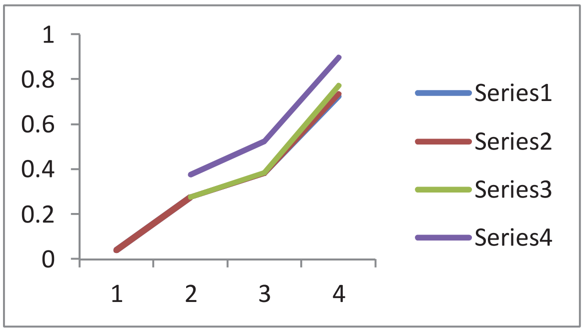
| | | GD | | |
|---|---|---|---|---|
| I | n=100 | n=40 | n=30 | n=10 |
| 20 | 0.583919 | 0.80873 | 0.96823 | 0.965051 |
| 10 | 0.585363 | 0.8266 | 0.91239 | 0.979411 |
| 5 | 0.60529 | 0.8657 | 0.87836 | 0.986644 |
| 1 | 0.76784 | 0.94669 | 0.8564 | 0.996003 |




$P_a(p)$ for Chsp-1 using GED




b. $P_a(p)$ for Chsp-1 using GD




c.$P_a(p)$ for Chsp-1 using BGED

**Results :** The paper presented here explains chain sampling plan with various distributions are used and probability of acceptance are found for given sample size and acceptance number. From the three distributions it is found that Gumbell distribution is comparatively better than other distributions. It works well compared to marshell olkin extended exponential distribution also.

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