SOFT COMPLEX FUZZY SET

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Abstract: The objective of this paper is to investigate the innovative concept of soft complex fuzzy set. The novelty of the complex fuzzy set lies in the range of values its membership function may attain. In contrast to a traditional fuzzy membership function, this range is not limited to [0,1], but extended to the unit circle in the complex plane. Thus, the complex fuzzy set provides a mathematical framework for describing membership in a set in terms of a complex number. Consequently, a major part of this work is dedicated to a discussion of the intuitive interpretation of parameterized sets in soft complex fuzzy set, Examples of possible applications, which demonstrate the soft complex fuzzy set. Basic set theoretic operations on soft complex fuzzy sets, such as soft complex fuzzy union, soft complex fuzzy intersection and soft complex fuzzy complement are discussed at length.

Keywords: complex fuzzy set, operations of soft complex fuzzy set, soft complex fuzzy set, soft fuzzy set.

Introduction: Soft set theory is a generalization of fuzzy set theory, which was proposed by Molodtsov [2] in 1999 to deal with uncertainty in a non-parametric manner. One of the most important steps for the theory of Soft Sets was to define mappings on soft sets, this was achieved in 2009 by mathematician Athar Kharal, though the results were published in 2011. Soft sets have also been applied to the problem of medical diagnosis for use in medical expert systems. Fuzzy soft sets have also been introduced in [1]. Mappings on fuzzy soft sets were defined and studied in the ground breaking work of Kharal and Ahmad.

Complex fuzzy set (CFS) [3]-[4] is a new development in the theory of fuzzy systems [7]. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state.

The paper is organized as follows: Section 2 reviews the notions of soft sets, fuzzy sets, soft fuzzy set, complex fuzzy set and relevant definitions used in the proposed work. In section 3 we introduce the concept of soft complex fuzzy sets and define some operations such as union, intersection, complement all explained with examples. In section 4 we propose an application of soft complex fuzzy set. Finally we conclude the paper with a summary in section 5.

Preliminaries:

Definition 2.1: [9] A pair (F, E) is called a soft set [2] (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set F(ε), ε∈E, from this family may be considered as the set of ε- elements of the soft set (F,E), or as the set of ε – approximate elements of the soft set.

Example: 2.2: Let U={h_1,h_2,h_3,h_4} be the set of four houses under consideration and E={p_1(costly),p_2(Beautiful),p_3(Modern Technology),p_4(luxurious),p_5(facility)}be the set of parameters and A={p_1,p_2,p_4}$\subseteq$E. Then (F,A) = {F(p_1)={h_1,h_3}, F(p_2)={h_1,h_2,h_4},F(p_3={h_2}} is the soft set representing the 'attractiveness of the house' which a person is going to buy.

Definition 2.3: [6,8]

A pair (F,A) is called a fuzzy soft set over U where F: A→P(U) is a mapping from A into P(U), where P(U) is the fuzzy power set.

Example: 2.4: Let U={h_1,h_2,h_3,h_4} be the set of four houses under consideration and E={p_1(costly),p_2(Beautiful),p_3(Modern Technology),p_4(luxurious),p_5(facility)}be the set of parameters and A={p_1,p_2,p_4}$\subseteq$E.Then (F,A)={F(p_1)={0.4/h_1,0.2/h_2,0.6/h_3,0.7/h_4}, F(p_2)={0.6/h_1,0.2/h_2,0.6/h_3,0.7/h_4},F(p_4)={0.4/h_1,0.3/h_2, 0.7/h_3, 0.3/h_4}} is the fuzzy soft set representing the 'attractiveness of the house' which a person is going to buy.

Definition 2.5: Ramot et al. [3] recently proposed an important extension of these ideas, the *Complex Fuzzy Sets,* where the membership function μ instead of being a real valued function with the range [0,1] is replaced by a complex-valued function of the form

$$\mu_s(x) = r_s(x)e^{j\varpi_s(x)}; \quad \mathrm{j} = \sqrt{-1}$$

where $r_S(x)$ and $\omega_S(x)$ are both real valued giving the range as the unit circle. However, this concept is different from fuzzy complex number introduced and discussed by Buckley and Zhang. Essentially as explained in [3] this still retains the characterization of the uncertainty through the amplitude of the grade of membership having a value in the range of [0, 1] whilst adding the membership phase captured by fuzzy sets. As explained in Ramot et al [3], the key feature of complex fuzzy sets is the presence of phase and its membership.

Example: 2.6: Solar cycle [3]

Every eleven years the sun undergoes a period of activity called the "solar maximum," followed by a period of quiet called the "solar minimum." During the solar maximum, there are many sunspots, solar flares, and coronal mass ejections, all of which can affect communications and weather on Earth. During the solar minimum, there are few sunspots. One way solar activity may be tracked is by observing sunspots. Sunspots are relatively cool areas that appear as dark blemishes on the face of the sun, and are sites where solar flares are observed to occur[3] Fig.2.1.shows the monthly average of the number of sunspots observed since 1749.

Let S denote the ordinary fuzzy set *high solar activity*. Assume the grade of membership of a particular month in S is derived from the average number of sunspots observed during this month. Clearly, an average sunspot number of 200 is associated with a large grade of membership, while an average sunspot number of 2 is associated with a small grade of membership. Consider a month with an average sunspot number of 50.Judging by the data in Fig. 2.1, it seems reasonable to assign this month a grade of membership of 0.25. However, the interpretation of an average sunspot number of 50 can vary considerably if the solar *cycle* is also considered. For example, in 1805 the sunspot number of 50 was the solar maximum, while in 1956 it was barely a quarter of the way "up" the solar cycle. Thus, for applications in which solar activity is a significant parameter, the representation of a sunspot number of 50 by the single grade of membership 0.25 may be insufficient. This may be particularly relevant to applications that require long-term planning, e.g., a mission to space expected to last several years.

There are several methods for incorporating the additional required information into the fuzzy representation of solar activity.

One method is to define a second fuzzy set, *close to solar maximum*, so that the solar activity in any particular month is characterized by its grade of membership in two fuzzy sets. Alternatively, it is possible to determine the grade of membership of a particular month in S by considering not only its absolute sunspot number, but also its phase in the solar cycle. Thus, a month that is a solar maximum would receive a large grade of membership in S even if its average sunspot number were only 50. However, a more elegant and comprehensive manner of representing all the necessary information pertaining to the solar activity in any particular month, is to define S as a **complex fuzzy set**. When S is a complex fuzzy set, each month is associated with a complex-valued grade of membership consisting of an amplitude term and a phase term. The role of the amplitude term is simple: it signifies the degree to which a particular month x is a member of . The amplitude term is derived from the sunspot number in the same manner the real-valued grade of membership was determined when S was considered an ordinary fuzzy set. The phase term

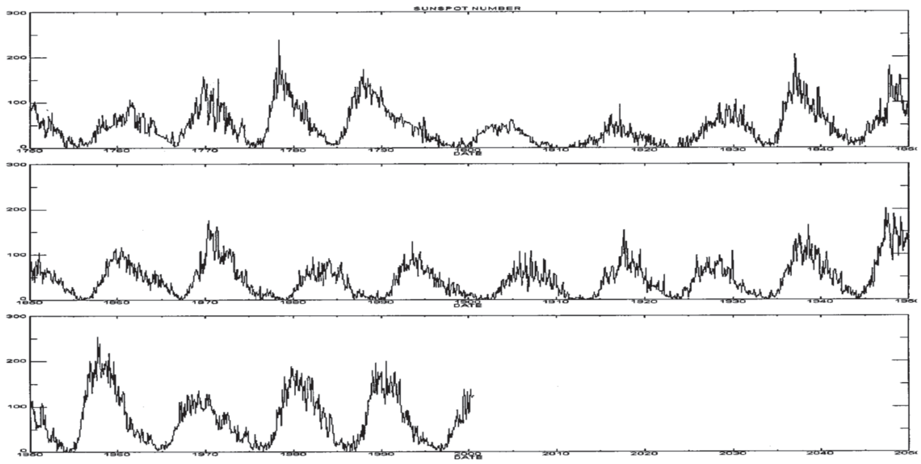

Fig.2.1: Monthly average of sunspots observed since 1749

contains information regarding the position of the month under consideration in the solar cycle. Thus, if the month in question, , is the solar minimum, it is attributed a membership phase of zero. If however, is the solar maximum, it is given a membership phase of π. Accordingly, any position between the solar minimum and the solar maximum is associated with a membership phase in the range$(0, \pi)$ if solar activity is on the rise, or a value in the range$(\pi, 2\pi)$ if solar activity is decreasing. The complex fuzzy representation of S is succinct, incorporating all required information in a single grade of membership. It is also more complete than the two traditional fuzzy representations previously suggested.

- The complex fuzzy representation conveys the precise Phase of in the solar cycle, rather than just its proximity to the solar maximum. The fuzzy set *close to solar maximum* does not consider the phase of x relative to the maximum, i.e., whether the maximum is past or whether it is approaching.
- The traditional fuzzy representation that uses a single fuzzy set to represent both the absolute sunspot number and its phase in the solar cycle is often ambiguous.

  For example, a grade of membership of 0.9 may imply a sunspot number of 150, a sunspot number of 50 that is close to the solar maximum, or several other combinations of sunspot number and "solar phase." In contrast, using the complex representation it is possible to describe both the absolute sunspot number and its phase in the solar cycle without risk of ambiguity.

3. Soft Complex Fuzzy Set

Definition 3.1: Let U be an initial set and E be set of parameters. C(U) denotes complex fuzzy power set of U, and let $A \subset E$. A pair (C, A) is called a **soft complex fuzzy set** over U, where C is a mapping given by C: A→C (U).

Example 3.2: [10]

LetU={H_1(India),H_2(Australia),H_3(UK),H_4(USA)} be an initial set, considerE={e_1(Inflationrate),e_2(populationgrowth),e_3(Unemployment rate),e_4(share market index)} be an country's growth parameters set and $A \subseteq E$,A={e_1,e_3}, then **soft complex fuzzy set** (C,A), where C: A→C(U), may be represented as,

(C,A)=

$$\left\{\begin{array}{l} F(e_1)=\left\{\dfrac{0.4e^{j2.7\pi}}{H_1},\dfrac{0.8e^{j3.8\pi}}{H_2},\dfrac{0.8e^{j3.7\pi}}{H_3},\dfrac{1.0e^{j2.75\pi}}{H_4}\right\}, \\ F(e_3)=\left\{\dfrac{0.6e^{j0.7\pi}}{H_1},\dfrac{0.9e^{j0.9\pi}}{H_2},\dfrac{0.7e^{j0.95\pi}}{H_3},\dfrac{0.75^{j0.95\pi}}{H_4}\right\} \end{array}\right\}$$

Definition 3.3: Let U be a universe and E a set of attributes. Then the pair (U,E) denotes the collection of all soft complex fuzzy sets on U with attributes from E and is called a soft complex fuzzy class.

Definition 3.4: The complement of a soft complex fuzzy set (C,A) is denoted by $(C,A)^c$ and is defined by$(C,A)^c$ = $(C^c, \neg A)$, where C^c: $\neg A$→C(U) is a mapping given by $C^c(x)=(C(\neg x))^c$ for all $x \in \neg A$ and C(U) is complex fuzzy power set.

Definition 3.5: [5] Union of two soft complex fuzzy sets (C_1, A) and (C_2,B) in a soft class (U,E) is a soft complex fuzzy set ($C,A\bigcup B$) where C_1:A→C_1(U), C_2:A→C_2(U) , and for all $\varepsilon \in A\bigcup B$,

$$C(\varepsilon) = \begin{cases} C_1(\varepsilon), & if\ \varepsilon \in \mathrm{A-B} \\ C_2(\varepsilon), & if\ \varepsilon \in \mathrm{A-B} \\ C_1 \cup C_2 = [r_A(\varepsilon) \oplus r_B(\varepsilon)]e^{i\omega_{AUB}(\varepsilon)}, & \\ \quad \text{if } \varepsilon \in \mathrm{A} \bigcap \mathrm{B} & \end{cases}$$

Here C_1(U) and C_2(U) is complex fuzzy power set $[r_A(\varepsilon) \oplus r_B(\varepsilon)]$ defined in [3], ω_{AUB} is defined as follows.

a) (Sum) $\omega_{AUB} = \omega_A + \omega_B$

b) (Max) ω_{AUB} = max (ω_A, ω_B)

c) (Min) ω_{AUB} =min (ω_A, ω_B)

d) (Winner takes all)

$$\omega_{AUB} = \begin{cases} \omega_A, if \ldots r_A > r_B \\ \omega_B, if \,..r_B \geq r_A \end{cases}$$

Example 3.6:

Consider U={H_1,H_2,H_3,H_4} ; E={e_1,e_2,e_3,e_4}; A={e_1,e_2}$\subseteq$ E, B={e_1,e_3}$\subseteq$E and C_1:A→C_1(U), C_2:A→C_2(U) ; Here C_1(U) and C_2(U) is complex fuzzy power set, then

(C_1, A)=

$$\left\{\begin{array}{l} C(e_1)=\left\{\dfrac{0.6e^{j1.2\pi}}{H_1},\dfrac{0.7e^{j0.2\pi}}{H_2},\dfrac{0.8e^{j0.4\pi}}{H_3},\dfrac{0.5e^{j0.6\pi}}{H_4}\right\} \\ ,C(e_2)=\left\{\dfrac{0.4e^{j1.2\pi}}{H_1},\dfrac{0.5e^{j1.6\pi}}{H_2},\dfrac{0.7e^{j0.2\pi}}{H_3},\dfrac{0.9e^{j0.2\pi}}{H_4}\right\} \end{array}\right\}$$

(C_2, B)=

$$\left\{\begin{array}{l} C(e_1)=\left\{\dfrac{0.6e^{j1.2\pi}}{H_1},\dfrac{0.8e^{j2\pi}}{H_2},\dfrac{1.0e^{j1.6\pi}}{H_3},\dfrac{0.3e^{j0.6\pi}}{H_4}\right\}, \\ C(e_3)=\left\{\dfrac{0.7e^{j0.4\pi}}{H_1},\dfrac{0.5e^{j0.2\pi}}{H_2},\dfrac{0.3e^{j0.4\pi}}{H_3},\dfrac{0.2e^{j0.7\pi}}{H_4}\right\} \end{array}\right\}$$

(C_1, A)$\bigcup$ (C_2, B)=

$$\left\{\begin{array}{l} C(e_1)=\left\{\dfrac{0.6e^{j1.2\pi}}{H_1},\dfrac{0.8e^{j0.2\pi}}{H_2},\dfrac{1.0e^{j1.6\pi}}{H_3},\dfrac{0.5e^{j0.6\pi}}{H_4}\right\}, \\ C(e_2)=\left\{\dfrac{0.4e^{j1.2\pi}}{H_1},\dfrac{0.5e^{j1.6\pi}}{H_2},\dfrac{0.7e^{j0.2\pi}}{H_3},\dfrac{0.9e^{j0.2\pi}}{H_4}\right\}, \\ C(e_3)=\left\{\dfrac{0.7e^{j0.4\pi}}{H_1},\dfrac{0.5e^{j0.2\pi}}{H_2},\dfrac{0.3e^{j0.4\pi}}{H_3},\dfrac{0.2e^{j0.7\pi}}{H_4}\right\} \end{array}\right\}$$

From,

$$\mu_{AUB}(x) = r_{A\cup B}(x)e^{j\arg_{AUB}(x)}$$
$$= \max(r_A(x), r_B(x))e^{j\max(\arg_A(x),\arg_B(x))} \tag{3.1}$$

Definition 3.7: [5]

Intersection of two soft complex fuzzy sets (C_1, A) and (C_2,B) in a soft class (U,E) is a soft complex fuzzy set (C,A∩B) where C_1:A→C_1(U), C_2:A→C_2(U) , and for all

ε ∈ A∩B,
C (ε) =

$$\begin{cases} C_1(\varepsilon), & if\ \varepsilon \in \text{A - B} \\ C_2(\varepsilon), & if\ \varepsilon \in \text{A - B} \\ C_1 \cap C_2 = [r_A(\varepsilon) * r_B(\varepsilon)] e^{i\omega_{A\cap B}(\varepsilon)}, & \text{if } \varepsilon \in \text{A} \bigcap \text{B} \end{cases}$$

$r_A(\varepsilon) * r_B(\varepsilon)$ defined in [3], $\omega_{A\cap B}$ is defined as follows.

a) (Sum) $\omega_{A\cap B} = \omega_A + \omega_B$

b) (Max) $\omega_{A\cap B}$ =max (ω_A, ω_B)

c) (Min) $\omega_{A\cap B}$ = min (ω_A, ω_B)

d) (Winner takes all)

$$\omega_{A\cap B} = \begin{cases} \omega_A, if \ldots r_A > r_B \\ \omega_B, if ..r_B \geq r_A \end{cases}$$

Here $C_1(U)$ and $C_2(U)$ is complex fuzzy power set

Example 3.8: Consider U={H_1,H_2,H_3,H_4}; E={e_1,e_2,e_3,e_4}; A={e_1,e_2}$\subseteq$E, B={e_1,e_3}$\subseteq$E and C_1:A→$C_1(U)$, C_2:A→$C_2(U)$; Here $C_1(U)$ and $C_2(U)$ is complex fuzzy power set, then

(C_1, A)=

$$\left\{ \begin{array}{l} C(e_1) = \left\{ \dfrac{0.6e^{j1.2\pi}}{H_1}, \dfrac{0.7e^{j0.2\pi}}{H_2}, \dfrac{0.8e^{j0.4\pi}}{H_3}, \dfrac{0.5e^{j0.6\pi}}{H_4} \right\}, \\ C(e_2) = \left\{ \dfrac{0.4e^{j1.2\pi}}{H_1}, \dfrac{0.5e^{j1.6\pi}}{H_2}, \dfrac{0.7e^{j0.2\pi}}{H_3}, \dfrac{0.9e^{j0.2\pi}}{H_4} \right\} \end{array} \right\}$$

(C_2, B)=

$$\left\{ \begin{array}{l} C(e_1) = \left\{ \dfrac{0.6e^{j1.2\pi}}{H_1}, \dfrac{0.8e^{j2\pi}}{H_2}, \dfrac{1.0e^{j1.6\pi}}{H_3}, \dfrac{0.3e^{j0.6\pi}}{H_4} \right\}, \\ C(e_3) = \left\{ \dfrac{0.7e^{j0.4\pi}}{H_1}, \dfrac{0.5e^{j0.2\pi}}{H_2}, \dfrac{0.3e^{j0.4\pi}}{H_3}, \dfrac{0.2e^{j0.7\pi}}{H_4} \right\} \end{array} \right\}$$

(C_1, A)∩ (C_2, B)=

$$\left\{ \begin{array}{l} C(e_1) = \left\{ \dfrac{0.6e^{j1.2\pi}}{H_1}, \dfrac{0.7e^{j0.2\pi}}{H_2}, \dfrac{0.8e^{j0.4\pi}}{H_3}, \dfrac{0.3e^{j0.6\pi}}{H_4} \right\}, \\ C(e_2) = \left\{ \dfrac{0.4e^{j1.2\pi}}{H_1}, \dfrac{0.5e^{j1.6\pi}}{H_2}, \dfrac{0.7e^{j0.2\pi}}{H_3}, \dfrac{0.9e^{j0.2\pi}}{H_4} \right\}, \\ C(e_3) = \left\{ \dfrac{0.7e^{j0.4\pi}}{H_1}, \dfrac{0.5e^{j0.2\pi}}{H_2}, \dfrac{0.3e^{j0.4\pi}}{H_3}, \dfrac{0.2e^{j0.7\pi}}{H_4} \right\} \end{array} \right\}$$

from

$$\mu_{A\cap B}(x) = r_{A\bigcap B}(x) e^{j \arg_{A\bigcap B}(x)}$$

$$= \min(r_A(x), r_B(x)) e^{j \min(\arg_A(x), \arg_B(x))} \qquad (3.2)$$

Applications of Soft Complex Fuzzy Set: It is much easier to describe a human behavior directly showing the set of strategies which a person may choose in a particular situation especially a periodic phenomenon. The situation may be more complicated in real world because of the fuzzy characters of the parameters [6, 8]. In fuzzy set, the complex fuzzy set is extended to a fuzzy one; the complex fuzzy membership is used to describe the parameter approximate elements of soft complex fuzzy set.

This theory also used in data mining applications and decision making problem. This theory used in study of smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability, Theory of measurement, etc.

Conclusion: This paper presented a new concept of soft complex fuzzy set, which is generalized from the innovative concept of a complex fuzzy set. A soft complex fuzzy set is a mapping from parameter to the crisp subset of universe. We crisply summarized the basic concepts of soft set theory with example and also explained the operations of soft complex fuzzy set, such as Union, Intersection and Complement with example. Finally, we enumerate some of its various applications in different field. In future, Soft complex fuzzy set extended to as an intuitionistic soft complex fuzzy set [6] for using various applications.

This paper should be considered an introduction to soft complex fuzzy sets. Indeed, much research of this novel concept is still needed to fully comprehend its properties and potential. The concepts presented in this paper are entirely general and are not limited to specific application. Thus in all, the soft complex fuzzy set seems to be promising new concept, paving the way to numerous possibilities for future research.

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