HEURISTIC ALGORITHM FOR FINDING MORE-FOR-LESS OPTIMAL SOLUTION TO INTERVAL TRANSPORTATION PROBLEMS

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Abstract: This paper develops a heuristic algorithm for finding a more-for-less (MFL) optimal solution to interval transportation problems. In this algorithm, directed path connecting allotted cells is used for modifying the allotment of the allotted cells. Numerical example is presented to clarify the idea of the proposed approach. The MFL optimal solution obtained by the proposed method can be helpful for the decision makers to make economical oriented managerial decisions when they are handling various types of logistic problems having imprecise parameters.

Keywords: Interval transportation problem (ITP); fuzzy TP; MFL situation; directed path.

Introduction: The transportation problem (TP) is a special type of linear programming problem, which deals with shipping commodities from sources to destinations. The MFL paradox in a TP occurs when it is possible to ship more total goods for less (or equal) total cost, while shipping the same amount or more from each source and to each destination, keeping all shipping costs non-negative. The information of the occurrence of an MFL situation is useful to a manager in deciding which warehouse or plant capacities are to be increased, and which markets should be sought. The MFL paradox in the TP has been covered from a theoretical stand point by Charnes and Klingman [3], and Charnes et al. [4] and Robb [10]. Gupta et al. [6] and Arsham [2] developed an approach to post optimality analysis of the TPs. Adlakha and Kowalski [11,12] introduced a theory of absolute points for solving a TP and used these points for search opportunities to ship MFL in TP. An algorithm for finding feasible solution for linear fraction TP and condition for the existence of a paradoxical solution was presented by Ekezie [5]. Pandian and Anuradha [9] have introduced path method for finding a MFL optimal solution to a TP. (ITP) is a generalization of the TP in which input data are expressed as intervals instead of fixed values. This problem can arise when uncertainty exists in data problem and decision makers are more comfortable expressing it as intervals. An algorithm for finding an optimal MFL solution to fuzzy TP with mixed constraints was presented by Pandian and Natarajan [8]. Arpita and Bikash [1] have presented a solution procedure of cost varying interval TP under two vehicles. Jahir and Jayaraman [7] investigated the stability set of parameters corresponding to a vector fuzzy TP in interval integer form.

In this paper, a heuristic algorithm for finding a MFL optimal solution to TP in which costs, supplies (S) and demands (D) are intervals is proposed and the same is illustrated with the help of numerical example. Further, this method is extended to fuzzy TP in which all the parameters are trapezoidal fuzzy numbers (TFNs). The proposed method provides an optimal MFL solution to an ITP/FTP which helps the managers to evaluate the economical activities and make self satisfied managerial decisions when they are handling a variety of logistics problems.

(UB)	E. No	(LB)	E. No
Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} y_{ij}$		Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$	
S. to		S. to	
$\sum_{j=1}^{n} y_{ij} = a_i^2, i = 1,2,\ldots,m$	4	$\sum_{j=1}^{n} x_{ij} = a_i^1, i = 1,2,\ldots,m$	7
$\sum_{i=1}^{m} y_{ij} = b_j^2, j = 1,2,\ldots,n$	5	$\sum_{i=1}^{m} x_{ij} = b_j^1, j = 1,2,\ldots,n$	8
$y_{ij} \geq 0$ for all i and j	6	$x_{ij} \geq 0$ for all i and j	9

Preliminaries: Let D denote the set of all closed bounded intervals on the real line R. (i.e) $D = \{ [a,b], a \leq b \text{ and } a, b \in R\}$.

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [8,11].

Definition 1: Let A = [a, b] and B = [c, d] be in D. Then,

(i) $A \oplus B = [a+c, b+d]$;

(ii) $A \Theta B = [a-d, b-c]$;

(iii) kA = [ka, kb] ; k is a +ve real number; (iv) kA = [kb, ka]; k is a -ve real number;

(v) $A \otimes B = [p, q]$ where p = min {ac, ad, bc, bd} and q = max {ac, ad, bc, bd}.

Definition 2: Let A = [a, b] and B = [c, d] be in D. Then,

(i) $A \le B$ if $a \le c$ and $b \le d$;

(ii) $A < B$ if $a < c$ and $b < d$;

(iii) $A \ge B$ if $B \le A$, that is $a \ge c$, $b \ge d$

(iv) $A = B$ if $A \le B$; $B \le A$, i.e. $a = c$, $b = d$.

Interval Transportation Problem: Consider the following (ITP):

$$\text{Minimize}[z_1, z_2] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$$

$$\text{S. to } \sum_{j=1}^{n}[x_{ij}, y_{ij}] = [a_i^1, a_i^2],\ i = 1,2,\ldots,m \qquad (1)$$

$$\sum_{i=1}^{m}[x_{ij}, y_{ij}] = [b_j^1, b_j^2],\ j = 1,2,\ldots,n \qquad (2)$$

$$x_{ij} \ge 0,\ y_{ij} \ge 0,\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \qquad (3)$$

where c_{ij}, d_{ij}, a_i^1, a_i^2, b_j^1, b_j^2 are positive real numbers for all i and j.

A set $\{[x_{ij}, y_{ij}],\ \forall i = 1,2,\ldots,m;\ j = 1,2,\ldots,n\}$ is said to be a feasible solution of (ITP) if they satisfy the equations (1), (2) and (3).

A feasible solution of (ITP) which minimizes the total shipping cost, that is, $\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$ is called an optimal solution to the problem (ITP).

We consider the following two problems as an upper bound (UB) problem and a lower bound (LB) problem of the (ITP):

Heuristic Algorithm: In [2], Arsham proved that the existence of a MFL situation in a TP requires only one condition namely, the existence of a location with negative plant-to-market shipping shadow price (Modi index). The shadow prices are easily calculated from the solution of the TP. Modi index at a cell (i, j) is $u_i + v_j$ where u_i and v_j are shadow prices corresponding to the cell (i, j). The negative Modi index at a cell (i, j) indicates that we can increase i^th plant capacity/demand of j^th market at the maximum possible level. If Modi index is non-negative at each cell (i, j), this indicates that we can not increase i^th plant capacity/demand of j^th market, that is, the current level is their maximum level.

An algorithm for finding an optimal MFL solution to (ITP) is proposed below:

Step 1: Construct two individual problems of the given ITP namely, (UB) problem and (LB) problem.

Step 2: Obtain a feasible/an optimal solution to the (UB) problem.

Step 3: Create the MODI index matrix using solution obtained in Step 2 and select the most negative MODI index cell say (r,s). If not, that is, if there is no negative MODI index, then go to Step 7.

Step 4: Form a directed allocation path (DAP) for cell (r,s). DAP is a directed path from r^th row to s^th column such that each of its terminal cell is an allocated cell.

Step 5: Add maximum possible allocation to the initial and terminal cell in the DAP in such a way that the rim requirements remain satisfied. Then, a new MFL solution to the UB problem is obtained.

Step 6: If MODI indices for MFL solution obtained from Step 5 are positive, then go to Step 7. If not, repeat, Steps from 3 to 6.

Step 7: The current solution say $\{y_{ij}^{\circ}, \forall\, i \,\&\, j\}$ is an optimal MFL solution to the (UB) problem.

Step 8: Repeat the steps from 2 to 6 for the (LB) problem with the upper bound constraints $x_{ij} \le y_{ij}^{\circ}$, for all i and j. Let $\{x_{ij}^{\circ}, \forall\, i \,\&\, j\}$ be an optimal solution of the (LB) problem with $x_{ij} \le y_{ij}^{\circ}$.

Step 9: The set of intervals $\{[x_{ij}^{\circ}, y_{ij}^{\circ}], \forall\, i \,\&\, j\}$ is an optimal solution to the ITP.

Numerical Example: The proposed method is illustrated by the following example.

Example 1: Consider the following ITP

	D_1	D_2	D_3	D_4	D_5	S
O_1	[4,6]	[15,17]	[6,8]	[13,15]	[14,16]	[7,9]
O_2	[16,18]	[9,11]	[22,24]	[13,15]	[16,18]	[18,24]
O_3	[8,10]	[5,7]	[11,13]	[4,6]	[5,7]	[6,10]
O_4	[12,14]	[4,6]	[18,20]	[9,11]	[10,12]	[15,19]
D	[4,6]	[11,15]	[12,16]	[8,10]	[11,15]	

Now, the (UB) problem of the given problem (ITP) is given below:						
	D_1	D_2	D_3	D_4	D_5	S
O_1	6	17	8	15	16	9
O_2	18	11	24	15	18	24
O_3	10	7	13	6	7	10
O_4	14	6	20	11	12	19
D	6	15	16	10	15	

Now, we obtain the following optimal solution to the (UB) problem by the zero point method.						
	D_1	D_2	D_3	D_4	D_5	S
O_1	6	17	8(9)	15	16	9
O_2	18(6)	11(8)	24	15(10)	18	24
O_3	10	7	13(7)	6	7(3)	10
O_4	14	6 (7)	20	11	12(12)	19
D	6	15	16	10	15	

Therefore, the optimal solution to the (UB) problem is $y^{\circ}_{13}=9$, $y^{\circ}_{21}=6$, $y^{\circ}_{22}=8$, $y^{\circ}_{24}=10$, $y^{\circ}_{33}=7$, $y^{\circ}_{35}=3$, $y^{\circ}_{42}=7$, $y^{\circ}_{45}=12$ and all other $y^{\circ}_{ij}=0$, for a flow of 62 units with the minimum total transportation cost is 716. Therefore, the shipping rate of transportation per unit is 11.548

Now, MODI index matrix corresponding to the above solution is given below:						
	D_1	D_2	D_3	D_4	D_5	
O_1	3	-4	8	0	2	$u_1=-15$
O_2	18	11	23	15	17	$u_2=0$
O_3	8	1	13	5	7	$u_3=-10$
O_4	13	6	18	10	12	$u_4=-5$
	$v_1=18$	$v_2=11$	$v_3=23$	$v_4=15$	$v_5=17$	

Now, as in Step 3, the most negative MODI index cell is (1,2). According to Step 4, we construct a directed allocation path $L_1:(1,3)-(3,3)-(3,5)-(4,5)-(4,2)$ and by using Step 5, we can increase supply 1 by θ_1 and the demand 2 by θ_2 such that $0\le\theta_1\le 7$ and $\theta_1\le\theta_2\le 12$ and also, we can increase supply 3 by θ_2, because of the rim condition. Then, the maximum possible values of θ_1,θ_2 are 7,12 respectively.

Now, a new MFL solution to the given (UB) problem is						
	D_1	D_2	D_3	D_4	D_5	S
O_1	6	17	8(16)	15	16	16
O_2	18(6)	11(8)	24	15(10)	18	24
O_3	10	7	13	6	7(15)	15
O_4	14	6(19)	20	11	12	19
D	6	27	16	10	15	

Now MODI index matrix corresponding to the above solution is given below:						
	D_1	D_2	D_3	D_4	D_5	
O_1	18	11	8	15	11	$u_1=0$
O_2	18	11	8	15	11	$u_2=0$
O_3	14	7	4	11	7	$u_3=-4$
O_4	13	6	3	10	6	$u_4=-5$
	$v_1=18$	$v_2=11$	$v_3=8$	$v_4=15$	$v_5=11$	

From the above table, we notice that the MODI index matrix is positive, we stop the computations. Therefore, the MFL optimal solution to the (UB) problem is $y^{\circ}_{13}=16$, $y^{\circ}_{21}=6$, $y^{\circ}_{22}=8$, $y^{\circ}_{24}=10$, $y^{\circ}_{35}=15$, $y^{\circ}_{42}=19$ and all other $y^{\circ}_{ij}=0$, for a flow of 74 units with the minimum total transportation cost is 693. Therefore, the shipping rate of transportation per unit is 9.365.

Now, the (LB) problem of the (ITP) with the upper bound constraints is given below:						
	D_1	D_2	D_3	D_4	D_5	S
O_1	4	15	6	13	14	7
O_2	16	9	22	13	16	18
O_3	8	5	11	4	5	6
O_4	12	4	18	9	10	15
D	4	11	12	8	11	

and $x_{ij}\le y^{\circ}_{ij}, \forall\, i=1,2,\ldots,m; j=1,2,\ldots,n$.

Using the procedure followed as in the solution of (UB) problem, the MFL optimal solution to the (LB) problem is obtained as $x^{\circ}_{13}=12$, $x^{\circ}_{21}=4$, $x^{\circ}_{22}=6$, $x^{\circ}_{24}=8$, $x^{\circ}_{35}=11$, $x^{\circ}_{42}=15$ for a flow of 56 units with minimum total transportation cost is 409. Therefore, the shipping rate of transportation per unit is 7.304.

Thus, the optimal MFL solution to the given ITP is $[x_{13}^{\circ}, y_{13}^{\circ}] = [12,16]$, $[x_{21}^{\circ}, y_{21}^{\circ}] = [4,6]$, $[x_{22}^{\circ}, y_{22}^{\circ}] = [6,8]$, $[x_{24}^{\circ}, y_{24}^{\circ}] = [8,10]$, $[x_{35}^{\circ}, y_{35}^{\circ}] = [11,15]$, $[x_{42}^{\circ}, y_{42}^{\circ}] = [15,19]$ for a flow of [56,74] units with minimum total interval transportation cost is [409,693].

Fuzzy transportation problem:

Consider the following (FTP):

(FTP) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$

S. to $\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i$, $i = 1,2,\ldots,m$

$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j$, $j = 1,2,\ldots,n$

$\tilde{x}_{ij} \geq \tilde{0}$, for all i and j

where all the unit shipping costs $\tilde{c}_{ij}$; supply quantities $\tilde{a}_i$; demand quantities $\tilde{b}_j$ are assumed to be TFNs.

A TFN (a,b,c,d) can be represented as an interval number form as follows:

$$(a,b,c,d) - [a + (b-a)\alpha, d - (d-c)\alpha]; 0 \leq \alpha \leq 1$$

Using the above relation, we can convert the given (FTP) problem into an interval type problem, and by using the proposed algorithm, we obtain an optimal MFL interval solution to the (ITP). Again, using TFN relation, we can obtain an optimal MFL solution to the given problem (FTP).

The solution procedure of obtaining a MFL optimal solution to (FTP) using the proposed method is illustrated by the following example.

Example 2: The example FTP is given in Table 1.

The (ITP) of Table 1 is given in Table 2.

Now, the optimal MFL solution to the (ITP) by the proposed algorithm is obtained as $[x_{13}^{\circ}, y_{13}^{\circ}] = [7+3\alpha, 17-3\alpha]$, $[x_{21}^{\circ}, y_{21}^{\circ}] = [2+\alpha, 6-\alpha]$, $[x_{22}^{\circ}, y_{22}^{\circ}] = [3+2\alpha, 10-3\alpha]$, $[x_{24}^{\circ}, y_{24}^{\circ}] = [4+3\alpha, 12-3\alpha]$, $[x_{35}^{\circ}, y_{35}^{\circ}] = [4+3\alpha, 17-3\alpha]$, $[x_{42}^{\circ}, y_{42}^{\circ}] = [13+\alpha, 17-\alpha]$ for a flow of $[33+13\alpha, 79-14\alpha]$.

Therefore, the optimal MFL solution for the given (FTP) is $\tilde{x}_{13} = (7,11,13,17)$, $\tilde{x}_{21} = (2,3,5,6)$, $\tilde{x}_{22} = (3,5,7,10)$, $\tilde{x}_{24} = (4,7,9,12)$, $\tilde{x}_{35} = (4,7,14,17)$, $\tilde{x}_{42} = (13,14,16,17)$ for a flow of (33,47,64,79) units with transportation cost $\tilde{z} = (144, 287, 545, 807)$.

Conclusion: In this paper, we obtained a MFL optimal solution for an ITP/FTP. The MFL situation could be useful for managers in making strategic decisions such as increasing a ware-house stocking level or plant production capacity and advertising efforts to increase demand at certain markets. This method help decision makers in the logistics related issues of real life problems by aiding them in the decision making process and providing an optimal solution in an effective manner.

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Table 1						
	D_1	D_2	D_3	D_4	D_5	S
O_1	(2,3,5,6)	(13,14,16,17)	(3,5,7,10)	(10,11,15,16)	(11,13,15,17)	(3,6,8,12)
O_2	(12,15,17,19)	(7,8,10,11)	(19,21,23,25)	(10,11,15,16)	(12,15,17,19)	(16,17,19,20)
O_3	(4,7,9,12)	(3,4,6,7)	(4,7,14,17)	(2,3,5,6)	(3,4,6,7)	(3,5,7,10)
O_4	(7,11,13,17)	(2,3,5,6)	(16,17,19,20)	(7,8,10,11)	(8,9,11,12)	(13,14,16,17)
D	(2,3,5,6)	(4,7,14,17)	(7,11,13,17)	(4,7,9,12)	(4,7,14,17)	

Table 2						
	D_1	D_2	D_3	D_4	D_5	Supply
O_1	$[2+\alpha,6-\alpha]$	$[13+\alpha,17-\alpha]$	$[3+2\alpha,10-3\alpha]$	$[10+\alpha,16-\alpha]$	$[11+2\alpha,17-2\alpha]$	$[3+3\alpha,12-4\alpha]$
O_2	$[12+3\alpha,19-2\alpha]$	$[7+\alpha,11-\alpha]$	$[19+2\alpha,25-2\alpha]$	$[10+\alpha,16-\alpha]$	$[12+3\alpha,19-2\alpha]$	$[16+\alpha,20-\alpha]$
O_3	$[4+3\alpha,12-3\alpha]$	$[3+\alpha,7-\alpha]$	$[4+3\alpha,17-3\alpha]$	$[2+\alpha,6-\alpha]$	$[3+\alpha,7-\alpha]$	$[3+2\alpha,10-3\alpha]$
O_4	$[7+3\alpha,17-3\alpha]$	$[2+\alpha,6-\alpha]$	$[16+\alpha,20-\alpha]$	$[7+\alpha,11-\alpha]$	$[8+\alpha,12-\alpha]$	$[13+\alpha,17-\alpha]$
Demand	$[2+\alpha,6-\alpha]$	$[4+3\alpha,17-3\alpha]$	$[16+\alpha,20-\alpha]$	$[4+3\alpha,12-3\alpha]$	$[4+3\alpha,17-3\alpha]$	

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