ANISOTROPIC DARK ENERGY MODEL WITH CONSTANT DECELERATION PARAMETER IN BIANCHI TYPE-IX SPACE-TIMES

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Abstract: A Bianchi type-IX anisotropic dark energy cosmological model with constant deceleration parameter has been investigated. The field equations have been solved by applying variation law for generalized Hubble's parameter given by Berman [Nuovo Cimento 74, 182, 1983]. Some physical and geometrical properties of the models are also discussed.

Keywords: Dark Energy, Constant Deceleration Parameter, Bianchi type-IX.

Introduction: The universe is currently experiencing a phase of accelerated expansion. This is first observed from the experiments on a expansion history of the universe of type Ia-Supernovae (SNeIa)[1-5], Galaxy red-shift surveys [6] and confirmed later by cross-checks from the cosmic microwave background radiation [7-11]. The accelerating phase of the universe attributed to the fact that universe is dominated by an exotic energy with negative pressure called as dark energy (DE). There are several types of DE with negative pressure depending on their equation of state (EoS) parameter $\left(p=\omega\rho\right)$ which is not necessarily constant. The simplest DE candidate is the vacuum energy $\left(\omega=-1\right)$ which is mathematically equivalent to the cosmological constant $\left(\Lambda\right)$. When EoS $-1<\omega<-\frac{1}{3}$, the dust energy model is called quintessence [12] and when $\omega<-1$, it is phantom [13]. There are some other DE models which can cross the phantom divide $\omega=-1$ both sides are called quintom. Some other limits obtained from observational results coming from SN-Ia data [14] and SN-Ia data collaborated with CMBR anisotropy and galaxy clustering statistics [15] are $-1.67<\omega<-0.62$ and $-1.33<\omega<-0.79$ respectively. However, it is not at all obligatory to use a constant value of ω. Due to lack of observational evidence in making a distinction between constant and variable ω, usually the EoS parameter is considered as a constant [16, 17] with phase wise value -1, 0, -1/3 and +1 for vacuum fluid, dust fluid, radiation and stiff fluid dominated universe, respectively. But in general, ω is a function of time or redshift [18-20]. Chaplygin gas as well as generalized chaplygin gas have also been considered as possible DE sources due to negative pressure [21-25]. Many relativists [26-30] have studied anisotropic DE in different context.

Berman [31] proposed a special law of variation of Hubble parameter in FRW space-time which yields a constant value of deceleration parameter. Such a law of variation for Hubble parameter is not inconsistent with the observations and is also approximately valid for slowly time varying deceleration parameter model. The law provides explicit forms of scale-factor governing the FRW universe and facilitates to describe accelerating as well as decelerating models of evolution of the universe. Models with constant decelerating parameter have been extensively studied in the literature in different context. Kumar and Singh [32, 37], Akarsu and Kilinc [33-35], Yadav and Yadav [36], Pradhan *et al.* [38], Yadav and Rahaman [39] have studied various cosmological models with anisotrpic dark energy with constant deceleration parameter.

The study of Bianchi type-IX universe is important because familiar solutions like Robertson- Walker universe, the de Sitter universe, the Taub-Nut solutions etc., are of Bianchi type-IX space-times. Chakraborty [40], Raj Bali and Yadav [41] studied Bianchi type-IX string as well as viscous fluid models in general relativity. Pradhan [42] have studied some homogeneous Bianchi type-IX viscous fluid cosmological models with varying Λ. Bianchi type-IX stiff fluid tilted cosmological models with bulk viscosity have been investigated by Bali and Kumawat [43]. Rahaman *et al.* [44] have studied Bianchi type-IX string cosmological model in Lyra geometry. Recently, Ghate and Sontakke [45, 46] have studied Bianchi type-IX cosmological models with binary mixture of anisotropic DE and perfect fluid and DE in Brans-Dicke theory of gravitation.

In this paper, we investigate the effects of constant deceleration parameter on Bianchi type-IX model in the presence of anisotropic DE. The solutions to the Einstein field equations are obtained using the condition that expansion scalar is proportional to the shear scalar given by Berman. The physical and geometrical properties of Bianchi type-IX model are also discussed.

Metric and Field Equations: Bianchi type-IX metric is considered in the form,

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + \left(b^2\sin^2 y + a^2\cos^2 y\right)dz^2 - 2a^2\cos y\,dxdz \qquad (1)$$

where a, b are scale factors and are functions of cosmic time t. The energy momentum tensor of the fluid which is taken as $T_i^j = \left[T_0^0, T_1^1, T_2^2, T_3^3\right]$

The simplest generalization of EoS parameter of perfect fluid is to determine it separately on each spatial axis by preserving diagonal form of the energy momentum tensor in a consistent way with the considered metric. Hence one can parameterize energy momentum tensor as follows:

$$T_i^j = \left[-\rho, p_x, p_y, p_z\right]$$

$$T_i^j = \left[-1, \omega_x, \omega_y, \omega_z\right]\rho$$

$$T_i^j = \left[-1, \omega, \omega+\delta, \omega+\gamma\right]\rho . \qquad (2)$$

Here ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and ω_x, ω_y and ω_z are the directional EoS parameters along the *x, y* and *z* axes respectively, ω is the deviation free EoS parameter of the fluid.

Now, parameterizing the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameters δ and γ which are deviations from ω on *y* and *z* axes respectively. Here δ and γ is not necessarily constants and can be functions of the cosmic time t.

The Einstein field equations in gravitational units ($8\pi G = 1$ & $c = 1$) are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad . \qquad (3)$$

Here R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is the energy tensor.

In the co-moving coordinate system the field equations (3) for the metric (1) and with the help of energy momentum tensor (2) can be written as

$$2\frac{\dot a}{a}\frac{\dot b}{b} + \frac{\dot b^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4} = \rho , \qquad (4)$$

$$2\frac{\ddot b}{b} + \frac{\dot b^2}{b^2} + \frac{1}{b^2} - \frac{3a^2}{4b^4} = -\omega\rho , \qquad (5)$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{a} + \frac{\dot a}{a}\frac{\dot b}{b} + \frac{a^2}{4b^4} = -(\omega+\delta)\rho , \qquad (6)$$

$$\frac{\ddot a}{a} + \frac{\ddot b}{a} + \frac{\dot a}{a}\frac{\dot b}{b} + \frac{a^2}{4b^4} = -(\omega+\gamma)\rho , \qquad (7)$$

where the overdot $(^{\cdot})$ denotes the differentiation with respect to *t*.

From equations (6) and (7) we see that, the deviations from ω along y and z axes are same i.e. $\gamma = \delta$.

Solutions of Field Equations

We have three linearly independent equations (4) – (6) and five unknown parameters $a, b, \omega, \rho, \gamma$. Two additional constraints relating these parameters are required to obtain explicit solutions of the systems.

(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ . This condition leads to

$$a = b^m , \quad (8)$$

where *m* is proportionality constant.

The motive behind assuming condition is explained with reference to Thorne [47], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropy today within ≈ 30 percent (Kantowski and Sachs [48]; Kristian and Sachs [49]). To put more precisely, red-shift studies place the limit $\frac{\sigma}{H} \le 0.3$ on the ratio of shear σ to Hubble constant *H* in the neighborhood of our galaxy today. Collin *et al.* [50] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

(ii) Secondly, the law of variation of Hubble's parameter that yields a constant value of deceleration parameter. Such type of relations have already been considered by Berman [31] for solving FRW models.

The average scale factor of Bianchi type-IX metric is given by

$$R = \left(ab^2\right)^{\frac{1}{3}} . \qquad (9)$$

We define, the generalized mean Hubble's parameter *H* as

$$H = \frac{1}{3}(H_x + H_y + H_z) , \qquad (10)$$

where $H_x = \frac{\dot a}{a}$ & $H_y = H_z = \frac{\dot b}{b}$.

Using equation (9), equation (10) reduces to

$$H = \frac{\dot R}{R} = \frac{1}{3}\left(\frac{\dot a}{a} + 2\frac{\dot b}{b}\right) , \qquad (11)$$

Since, the line-element (1) is completely characterized by Hubble's parameter *H*, therefore, let us consider that mean Hubble parameter *H* is related to average scale-factor R by following relation

$$H = k_1 R^{-s} , \qquad (12)$$

where $k_1 > 0$ and $s \geq 0$ are constants.
We consider the constant deceleration parameter model defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \text{constant}, \qquad (13)$$

where the scale factor R is given by (9).
Here the constant is taken as negative (*i.e.*, it is accelerating model of the universe).
From equations (11) and (12), we obtain

$$\dot{R} = k_1 R^{-s+1}, \qquad (14)$$

$$\ddot{R} = k_1^2 (s-1) R^{-2s+1}. \qquad (15)$$

From equation (13), we obtain the law of average scale factor as

$$R = (Dt + c_1)^{\frac{1}{s}}, \qquad \text{for } s \neq 0 \quad (16)$$

$$R = c_2 e^{k_1 t}, \qquad \text{for } s = 0 \quad (17)$$

where c_1 and c_2 are the constants of integration.
From equations (13), (14) and (15), we obtain

$$q = s - 1 \qquad (18)$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign of q indicates inflation. However, the current observations of SN-Ia and CMBR favor accelerating models *i.e.* $q < 0$.

3.1 Case (i): When $s \neq 0$ $(q \neq -1)$:
Equations (8), (9) and (16) lead to

$$a = (Dt + c_1)^{\frac{3m}{(m+2)s}}, \qquad (19)$$

$$b = (Dt + c_1)^{\frac{3}{(m+2)s}}. \qquad (20)$$

Thus the Bianchi type-IX cosmological model can be written as

$$ds^2 = -dt^2 + (Dt + c_1)^{\frac{6m}{(m+2)s}} dx^2 + (Dt + c_1)^{\frac{6}{(m+2)s}} dy^2 + \left((Dt + c_1)^{\frac{6}{(m+2)s}} \sin^2 y + (Dt + c_1)^{\frac{6m}{(m+2)s}} \cos^2 y \right) dz^2 - 2(Dt + c_1)^{\frac{6m}{(m+2)s}} \cos y\, dx\, dz \qquad (21)$$

The physical quantities that are important in cosmology are spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2 which have the following expressions for the model (21):

Spatial volume, $V = ab^2 = (Dt + c_1)^{\frac{3}{s}}$. (22)

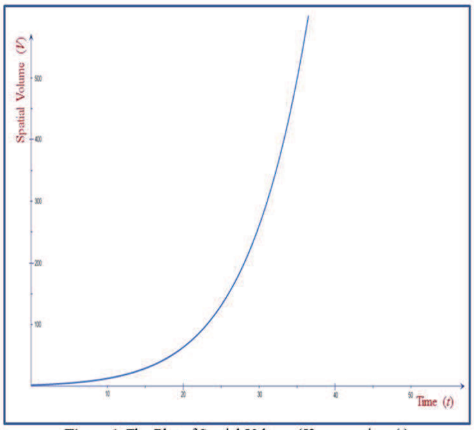

Figure 1. The Plot of Spatial Volume (V) versus time (t) with $D = 1$, $s = 0.25$ $c_1 = 0.1$

Hubble parameter, $H = \dfrac{k_1}{(Dt + c_1)}$. (23)

Expansion scalar, $\theta = 3H = \dfrac{3k_1}{(Dt + c_1)}$. (24)

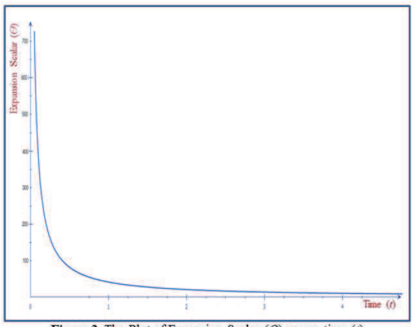

Figure 2. The Plot of Expansion Scalar (Θ) versus time (t) with $D = 1$, $k_1 = 25$ $c_1 = 0.1$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2 D^2}{k_1^2 (m+2)^2 s^2} \qquad (25)$$

Shear scalar, $\sigma^2 = \dfrac{3(m-1)^2 D^2}{(m+2)^2 s^2 (Dt + c_1)^2}$. (26)

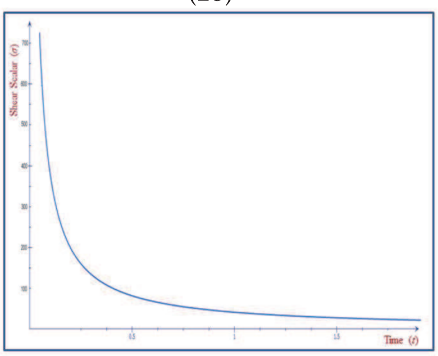

Figure 3. The Plot of Shear Scalar (σ) versus time (t) with $m = 3$, $D = 1$, $s = 0.25$ $c_1 = 0.1$

Also, $\dfrac{\sigma}{\theta} = \dfrac{D(m-1)}{\sqrt{3}\, k_1 s (m+2)} \neq 0$. (27)

The energy density,

$$\rho = \frac{9D^2(2m+1)}{(m+2)^2\ s^2 (Dt+c_1)^2} + (Dt+c_1)^{\frac{-6}{(m+2)s}} - \frac{1}{4}(Dt+c_1)^{\frac{6(m-2)}{(m+2)s}} . \quad (28)$$

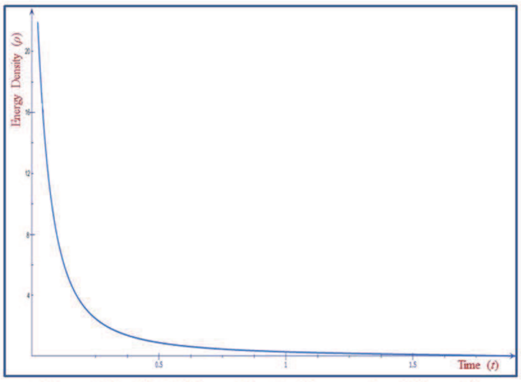

Figure 4. The Plot of Energy Density (ρ) versus time (t) for q < 0 with $m = 3$, $D = 1$, $s = 0.25$ $c_1 = 0.1$

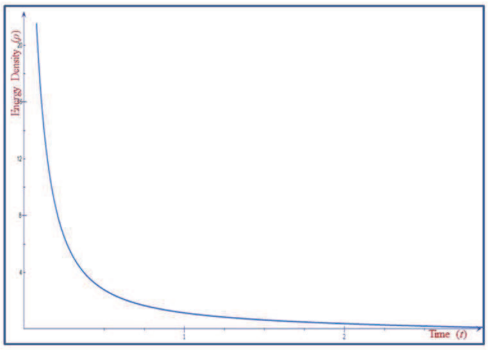

Figure 5. The Plot of Energy Density (ρ) versus time (t) for q > 0 with $m = 3$, $D = 1$, $s = 2$, $c_1 = 0.1$

EoS parameter,

$$\omega = -\frac{\left\{ \dfrac{3D^2(9-2ms+4s)}{(m+2)^2\ s^2 (Dt+c_1)^2} + (Dt+c_1)^{\frac{-6}{(m+2)s}} - \dfrac{3}{4}(Dt+c_1)^{\frac{6(m-2)}{(m+2)s}} \right\}}{\left\{ \dfrac{9D^2(2m+1)}{(m+2)^2\ s^2 (Dt+c_1)^2} + (Dt+c_1)^{\frac{-6}{(m+2)s}} - \dfrac{1}{4}(Dt+c_1)^{\frac{6(m-2)}{(m+2)s}} \right\}} . \quad (29)$$

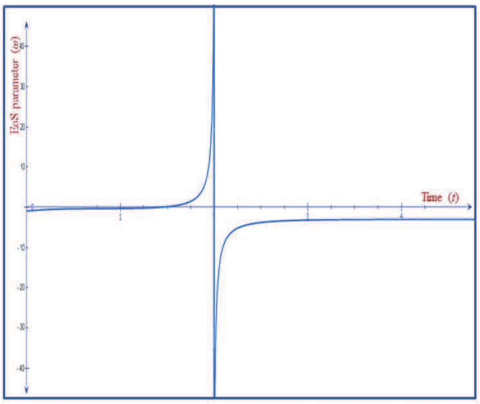

Figure 6. The Plot of EoS parameter (ω) versus time (t) for q < 0 with $m = 3$, $D = 1$, $s = 0.25$, $c_1 = 0.1$

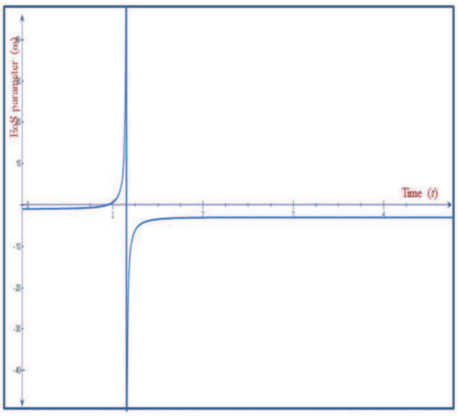

Figure 7. The Plot of EoS parameter (ω) versus time (t) for q > 0 with $m = 3$, $D = 1$, $s = 2$, $c_1 = 0.1$

Skewness parameter,

$$\gamma = -\frac{\left\{ \frac{3D^2(m^2+m-2)(3-s)}{(m+2)^2\ s^2(Dt+c_1)^2} - (Dt+c_1)^{\frac{-6}{(m+2)s}} + (Dt+c_1)^{\frac{6(m-2)}{(m+2)s}} \right\}}{\left\{ \frac{9D^2(2m+1)}{(m+2)^2\ s^2(Dt+c_1)^2} + (Dt+c_1)^{\frac{-6}{(m+2)s}} - \frac{1}{4}(Dt+c_1)^{\frac{6(m-2)}{(m+2)s}} \right\}} \quad (30)$$

In absence of any curvature, matter energy density (Ω_m) and dark energy density (Ω_Λ) are related by the equation

$$\Omega_m + \Omega_\Lambda = 1, \quad (31)$$

where $\Omega_m = \frac{\rho}{3H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$.

Thus, (31) reduce to

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1 \quad (32)$$

Using (23) and (28), therein (32) lead to

$$\Lambda = \frac{3k_1^2}{(Dt+c_1)^2} - \frac{9D^2(2m+1)}{(m+2)^2\ s^2(Dt+c_1)^2} - (Dt+c_1)^{\frac{-6}{(m+2)s}} + \frac{1}{4}(Dt+c_1)^{\frac{6(m-2)}{(m+2)s}} \quad (33)$$

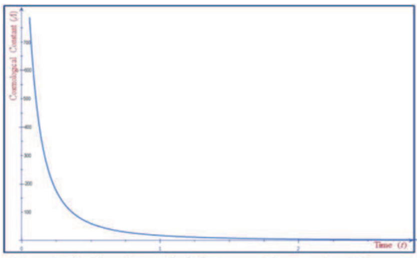

Figure 8. The Plot of Cosmological Constant (Λ) versus time (t) for q < 0 with $m = 3$, $D = 1$, $s = 0.25$, $c_1 = 0.1$, $k_1 = 25$

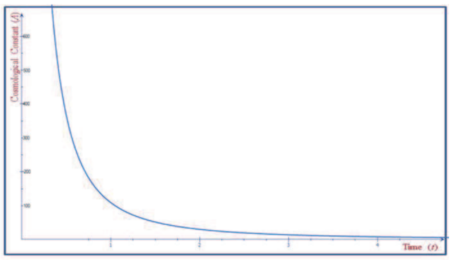

Figure 9. The Plot of Cosmological Constant (Λ) versus time (t) for q > 0 with $m = 3$, $D = 1$, $s = 2$, $c_1 = 0.1$, $k_1 = 25$

Physical behavior of the model: It is observed that, a spatial volume is zero at $t = t_0$ where $t_0 = -\frac{c_1}{D}$. At this epoch the energy density is infinite, both the scale factors a and b vanishes. Therefore the model has singularity at $t_0 = -\frac{c_1}{D}$. The expansion scalar (θ) and shear scalar (σ) are infinite at $t_0 = -\frac{c_1}{D}$ showing that the universe starts evolving with zero volume at $t = t_0$ and expands with time t. Further the values of θ σ tend to infinity for large values of t ($t \to \infty$) showing that the universe is expanding with increase of time and the rate of expansion decreases with increase of time. Also, $\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} \neq 0$ (= constant) indicating that the model does not approach isotropy for large time. The energy density (ρ) also tends to zero as $t \to \infty$. Thus the present model gives essentially an empty space for large time.

In fig. 6 and fig. 7, the equation of state (EoS) parameter ω versus cosmic time t of the evolution of the universe in two models ($i.e.\ q < 0\ \&\ q > 0$) have represented with appropriate choice of constants of integration and other physical parameters. It is observed that in early stage of evolution of the universe, the EoS parameter ω was positive *i.e.* the universe was matter dominated and late time it is evolving with negative value *i.e.* at the present time indicating that the dark energy dominate phase of the universe in both accelerating and decelerating models.

The plot of cosmological constant (Λ) versus time (t) in accelerating ($q < 0$) and decelerating ($q > 0$) phase of the universe have represented in fig. 8 and fig. 9. It is seen that, the cosmological term Λ is initially infinite. It is decreasing function of time and approaches to zero at late time which is supported by results of supernovae observations recently obtained by High-z supernovae Team and supernovae cosmological project [1-3].

3.2 Case (ii): When $s = 0$ $(q = -1)$:

Equations (8), (9) and (17) lead to

$$a = e^{\frac{3mk_1 t}{(m+2)}}, \quad (34)$$

$$b = e^{\frac{3k_1 t}{(m+2)}}. \quad (35)$$

Thus the Bianchi type-IX cosmological model can be written as

$$ds^2 = -dt^2 + e^{\frac{6mk_1t}{(m+2)}}dx^2 + e^{\frac{6k_1t}{(m+2)}}dy^2 + \left(e^{\frac{6k_1t}{(m+2)}}\sin^2 y + e^{\frac{6mk_1t}{(m+2)}}\cos^2 y\right)dz^2 - 2e^{\frac{6mk_1t}{(m+2)}}\cos y\,dxdz \quad (36)$$

The physical quantities that are important in cosmology are spatial volume V , Hubble parameter H , expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2 which have the following expressions for the model (36):

Spatial volume, $V = ab^2 = e^{k_1 t}$. (37)

Hubble parameter, $H = k_1$. (38)

Expansion scalar, $\theta = 3H = 3k_1$. (39)

Mean Anisotropy Parameter, $A_m = \frac{2(m-1)^2}{(m+2)^2}$ (40)

Shear scalar, $\sigma^2 = \frac{3k_1^2(m-1)^2}{(m+2)^2}$. (41)

Also, $\frac{\sigma}{\theta} = \frac{(m-1)}{\sqrt{3}(m+2)} \neq 0$. (42)

The energy density,

$$\rho = \frac{9k_1^2(2m+1)}{(m+2)^2} + e^{\frac{-6k_1t}{(m+2)}} - \frac{1}{4}e^{\frac{6k_1(m-2)t}{(m+2)}} \quad (43)$$

EoS parameter,

$$\omega = -\frac{\frac{27k_1^2}{(m+2)^2} + e^{\frac{-6k_1t}{(m+2)}} - \frac{3}{4}e^{\frac{6k_1(m-2)t}{(m+2)}}}{\frac{9k_1^2(2m+1)}{(m+2)^2} + e^{\frac{-6k_1t}{(m+2)}} - \frac{1}{4}e^{\frac{6k_1(m-2)t}{(m+2)}}} . \quad (44)$$

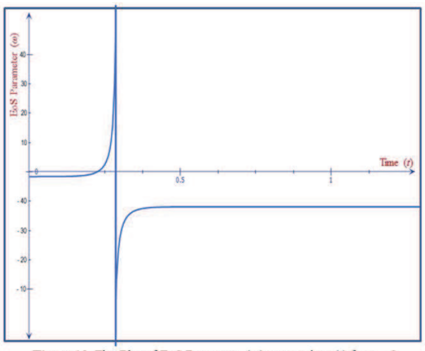

Figure 10. The Plot of EoS Parameter (ω) versus time (t) for s = 0 with m = 3, k_1 = 25

Skewness parameter,

$$\delta = -\frac{\frac{9k_1^2(2m^2+m-2)}{(m+2)^2} - e^{\frac{-6k_1t}{(m+2)}} + e^{\frac{6k_1(m-2)t}{(m+2)}}}{\frac{9k_1^2(2m+1)}{(m+2)^2} + e^{\frac{-6k_1t}{(m+2)}} - \frac{1}{4}e^{\frac{6k_1(m-2)t}{(m+2)}}} . \quad (45)$$

Using (38) and (43), therein (32) lead to

$$\Lambda = 3k_1^2 - \frac{9k_1^2(2m+1)}{(m+2)^2}e^{\frac{-6k_1t}{(m+2)}} + \frac{1}{4}e^{\frac{6k_1(m-2)t}{(m+2)}} . \quad (46)$$

Physical behavior of the model:_ The spatial volume increases with increase in time. Expansion scalar θ and shear scalar σ are constants showing that the rate of expansion remains constant with increase in time. Also $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ (= constant) indicating that the model does not approach isotropy for large time. In fig. 10 EoS parameter (ω) versus time (t) is represented with appropriate choice of constants of integration and other physical parameters. It is observed that in early stages of evolution of the universe, the EoS parameter was positive *i.e.* the universe was matter dominated and at late time it is evolving with negative value indicating that the dark energy dominated phase of the universe.

For $s = 0$, we get $q = -1$; incidentally this value of deceleration parameter leads to $\frac{dH}{dt} = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore the universe exhibits the fastest possible rate of the expansion among the all possible values of s.

Conclusions: Bianchi type-IX anisotropic dark energy model has been investigated using the law of variation for the Hubbles parameter which yields a constant value of decelerating parameter stated by Berman which gives two type of cosmologies $s \neq 0$ and $s = 0$ respectively.

For the model $s \neq 0$ it has singularity for the finite time $t = t_0$ where $t_0 = -\frac{c_1}{D}$. At this epoch the model starts evolving with zero volume and expands for large time where the rate of expansion decreases with increase of time. Since, $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ (= constant) indicating that the model does not approach isotropy for large time. The present model gives essentially an empty space for large time.

For the model $s = 0$ there is no initial and finite time singularity. The model is expanding at a constant rate with increase of time. Further the model does not approach isotropy for large time.It is observed that, in both cases, EoS parameter ω is variable function of time which has been supported by recent observations [14,15] and also analogous with the model investigated by Yadav and Yadav [36].

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