STAR – ACYCLIC COLORING OF GENERALIZED PETERSEN GRAPHS P(n , 1)

S. SUDHA,R. RAMACHANDRAN

Abstract: A star coloring of an undirected graph G is a proper vertex coloring of the vertices of G such that no two adjacent vertices receive the same color and no path of length 3 is bicolored. The star chromatic number χ_s(G) is the minimum number of colors needed for star coloring. The acyclic chromatic number $\chi_a(G)$ is the minimum number of colors needed for acyclic coloring G. In this paper, we have defined star-acyclic coloring of graphs and discussed the star-acyclic coloring of the generalized Petersen graphs $P(n,1), n \geq 4$.

Keywords: Vertex coloring, star coloring and acyclic coloring.

Introduction: All our graphs in this paper are simple, finite and undirected. A proper coloring of a graph G is a labeling of the vertices of G such that no two neighbours in G are assigned the same label. The minimum number of colors needed for coloring a graph G is called its chromatic number which is denoted by $\chi(G)$. K.Thilagavathi and P.Shanas Babu [1] has given the acyclic coloring of star graph families. We have considered both star coloring and acyclic coloring together and defined the star-acyclic coloring of graphs in this paper. We have found the general pattern of star-acyclic coloring of generalized Petersen graphs $P(n,1), n \geq 4$ and obtained the result that the star acyclic chromatic number is 4 for all $P(n,1), n \geq 4$.

Definition 1. *A vertex coloring (or simply coloring) of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color. The chromatic number of vertex coloring G is denoted by $\chi(G)$ and is the minimum numbers of colors required to vertex coloring.*

Definition 2. *An acyclic coloring of a graph G is a proper coloring of G such that there is no bicolored cycle in G(that is union of any two color classes induces a forest).The acyclic chromatic number of G is denoted by $\chi_a(G)$ and is the minimum number of colors required for acyclic coloring G.*

Definition 3. *A star coloring of a graph G is a proper coloring of G such that no path of length 3 in G is bicolored (that is the union of any two color classes induces a star forest). The star chromatic number of G is denoted by $\chi_s(G)$ and is the minimum number of colors required to star color G.*

Definition 4. *If a graph G admits both acyclic coloring and star coloring, then we say that the graph is star-acyclic coloring and its minimum chromatic number for star-acyclic coloring is denoted by $\chi_{sa}(G)$.*

In this paper, we have discussed the star-acyclic coloring of the generalized Petersen graphs $P(n,1)$ for even n and $n \geq 4$.

Theorem 1. *The generalized Petersen graphs P(n,1), n even and n ≥ 4 admit star-acyclic coloring and its star-acyclic chromatic number is four for all even n and n ≥ 4.*

Proof. Consider the generalized Petersen graph P(n,1) and n even, n ≥ 4. Let $v_1, v_2, \ldots\ldots, v_n$ be its inner vertices and $u_1, u_2, \ldots\ldots, u_n$ be its corresponding outer vertices as shown in fig 1. Let the colors be denoted by 1,2,3,.... . The graph P(n,1) consists of 2n vertices and 3n edges.

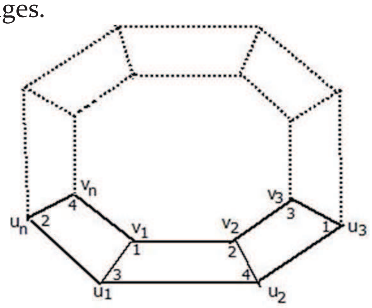

FIGURE 1

There are four cases.

Case 1: Let $n \equiv 0(mod\ 4)$

We define

$$f(v_i) = \begin{cases} v_i = 1,\ i \equiv 1(mod\ 4) \\ v_i = 2,\ i \equiv 2(mod\ 4) \\ v_i = 3,\ i \equiv 3(mod\ 4) \\ v_i = 4,\ i \equiv 0(mod\ 4) \end{cases}$$

$$f(u_i) = \begin{cases} u_i = 1,\ i \equiv 3(mod\ 4) \\ u_i = 2,\ i \equiv 4(mod\ 4) \\ u_i = 3,\ i \equiv 1(mod\ 4) \\ u_i = 4,\ i \equiv 2(mod\ 4) \end{cases}$$

Case 2: Let $n \equiv 0(mod\ 6)$.

We define

$$f(v_i) = \begin{cases} v_i = 1, 1 \leq i \leq n,\ \forall\ odd\ i \\ v_i = 2,\ i \equiv 2(mod\ 6) \\ v_i = 3,\ i \equiv 4(mod\ 6) \\ v_i = 4,\ i \equiv 0(mod\ 6) \end{cases}$$

$$f(u_i) = \begin{cases} u_i = 1,\ 1 \leq i \leq n,\ \forall\ even\ i \\ u_i = 2,\ i \equiv 5(mod\ 6)\ 1 \leq i \leq n \\ u_i = 3,\ i \equiv 1(mod\ 6)\ 1 \leq i \leq n \\ u_i = 4,\ i \equiv 3(mod\ 6) \\ \qquad 1 \leq i \leq n-1 \end{cases}$$

Case 3: Let $n = 10 + 12j, \quad j = 0,1,2,\ldots$
We define

$$f(v_i) = \begin{cases} v_i = 1, & \forall\, odd\ i \\ v_i = 2, & i \equiv 2(mod\ 6) \\ v_i = 3, & i \equiv 4(mod\ 6) \\ v_i = 4, & i \equiv 0(mod\ 6) \end{cases}$$

$$f(u_i) = \begin{cases} u_i = 1,\ 4 \le i \le n-2,\ \forall\ even\ i \\ u_i = 2,\ i \equiv 5(mod\ 6) \\ u_i = 3,\ i \equiv 1(mod\ 6) \\ \qquad 1 \le i \le n-3 \\ u_i = 4,\ i \equiv 3(mod\ 6) \end{cases}$$

$$u_1 = 4, u_2 = 3, u_{10} = 2$$

Case 4: Let $n = 14 + 12j, \quad j = 0,1,2,\ldots.$
We define

$$f(v_i) = \begin{cases} v_i = 1,\ \forall\ odd\ i \\ v_i = 2,\ i \equiv 2(mod\ 6) \\ \qquad 2 \le i \le n-1 \\ v_i = 3,\ i \equiv 4(mod\ 6)\ 4 \le i \le n \\ v_i = 4,\ i \equiv 0(mod\ 4)\ 1 \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} u_i = 1,\ 4 \le i \le n-3,\ \forall\ even\ i \\ u_i = 2,\ i \equiv 5(mod\ 6)\ 1 \le i \le n \\ u_i = 3,\ i \equiv 1(mod\ 6) \\ \qquad 2 \le i \le n-3 \\ u_i = 4,\ i \equiv 3(mod\ 6) \\ \qquad 1 \le i \le n-3 \end{cases}$$

and $u_1 = 4,\ u_2 = 3,\ u_n = 1,$
$u_{n-1} = 2,\ u_{n-2} = 3$

With this type of coloring, the graph P(n,1) is star-acyclic colored and the colors 1,2,3,4 are needed for star-acyclic coloring. Hence the star-acyclic chromatic number of the Petersen graph $\chi_{sa}(P(n;\ 1))$ is 4 for all the cases.

Remarks.

1. *If* $n \equiv 0(mod\ 12)$, *then any one of the first two cases may be taken.*
2. *The consecutive even numbers are all covered for n by these 4 cases.*

Illustration 1:

Let $n \equiv 0(mod\ 4)$.
Consider the Petersen graph P(8,1).

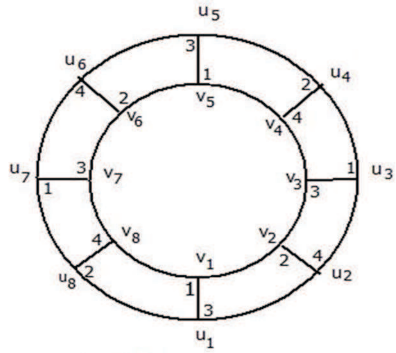

FIGURE 2

By case 1, The inner vertices $v_1, v_2, \ldots.,$ and v_8 are colored with 1,2,3,4,1,2,3 and 4 respectively; the outer vertices $u_1, u_2, \ldots.,$ and u_8 are colored with 3,4,1,2,3,4,1, and 2 respectively. With this choice of coloring the vertices, the condition for star- acyclic coloring is satisfied for $P(8,1)$. The star-acyclic chromatic number of $P(8,1)$ is 4.

Illustration 2:

Let $n \equiv 0(mod\ 6)$.
Consider the Petersen graph $P(12,1)$.

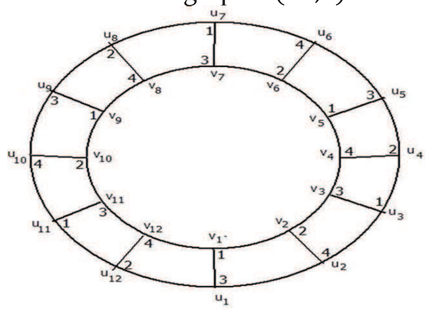
FIGURE 3

By case 2, The inner vertices $v_1, v_2, \ldots.,$ and v_{12} are colored with 1,2,3,4,1,2,3,4,1,2,3, and 4 respectively ; the outer vertices $u_1, u_2, \ldots.,$ and u_{12} are colored with 3,4,1,2,3,4,1,2,3,4,1, and 2 respectively. With this choice of coloring the vertices, the condition for star-acyclic coloring is satisfied for $P(12,1)$. The star-acyclic chromatic number of $P(12,1)$ is 4.

Illustration 3:

Let $n = 10 + 12j,\ j = 0,1,2,\ldots$
Consider the Petersen graph $P(10,1)$.

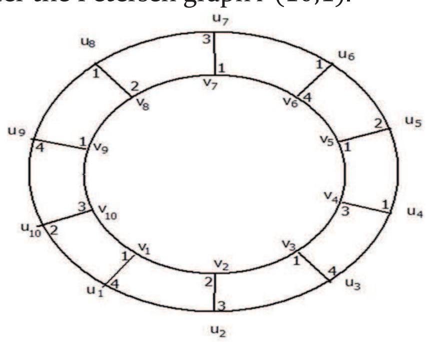
FIGURE 4

By case 3, The inner vertices $v_1, v_2, \ldots\ldots,$ and v_{10} are colored with 1,2,1,3,1,4,1,2,1, and 3 respectively; the outer vertices $u_1, u_2, \ldots\ldots,$ and u_{10} are colored with 4,3,4,1,2,1,3,1,4, and 2 respectively. With this choice of coloring the vertices, the condition for star- acyclic coloring is satisfied for $P(10,1)$. The star-acyclic chromatic number of $P(10,1)$ is 4.

Illustration 4:

Let $n = 14 + 12j$, $j = 0,1,2, \ldots.$

Consider the Petersen graph $P(14,1)$.

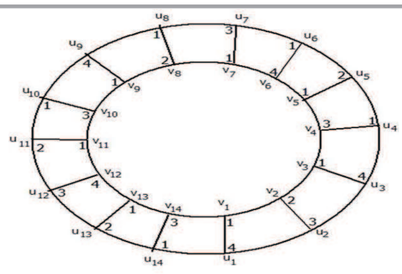

FIGURE 5

By case 4, The inner vertices $v_1, v_2, \ldots\ldots,$ and v_{14} are colored with 1,2,1,3,1,4,1,2,1,3,1,4,1, and 3 respectively ; the outer vertices $u_1, u_2, \ldots\ldots,$and u_{14} are colored with 4,3,4,1,2,1,3,1,4,1,2,3,2, and 1 respectively. With this choice of coloring the vertices, the condition for star- acyclic coloring is satisfied for $P(14,1)$. The star-acyclic chromatic number of $P(14,1)$ is 4.

References:

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