MULTI CRITERIA ACADEMIC RESOURCE ALLOCATION USING FUZZY MULTI OBJECTIVE INTEGER PROGRAMMING APPROACH

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Abstract : In this paper, the problem of faculty allocation to different departments according to the department's need and budget of the institute has been considered. Most of the real world problems consist of imprecise data. The problem of academic resource allocation has been considered as a fuzzy multi objective linear integer programming problem. The main objective of the paper is that the number of regular faculties and contractual faculties of the institute is optimized according to the actual need of the institute. Fuzzy integer linear programming problem has been proposed to solve the above decision making problem. The proposed approach serves as an effective decision tool for the above mentioned faculty allocation decision making problem of the institute.

Keywords: Fuzzy Goals, Crisp goals, Multi Objective Programming, Membership function, integer programming.

Introduction: Any Institutes are responsible for the allocation and alignment of limited resources so that the university serves its mission and meets its objectives. It is strategic analysis that guides this resource allocation and alignment so that the institution positions itself to leverage its assets, minimize its risks, and satisfy the expectations of its varied stakeholders.Multi criteria decision making problem has become more challenging problem in the current scenario where there will be more than one criterion which should be satisfied in an optimised way.Most of the real world decision problems involve multi criteria that are often conflicting in nature. Now, in the real world, elements are perturbed by imperfection and thus there exists no element that is perfectly round. Mathematical modeling essentially consists of translating real world problems into mathematical problems, solving the mathematical problems and interpreting these solutions in the language of real world.Also to capture the reality, the traditional modeling methodology is perhaps not relevant, and at least not powerful enough to satisfy the requirements of decision maker. Furthermore, considering the fuzziness of human's subjective judgement in real world decision making it is necessary to deal with the large scale programming problems in the fuzzy environment. In a classical paper, Bellmann and Zadeh (1970) introduced the theory of fuzzy sets, which is the generalization of abstract set theory, as a powerful and useful conceptual framework for dealing quantitatively with imprecision in a decision problem. In this paper, we have considered the resource allocation of any academic institution as a multi criteria decision making problem. We have solved multi criteria resource allocation problem using multi objective fuzzy integer programming approach. **Literature Survey :**Multi criteria fuzzy Linear Programming problems have an essential role in fuzzy modelling for multi criteria optimization problem, which can formulate uncertainty in actual environment. It has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications.Fuzzy data was first introduced byZadeh [17].Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment.Tanaka et al. [14] adopted this concept for solving mathematical programming problems.Zimmerman [18] proposed the first formulation of fuzzy linear programming. Chanas [6]proposed the possibility of the identification of a complete fuzzy decision in fuzzy linearprogramming by use of the parametric programming technique. Chakraborty et al. in [2, 3, 4] shows various applications in the field of multi criteria fuzzy linear programming. Various authors have done work in the field of fuzzy integer programming and the technique has been applied to a large number of application problems. In [8], the authors proposed three models of fuzzy integer linear programming. In [5], Chakraborty et al. has applied multi objective Fuzzy 0-1 programming in one application problem. Saad et al. described a solution algorithm for multi objective integer nonlinear fractional programming problems under fuzziness [12]. In [9] Kavitha et al. implemented multi objective fuzzy integer programming technique in supply chain management. In [13], Sharma et al described fuzzy integer programming problem for trapezoidal fuzzy number with the help of simple ranking function. Gomory's algorithm is used to find the fuzzy optimal solution. A new method named decomposition method has been proposed for solving integer linear programming problem with fuzzy variables by Pandian et al [11]. In paper [15], a fuzzy mixed-integer mathematical programming model is developed to formulate the fuzzy mixed-integer optimization problem. In [10], Lee and Clayton have given goal programming model for academic resource allocation problem. In [16], Xiong et al. formulates a bilevel programming model for allocating resources between pavement and bridge deck maintenances. In [7], a new procedure of weighted fuzzy multi objective linear fractional goal programming problem is used to solve the fuzzy multi objective linear fractional goal programming problem. In this paper, we have solved the academic resource allocation problem with limited resources like infrastructure, human resources and equipments in any institute. Data are considered as imprecise in nature. Fuzzy integer linear programming technique has been used to solve the problem and optimised solution has been obtained.

Problem Discussion :In this paper we have considered faculty allocation in different departments of any engineering institute. Suppose in any institute departments like Computer Science and Engineering (CSE), Electronics and Communication (ECE), Information Science (IS), Telecommunication (TE), Mathematics, Physics, Chemistry and Humanities have been considered. According to the UGC (University Grants Commission) rule, ratio of regular faculty and student should be about 1:12. Ratio between the regular faculty and weekly class load should be 1:14. According to the survey of an engineering institute, the total number of student intake is 380. The main objective of the paper is that the number of regular faculties and contractual faculties of the institute is optimized according to the actual need of the institute. The proposed approach serves as an effective decision tool for the above mentioned faculty allocation decision making problem of the institute. Fuzzy Integer Programming Model: Fuzzy multi objective integer programming problem can be defined as :

$$Max \quad \mathrm{Z_i} = C_{ij}x_j$$

$$\sum_{j\in N} a_{ij}x_j \tilde{\leq} \mathrm{b_i}$$

Subject to $x_j \geq 0, j \in N$

$$x_j \in N, j \in N$$

The following parameters have been used to solve the problem.

Parameters: R_{ij} - Regular faculties (RF) in the ith department of jth rank at time t.

C_i - Contractual faculties (CF) in the ith department at time t.

T_i - Total weekly class load in the ith department at time t.

St_i -Total number of student in the ith department at time t.

S_{ij} - Annual Salary of the regular faculties in the ith department of jth rank at time t.

A_i - Annual remuneration of contractual faculties in the ith department at time t.

Q_i -CF/RF ratio in the ith department at time t.

SR_i - RF/Student ratio in the ith department at time t.

B_t - Total pay budget at time t.

WC_i - RF/Weekly class load.

Decision Variables: Two types of decision variables can be considered.

$x_{ij} \rightarrow$ Number of regular faculty in the department

$y_i \rightarrow$ Number of contractual faculty required in the Department

Ratio of regular faculty and total number of students in the Department = 1/12

Ratio of regular faculty and weekly class load= 1/14

Description of Fuzzy Goals and Crisp Goals:

Fuzzy Goals

1. RF goal: Number of regular faculty in the ith department of jth rank at time t should be greater than total number of Regular faculties in the department.

$$x_{ij} \tilde{\geq} R_{ij}$$

2. CF/RF ratio: The ratio should be greater than the specified value

$$y_i - Q_i \sum_{j=1}^{J} x_{ij} \tilde{\geq} C_i$$

3. Budget: The Budget of the institute should be less than the prescribed value of the budget.

$$\left[\sum S_{ij}x_{ij} + \sum_{i=1}^{I} A_i y_i\right] \tilde{\leq} B_t$$

Crisp Goals

1. RF/ Student ratio should be maintained.
2. RF/ Weekly class load should be maintained.

Data Representation : The following data has been collected from an engineering institute. The institute offers Computer science, Electronics and Communication, Telecommunication and Information Science for B. Tech degree. The table shows the ranges of the number of faculty requirement in the respective departments.

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Departments / Faculty Positions	CSE	EC	TE	IS	Maths	Physics	Chemistry	Humanities
Professor	2- 4	1-4	0-3	1-2	0-3	1-2	0-1	0-1
Associate Professor	5-7	4-6	0-6	2-4	4-6	2-3	2-3	1-2
Asst. Professor	7-9	6-9	6-10	4-8	6-8	3-6	3-6	2-4
Contractual Faculty	1-2	2-3	2-3	1-2	1-2	1-2	0	0

The following table represents the ratio between the contractual and regular faculty and the number of student in different departments.

Departments	Number of Students	Ratio Between contractual and Regular faculty
CSE	290	1:25
EC	250	2:20
TE	200	2:25
IS	200	1:20
Departments	No. of Classes	Ratio Between contractual and Regular faculty
Maths	140	1:10
Physics	70	1:15
Chemistry	70	0
Humanities	45	0

The ratio between the regular faculty and the weekly teaching learning process hour per teacher for all the subjects is 1:14. The budget of the institute is around 1300 and can be considered as the aspiration level of the fuzzy budget goal. The upper tolerance level is determined by increasing upto about 15 % of the current budget that is upto 1500 Lakhs. Considering all the fuzzy and crisp goals under the given constraints of the above problem, it can be formulated mathematically as follows.

Mathematical Formulation Of The Problem :

Mathematically the above problem can be formulated as a multi objective linear programming problem as follows:

Maximize x_{ij}

Maximize $y_i - Q_i \sum_{j=1}^{J} x_{ij}$

Minimize $\left[\sum S_{ij} x_{ij} + \sum_{i=1}^{I} A_i y_i \right]$

Subject to

$$290 - 12\left(\sum_{j=1}^{3} x_{1j}\right) \geq 0$$

$$250 - 12\left(\sum_{j=1}^{3} x_{2j}\right) \geq 0$$

$$200 - 12\left(\sum_{j=1}^{3} x_{3j}\right) \geq 0$$

$$200 - 12\left(\sum_{j=1}^{3} x_{4j}\right) \geq 0$$

$$140 - 14\left(\sum_{j=1}^{3} x_{5j}\right) \leq 0$$

$$70 - 14\left(\sum_{j=1}^{3} x_{6j}\right) \leq 0$$

$$70 - 14\left(\sum_{j=1}^{3} x_{7j}\right) \leq 0$$

$$45-14\left(\sum_{j=1}^{3}x_{8j}\right)\le 0$$

$$x_{ij}\ge 0,\ x_{ij}\in N.$$

The above multi objective problem can be solved by fuzzy programming approach by assigning some aspiration level to all objectives. The fuzzy linear programming problem for the above problem can be defined as follows:

Find x_{ij} where $x_{ij}\in N$

Subject to $x_{11}\tilde{\ge}4$ $x_{12}\tilde{\ge}5$ $x_{13}\tilde{\ge}9$

$x_{21}\tilde{\ge}4$ $x_{22}\tilde{\ge}6$ $x_{23}\tilde{\ge}9$

$x_{31}\tilde{\ge}3$ $x_{32}\tilde{\ge}6$ $x_{33}\tilde{\ge}10$

$x_{41}\tilde{\ge}2$ $x_{42}\tilde{\ge}4$ $x_{43}\tilde{\ge}8$

$x_{51}\tilde{\ge}3$ $x_{52}\tilde{\ge}6$ $x_{53}\tilde{\ge}8$

$x_{61}\tilde{\ge}2$ $x_{62}\tilde{\ge}3$ $x_{63}\tilde{\ge}6$

$x_{71}\tilde{\ge}1$ $x_{72}\tilde{\ge}3$ $x_{73}\tilde{\ge}6$

$x_{81}\tilde{\ge}1$ $x_{82}\tilde{\ge}2$ $x_{83}\tilde{\ge}4$

$$25y_1-\sum_{j=1}^{3}x_{1j}\tilde{\ge}2\quad 20y_1-2\sum_{j=1}^{3}x_{2j}\tilde{\ge}2$$

$$25y3-2\sum_{j=1}^{3}x_{2j}\tilde{\ge}3$$

$$20y4-\sum_{j=1}^{3}x_{4j}\tilde{\ge}2$$

$$20y5-\sum_{j=1}^{3}x_{5j}\tilde{\ge}2$$

$$15y6-\sum_{j=1}^{3}x_{6j}\tilde{\ge}2$$

$$290-12\left(\sum_{j=1}^{3}x_{1j}\right)\ge 0$$

$$250-12\left(\sum_{j=1}^{3}x_{2j}\right)\ge 0$$

$$200-12\left(\sum_{j=1}^{3}x_{3j}\right)\ge 0$$

$$200-12\left(\sum_{j=1}^{3}x_{4j}\right)\ge 0$$

$$140-14\left(\sum_{j=1}^{3}x_{5j}\right)\le 0$$

$$70-14\left(\sum_{j=1}^{3}x_{6j}\right)\le 0$$

$$70-14\left(\sum_{j=1}^{3}x_{7j}\right)\le 0$$

$$45-14\left(\sum_{j=1}^{3}x_{8j}\right)\le 0$$

$$0.05\sum_{i=1}^{8}x_{i1}+0.025\sum_{i=1}^{8}x_{i2}+0.025\sum_{i=1}^{8}x_{i3}\tilde{\le}1300$$

$$0\le\lambda\le 1,\ x_{ij}\ge 0,\ x_{ij}\in N.$$

The membership functions $\mu_i(x)$ for the fuzzy sets $Z_i(x)\tilde{\ge}p_i$ may respectively be constructed as:

$$\mu_i(x)=\begin{cases}0 & \text{if } Z_i(x)\le \underline{Z}_i\\ \dfrac{Z_i(x)-\underline{Z}_i}{p_i-\underline{Z}_i} & \text{if } \underline{Z}_i<Z_i(x)<p_i\\ 1 & \text{if } Z_i(x)\ge p_i\end{cases}$$

Where p_i is the assigned aspiration level and $\underline{Z}_i$ is the lower bound of the i[th] fuzzy goal.Similarly, for $Z_i(x)\tilde{\le}p_i$ the membership function is

$$\mu_i(x)=\begin{cases}1 & \text{if } Z_i(x)\le p_i\\ \dfrac{\bar{Z}_i-Z_i(x)}{\bar{Z}_i-p_i} & \text{if } p_i<Z_i(x)<\bar{Z}_i\\ 0 & \text{if } Z_i(x)\ge \bar{Z}_i\end{cases}$$

Where p_i is the assigned aspiration level and $\bar{Z}_i$ is the upper bound of the i[th] fuzzy goal.

The membership function of the fuzzy goals can be formed according to the above Definition. The membership function for the budget can be given as

$$\mu=\begin{cases}1 & \text{if} B_t\le 1300\\ \dfrac{1500-F}{200} & \text{if } 1300\le B_t\le 1500\\ 0 & \text{if} B_t\ge 1500\end{cases}$$

After calculating the membership function for each fuzzy constraint, we solve the problem using Lingo 13.0. The solution of the above problem is given below.

Solution:

Global optimal solution found.

Objective value:	0.6666667
Objective bound:	0.6666667
Infeasibilities:	0.000000
Extended solver steps:	0

Total solver iterations: 21
Model Class: MILP
Total variables: 31
Nonlinear variables: 0
Integer variables: 30
Total constraints: 41
Nonlinear constraints: 0
Total nonzeros: 128
Nonlinear nonzeros: 0

Variable	Value
LAMBDA	0.6666667
X11	4.000000
X12	7.000000
X13	9.000000
X21	4.000000
X22	6.000000
X23	9.000000
X31	3.000000
X32	4.000000
X33	9.000000
X41	2.000000
X42	2.000000
X43	5.000000
X51	3.000000
X52	6.000000
X53	8.000000
X61	1.000000
X62	3.000000
X63	6.000000
X71	2.000000
X72	3.000000
X73	4.000000
X81	1.000000
X82	2.000000
X83	4.000000
Y1	1.000000
Y2	2.000000
Y3	2.000000
Y4	1.000000
Y5	2.000000
Y6	1.000000

Conclusion: In this paper, an effort has been made to develop a model for the faculty allocation in different departments according to the department's need and the budget of the institute. The problem has been considered as Multi Criteria Decision Making problem. Using fuzzy set theoretic approach, a fuzzy programming model has been developed and solution to the problem has been obtained by considering data from an engineering institute. Here all the objectives have been considered with equal importance. The model proposed is competitive in nature. Out of three objectives, two objectives have been maximized and one objective has been minimized. On analyzing the solution, we can see that most of the objectives are achieved, either fully or reasonably. Since the objectives are conflicting in nature, we cannot expect cent percent result. The overall achievement of this model is approximately 67%. Also the solution has not violated any of the constraints and the values are also in the range. Hence the result obtained is satisfactory result. The capable result encourage the need for further research in the fuzzy multi objective programming problem

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