COMPLETELY PRIME AND COMPLETELY SEMIPRIME S-IDEALS OF SEMI NEAR-RINGS

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Abstract: In this paper we define and study the fuzzy point view of some properties of special ideals in semi near-rings. These ideals play a vital role in developing new substructures in semi near-rings and other similar generalized algebraic systems.

Keywords: Semi near-ring, Completely prime s-ideal, Completely semi prime s-ideal.-+

Introduction: In this paper an algebraic system semi near-ring is considered which is a generalization of both a semi ring and a near-ring.

Definition 1.1. A semi near-ring S is said to have an absorbing zero 0 if $a + 0 = 0 + a = a$ and $a \cdot 0 = 0 = 0 \cdot a$ for all $a \in S$.

Definition 1.2.A subset I of a semi near-ring Sis a right (respectively, left) s-ideal if (i) $x + y \in I$, for all $x, y \in I$.
(ii) $xr \in I$ (right s-ideal), $rx \in I$ (left s-ideal) for all $x \in I$ and $r \in S$.

Definition 1.3. Let μ be a fuzzy subset of a semi near-ring S. Then the set defined by

$$\mu_t = \{x \in S \mid \mu(x) \geq t,\ t \in [0,1]\}$$

is called the level subset of S with respect to μ.

PROPERTIES OF S-IDEALS

Definition 2.1.Let S be a semi near-ring with absorbing zero
(i) s-ideal I is completely prime if for any $a, b \in S$, $ab \in I$ implies $a \in I$ or $b \in I$.
(ii) s-ideal I is called completely semiprime if for all $a \in S$, $a^n \in I$ (for some positive integer n) implies that $a \in I$.

Definition 2.2. Let μ be a fuzzy s-ideal of a semi near-ring S
(i) μ is said to be completely prime if $\mu(a) \geq \mu(ab)$ or $\mu(b) \geq \mu(ab)$ for all $a, b \in S$.
(ii) μ is said to be completely semiprime if $\mu(a) \geq \mu(a^n)$ for all $a \in S$ and a positive integer n.

Proposition 2.3. Let μ be s-ideal of a semi near-ring S with absorbing zero
(i) μ is completely prime if and only if μ_t is completely prime for all $t \in (0, 1)$
(ii) μ is completely semiprime if and only if μ_t is completely semiprime, for all $t \in (0, 1)$

Proof: (i) Suppose μ is completely prime
We show that μ_t is completely prime for all $t \in (0, 1)$.
Let $a, b \in S$ and $ab \in \mu_t$. This implies $\mu(ab) \geq t$.
Since μ is completely prime,
we have $\mu(a) \geq \mu(ab) \geq t$ or $\mu(b) \geq \mu(ab) \geq t$.
Therefore, μ_t is completely prime.
Conversely, suppose that μ_t is prime for all $t \in (0, 1)$.
We show that μ is completely prime
Let $a, b \in S$ and $\mu(ab) = t$,
This means that $ab \in \mu_t$.
Since μ_t is completely prime,
we have $a \in \mu_t$ or $b \in \mu_t$, which implies that
$\mu(a) \geq t = \mu(ab)$ or $\mu(b) \geq t = \mu(ab)$.
Thus μ is completely prime.

(ii) Suppose μ is completely semiprime.
We show that μ_t is completely semiprime.
Let $a^n \in \mu_t$ for some positive integer n.
This implies $\mu(a^n) \geq t$. Since μ is completely semiprime,
we have $\mu(a) \geq \mu(a^n) \geq t$. This implies that $a \in \mu_t$.
Thus μ_t is completely semiprime.
Conversely,
suppose that μ_t is completely semiprime for all $t \in (0, 1)$.
Let $a \in S$. Since S is multiplicatively closed,
we have $a^n \in S$. Let $\mu(a^n) = t$.
This implies that $a^n \in \mu_t$, Since μ_t is completely semiprime,
we have $a \in \mu_t$, This means that $\mu(a) \geq t = \mu(a^n)$.
Thus μ is completely semiprime.

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