TWO-PHASE MAGNETOHYDRODYNAMIC FLOW AND HEAT TRANSFER IN A HORIZONTAL CHANNEL

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Abstract: The present investigation deals with the study of steady, mixed convective, laminar flow of incompressible, electrically conducting and heat absorbing two immiscible viscous fluids in a horizontal channel where the fluid in the region $-h < y < 0$ (region I) is saturated with porous medium and the region $0 < y < h$ (region II) is occupied by a clear viscous fluid. A uniform magnetic field is applied in the transverse direction, the fluids rise in the channel driven by uniform pressure, the heat transfer is influenced by thermal radiation. The equations are modeled using the fully developed flow conditions. An exact solution is obtained for the velocity and temperature distributions. The graphical results are presented and the physical aspects are discussed in detail to interpret the effect of various significant parameters of the problem. The effect of the skin-friction, rate of heat transfer coefficients at the channel walls is tabulated.

Keywords: MHD, mixed convection, heat absorption, thermal radiation, porous medium.

Introduction: The problem of MHD flows has wide range of applications in emerging fields due to an electro-magnetic field, are relevant to many practical applications in Geophysical and Astrophysical situations, the metallurgy industry, cooling of continuous strips and filaments drawn through a quiescent fluid [1-2]. The flow and heat transfer aspects of immiscible fluids is of special importance in the petroleum extraction and transport [3-4]. Convective heat transfer and fluid flow in a system simultaneously containing a fluid reservoir and a porous medium saturated with fluid is of great mathematical and physical interest. More specifically the existence of a fluid layer adjacent to a layer of fluid saturated porous medium is a common occurrence in both geophysical and engineering environments [5-6]. The hot walls and the working fluid are usually emitting the thermal radiation within the systems. The role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature and knowledge of radiative heat transfer becomes very important in nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles [7-8].

Motivated by the above mentioned investigations and applications, in this paper, we investigate the problem of MHD flow of two immiscible viscous fluids in a horizontal channel where the fluid in the region I is saturated with porous medium and the region II is occupied by a clear viscous fluid. An exact solution is obtained for the velocity and temperature distributions and the graphical results are discussed in detail.

Formulation of the problem: The fully developed, mixed convective, laminar flow of two immiscible viscous fluids in a horizontal channel where the fluid in the region $-h < y < 0$ (region I) is saturated with porous medium and the region $0 < y < h$ (region II) is occupied by a clear viscous fluid is shown in the Figure 1. A coordinate system is chosen such that the X-axis is taken horizontally and the Y-axis is perpendicular to it. The walls Y_1 and Y_2 are maintained constant temperatures T_{w1} and T_{w2} $\left(T_{w1} > T_{w2} \ \& \ T_{w2} = T_0\right)$ respectively. An external uniform magnetic field of strength B_0 is applied normal to the vertical walls, radiation and temperature dependent heat sink are taken into account. The transport properties of both fluids are assumed to be constant. It is worth to mention here that the two viscous fluids are immiscible and the constitutive equations for the fluids are different. Also, the viscosities, conductivities, densities $\rho_1 = \rho_0\left[1-\beta_1\left(T_1 - T_0\right)\right]$, $\rho_2 = \rho_0\left[1-\beta_2\left(T_2 - T_0\right)\right]$ and thermal expansions of both fluids are different. Since our model is general, one can choose any two different fluids which are immiscible. Under the assumptions stated above, we employ the Boussinesq approximation for the governing equations.

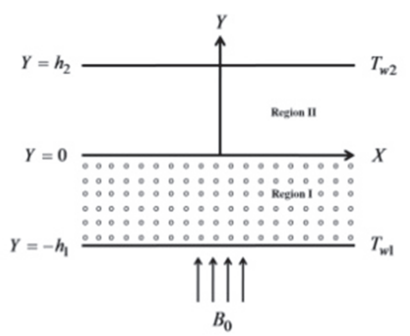

Fig 1: Flow geometry of the problem

It is assumed that the only non-zero components of the velocity q is the X-component $U_i\left(i=1,2\right)$. Thus, as a consequence of the mass balance equation, one can obtain

$$\frac{\partial U_i}{\partial X}=0 \qquad (1)$$

So that U_i depends only on Y. The momentum and energy balance equations are as follows

Region-I

$$\mu_1\frac{d^2U_1}{dY^2}-\rho_1\frac{dU_1}{dY}-\sigma_1B_0^2U_1-\frac{\mu_1}{K_0}U_1=\frac{dP}{dX} \qquad (2)$$

$$\frac{k_1}{\rho_1c_p}\frac{d^2T_1}{dY^2}-\frac{dT_1}{dY}-\frac{Q\left(T_1-T_0\right)}{\rho_1c_p}-\frac{1}{\rho_1c_p}\frac{dq_r}{dY}=0 \qquad (3)$$

Region-II

$$\mu_2\frac{d^2U_2}{dY^2}-\rho_2\frac{dU_2}{dY}-\sigma_2B_0^2U_2=\frac{dP}{dX} \qquad (4)$$

$$\frac{k_2}{\rho_2c_p}\frac{d^2T_2}{dY^2}-\frac{dT_2}{dY}-\frac{Q\left(T_2-T_0\right)}{\rho_2c_p}-\frac{1}{\rho_2c_p}\frac{dq_r}{dY}=0 \qquad (5)$$

The appropriate boundary and interface conditions on velocity and temperature are

$$U_1=0,\ T_1=T_{w1}\ \ at\ \ Y_1=-h_1 \qquad (6)$$

$$U_2=0,\ T_2=T_{w2}\ \ at\ \ Y_2=h_2 \qquad (7)$$

$$U_1=U_2,\ T_1=T_2\ \ at\ \ Y=0 \qquad (8a)$$

$$\mu_1\frac{dU_1}{dY}=\mu_2\frac{dU_2}{dY},\ k_1\frac{dT_1}{dY}=k_2\frac{dT_2}{dY}\ \ at\ \ Y=0 \qquad (8b)$$

The pressure gradient dP/dX in Eqns. (2) and (4) is unknown and must be evaluated via the overall mass conservation equation

$$\int_{Y=-h}^{Y=0}UdY=Q^* \qquad (9)$$

The radiative heat flux [8] is given by

$$\frac{\partial q_r}{\partial Y}=4\left(T_i-T_0\right)I' \qquad (10)$$

We introduce the following non-dimensional variables

$$x=\frac{X_i}{h_i},\quad y=\frac{Y_i}{h_i},\quad u_i=\frac{U_i}{U_0},\quad \theta_i=\frac{T_i-T_0}{T_{w1}-T_{w2}},$$

$$p=\frac{h_iP}{\mu_iU_0},\ R=\frac{U_0d}{\nu_i},\ M^2=\frac{\sigma_iB_0^2h_i^2}{\mu_i},$$

$$\frac{1}{K}=\frac{h_i^2}{K_0},\ Pr=\frac{\mu_ic_p}{k_i},\ \alpha=\frac{Qh_i^2}{k_i},$$

$$F=\frac{4I'h_i^2}{k_i},\ \ i=1,2 \qquad (11)$$

In view of Eqn. (11), the dimensionless form of the momentum and energy equations become

Region-I

$$\frac{d^2u_1}{dy^2}-R\frac{du_1}{dy}-\left(M^2+\frac{1}{K}\right)u_1=B \qquad (12)$$

$$\frac{d^2\theta_1}{dy^2}-PrR\frac{d\theta_1}{dy}-\left(\alpha+F\right)\theta_1=0 \qquad (13)$$

Region-II

$$\frac{d^2u_2}{dy^2}-R\frac{du_2}{dy}-M^2u_2=B \qquad (14)$$

$$\frac{d^2\theta_2}{dy^2}-PrR\frac{d\theta_2}{dy}-\left(\alpha+F\right)\theta_2=0 \qquad (15)$$

The dimensionless form of the boundary and interface conditions become

$$u_1=0,\quad \theta_1=1\quad at\quad y_1=-1 \qquad (16)$$

$$u_2=0,\quad \theta_2=0\quad at\quad y_2=1 \qquad (17)$$

$$u_1=u_2,\quad \theta_1=\theta_2\quad at\quad y=0 \qquad (18a)$$

$$\frac{du_1}{dy}=\frac{1}{ch}\frac{du_2}{dy},\ \frac{d\theta_1}{dy}=\frac{1}{kh}\frac{d\theta_2}{dy}\quad at\quad y=0 \qquad (18b)$$

Along with the overall mass conservation equation

$$\int_{y=-1}^{y=0}udy=1 \qquad (19)$$

Solution of the problem: Solutions of the velocity and temperature distributions are obtained by solving the Eqns. (12) –(15) using the boundary and interface conditions (16) – (18b) are

$$u_1(y)=B_5+B_6e^{l_5Y}+B_7e^{l_6Y} \qquad (20)$$

$$u_2(y)=B_8+B_{11}e^{l_7Y}+B_{12}e^{l_8Y} \qquad (21)$$

$$\theta_1(y)=B_1e^{l_1Y}+B_2e^{l_2Y} \qquad (22)$$

$$\theta_2(y)=B_3e^{l_3Y}+B_4e^{l_4Y} \qquad (23)$$

where $I'=\int_0^\infty K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T}d\lambda$; $Q^*=U_0h_i$ (say); $B=dp/dx$; $N=\left(M^2+\frac{1}{K}\right)$;

$$l_1=l_3=\frac{PrR+\sqrt{Pr^2R^2+4\left(\alpha+F\right)}}{2};$$

$$l_2=l_4=\frac{PrR-\sqrt{Pr^2R^2+4\left(\alpha+F\right)}}{2};$$

$l_5 = l_7 = \dfrac{R + \sqrt{R^2 + 4N^2}}{2}$;

$l_6 = l_8 = \dfrac{R - \sqrt{R^2 + 4N^2}}{2}$; $B_1 = \dfrac{b_2}{b_2 e^{-\beta_1} - b_1 e^{-\beta_2}}$;

$B_2 = \dfrac{1 - B_1 e^{-l_1}}{e^{-l_2}}$;

$B_3 = \dfrac{(B_1 + B_2) e^{l_4}}{e^{l_4} - e^{l_3}}$; $B_4 = \dfrac{-B_3 e^{l_3}}{e^{l_4}}$;

$B_5 = B_8 = \dfrac{-B}{N}$; $B_6 = \dfrac{b_7 b_{12} + b_{13} e^{-l_6}}{b_{11} e^{-l_6} - b_{12} e^{-l_5}}$;

$B_7 = \dfrac{b_7 + B_{13} e^{-l_5}}{-e^{-l_5}}$; $B_9 = \dfrac{b_3 b_8 + b_9 e^{-l_6}}{b_7 e^{-l_6} - b_8 e^{-l_5}}$;

$B_{10} = \dfrac{b_3 + B_7 e^{-l_5}}{-e^{-l_6}}$; $B_{11} = \dfrac{b_4 + (B_9 + B_{10}) e^{l_8}}{e^{l_8} - e^{l_7}}$;

$B_{12} = \dfrac{b_4 + B_{10} e^{l_7}}{-e^{l_8}}$;

$b_1 = khl_1 e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4 e^{l_3} - l_3 e^{l_4})$;

$b_2 = khl_2 e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4 e^{l_3} - l_3 e^{l_4})$;

$b_3 = B_5$; $b_4 = B_{11}$; $b_5 = -b_4 e^{l_8}$;

$b_6 = chl_5 e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8 e^{l_7} - l_7 e^{l_8})$;

$b_7 = chl_6 e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8 e^{l_7} - l_7 e^{l_8})$;

$b_8 = b_4(l_8 e^{l_7} - l_7 e^{l_8}) + b_5(e^{l_7} - e^{l_8})$,

$b_9 = khl_1 e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4 e^{l_3} - l_3 e^{l_4})$;

$b_{10} = khl_2 e^{l_4}(e^{l_3} - e^{l_4}) - e^{l_4}(l_4 e^{l_3} - l_3 e^{l_4})$;

$b_{11} = chl_{10} e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8 e^{l_7} - l_7 e^{l_8})$;

$b_{12} = chl_{11} e^{l_8}(e^{l_7} - e^{l_8}) - e^{l_8}(l_8 e^{l_7} - l_7 e^{l_8})$;

$b_{13} = b_8(l_8 e^{l_7} - l_7 e^{l_8}) + b_{10}(e^{l_7} - e^{l_8})$.

The skin friction (τ_i) and the Nusselt number (Nu_i) at the walls $y = -1$ and $y = 1$ are given by

$$\tau_1 = u_1'\big|_{y=-1} \qquad \tau_2 = u_2'\big|_{y=1} \tag{26}$$

$$Nu_1 = -\theta'\big|_{y=-1} \qquad Nu_2 = -\theta'\big|_{y=1} \tag{27}$$

where $i = 1, 2$ and the primes are the differential derivative with respect to y.

Result and discussion: In order to get a physical insight into the problem, factors such as velocity, temperature, skin friction and Nusselt number have been discussed by assigning numerical values to various parameters obtained in the mathematical formulation of the problem and the results are graphically displayed in Figs. 2–11. Throughout the computations we employ $M = 2$, $K = 0.1$, $B = -5$, $c = 0.5$, $R = 2$, $Pr = 0.71$, $\alpha = 1$, $F = 0.5$, $k = 1$ and $h = 1$ unless otherwise stated.

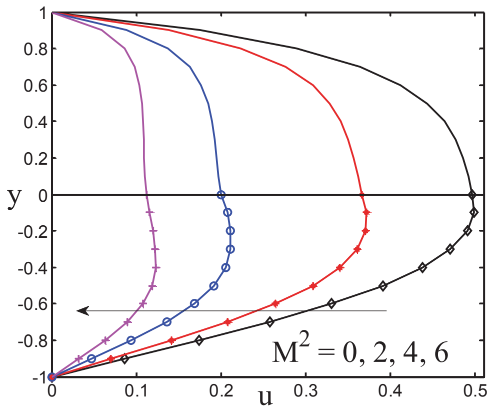

Fig 2: Effect of M on u

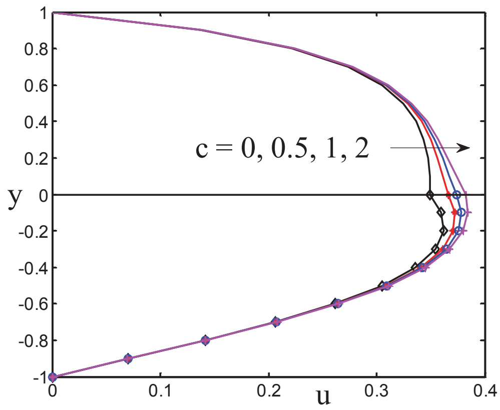

Fig 3: Effect of c on u

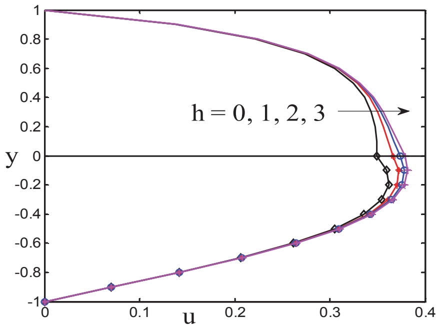

Fig 4: Effect of h on u

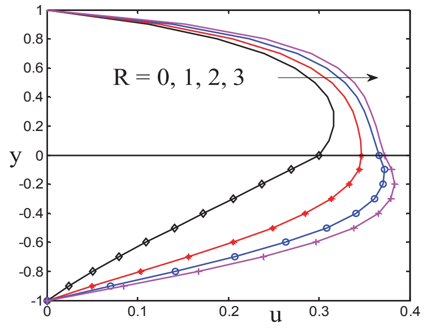

Fig 5: Effect of R on u

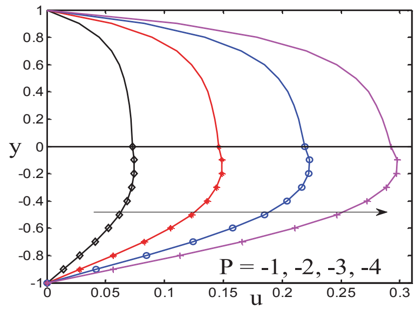

Fig 6: Effect of P on u

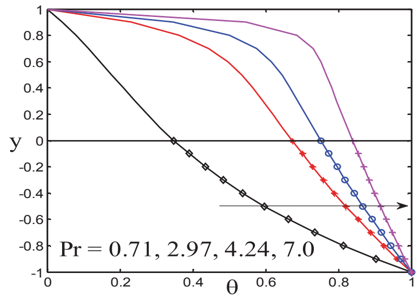

Fig 7: Effect of Pr on θ

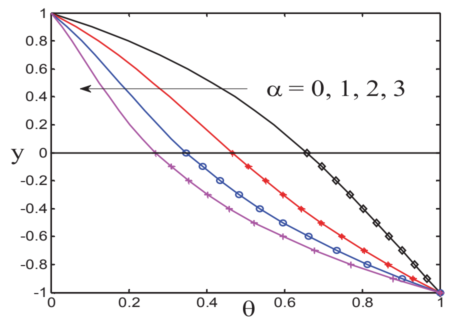

Fig 8: Effect of α on θ

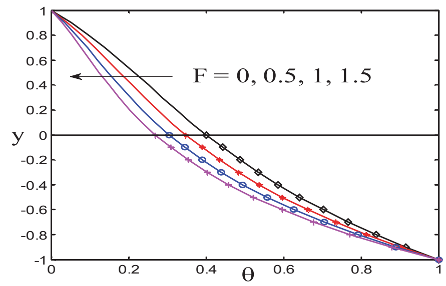

Fig 9: Effect of F on θ

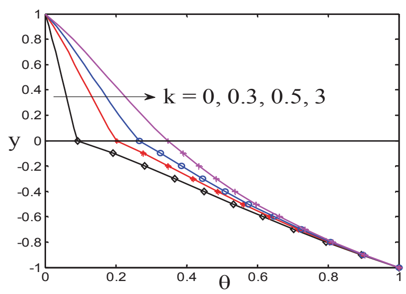

Fig 10: Effect of k on θ

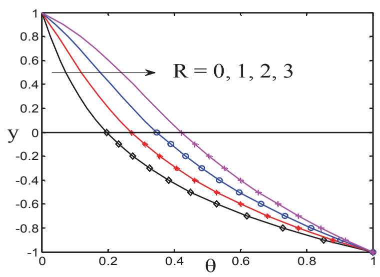

Fig 11: Effect of R on θ

Figures 2-6 presents the effect of M, c, h, R and P on velocity distribution respectively. Figure 2 shows that application of a magnetic field normal to the flow direction has the tendency to slow down the movement of the fluids in the channel. Figures 3 and 4 represent that increase in the density ratio and height ratio has the tendency to increase the fluid velocity in the channel. It is apparent from Fig. 5 that an increase in the values of suction parameter leads to increase the velocity. Figures 6 elucidates that fluid velocity of both fluids notably increases for the lower magnitudes of the pressure. It is observed from Figs. 2-6 that the fluid velocity in the Region I is higher than the fluid velocity in the Region II. The reason is region I is saturated with uniform porous medium in which the porosity parameter leads to enhance fluid velocity because it reduces the drag force. Figures 7-11 are plotted to show the influence of Pr, α, F, R and k on temperature distribution respectively. Figure 16 displays that increasing the Prandtl number increase the heat transfer. Figure 8 illustrates that the fluid temperature monotonically decreases for increasing the heat absorption parameter. Figure 9 represents that increase in the radiation parameter decreases the temperature distribution. Figure 10 shows that the thermal conductivity ratio is seemed to increase the temperature of both fields. Furthermore increases in boundary layer suction also increase the fluid temperature which is graphed in Fig. 11.

Conclusions: The key observations of the present study are as follows: It is found that increase in suction parameter enhances the velocity of both the fluids whereas increasing the magnetic field parameter reverse the effect. The higher values of viscosity ratio and channel width ratio parameters increase the viscous fluid velocity. Velocity of both fluids increases for the lower magnitudes of the pressure. The heat transfer increases for increasing conductivity ratio, suction parameter and Prandtl number but the trend is reversed for increasing heat absorption parameter.

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