LIFE-TIME AND DEGREE DEPENDENT CONNECTION PROBABILITY IN NETWORKS.

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Abstract: We consider a modification of the usual mechanism to generate scale-free networks. Our model incorporates life time dependent affinity, apart from the degree dependent affinity, for the new nodes. The model is studied numerically, as well as analytically for different weight factors for the two kinds of affinities. We show numerically that the model leads to scale-free behavior. Analytical results for the model show a good match with the numerics. It is seen that while the degree distribution at small values of degree doesn't get affected much with change in the weight factor, the tail of the distribution which contains the hubs, gets affected much more.

Keywords: evolving, life-time, network, scale-free.

Introduction: Since it's introduction in 1999 by Barabasi and Albert [1] [2],the scale free model for networks has been proven to be avery successful one, with applications ranging from the internet and telephone call networks to biologicalnetworks like protein interactions in a cell cycle and a variety of socialnetworks [3][4]. The main idea of the scale-free model is that it considers an evolving network where connection probability of a new node to an old node is decided by the degree of the old node. While the model has been successful in many set-ups, a usual feature of the empiricaldegree distributions [3] is the exponential tail which needs a moredetailed analysis. In [5] theory of a growing networkwas studied to explain thetail of the degree distribution. It was also shown that all models for scale-free networks show scale-free behavior only in a limited range of the degree k. Since in real life one always workswith a finite network, it is of interest to study models of differentgrowing networks. Different growing networks are expected to have differentkinds of tails in their degree distributions. Since the hubs of network arein the tail region, different types of tails would lead to differentkinds of network characteristics. Different types of networks also areexpected to have different fault tolerance, which provides us withmotivation to study different models for a growing networks.It has been well known that scale-free distributions are easy togenerate and quite abundant in various set-ups but their actualarchitecture in general can be quite different leading to differentnetwork properties [6][7].

In this paper we consider a natural modifications of the scale-freemodel. The models involve affinity which depends not only on thedegree of the old node but also on its life time. It appears natural to assume that an older node in a network is more likely to make a new connection than a new one. Especially if connection are made through arandom collision kind of mechanism which is quite possible in a social set-up, then one would expect a larger lifetime will correspond tohigher affinity. The basic network growth rules are as follows.

Our model assumes that the node affinity depends both on thedegree of the old node and on its life-time at the time of connection,linearly. The connection probability of the i^th^ node with degree k_i is given by,

$$P(k_i) = w \frac{k_i}{\sum_{all\ nodes} k_j} + (1-w)\frac{l_i}{\sum_{all\ nodes} l_j} \quad (1).$$

Where $l_i = t - t_{0i}$ is the life-time of the i^{th} node, t is the timeof connection and t_{0i} is the time at which the i^{th} node was introduced. w is an adjustable parameter, for w=1 we get the scale-free model, for w=0 we get a model with affinity that depends only on time. The intermediate cases are given by intermediate w values. If w is chosento be larger than 1 then connection to a node with long life is discouraged and for negative w values connection with a node with high degree is discouraged.

Theoretical Calculations : To analize the model, we follow the continuum approach in [3]. The rate at which the degree k_i of the i^{th} node increases in time is given by,

$$\frac{dk_i}{dt} = w \frac{k_i}{\sum_{all\ nodes} k_j} + (1-w)\frac{l_i}{\sum_{all\ nodes} l_j} \quad (2).$$

Since $\sum_{all\ nodes} k_j = 2t$ and $\sum_{all\ nodes} l_j = t(t+1)/2$, we get the first order inhomogeneous differential equation,

$$\frac{dk_i}{dt} = w\frac{k_i}{2t} + 2(1-w)\frac{t-t_{0i}}{t(t+1)} \quad (3).$$

Using a change of variables $k_i = t^{w/2} y$, we have,

$$\frac{dy}{dt} = 2(1-w)t^{-w/2}\frac{t-t_{0i}}{t(t+1)} \quad (4)$$

In ^general^ $t >> 1$ for most of the nodes in a large nework and so $t(t+1)$ can be approximated to t^2.

$$y(t) - y(t_{0i}) = -4(1-w)\left(\frac{t^{-w/2}}{w} - \frac{t_{0i}t^{-w/2-1}}{w+2}\right) + {t_{0i}}^{-w/2}\frac{8(1-w)}{w(w+2)} \quad (5).$$

The initial condition is $k_i = 1$ at $t = t_{0i}$, so $y(t_{0i}) = t_{0i}^{-w/2}$ thus,

$$k_i(t) = \left(\frac{t}{t_{0i}}\right)^{w/2}\left(1 + 8\,\frac{(1-w)}{w(w+2)}\right) - 4(1-w)\left(\frac{1}{w} - \frac{t_{0i}}{t(w+2)}\right) \quad (6)$$

For positive $1 > w > 0$, $\frac{dk_i(t)}{dt_{0i}}$ is negative thus k_i is a decreasing function of t_{0i} for a fixed t. Thus the condition $k_i > k$ is the same as $t_{0i} > t_b$ where t_b is a function of k and t that satisfies the equation,

$$k = \left(\frac{t}{t_b}\right)^{w/2}\left(1 + 8\,\frac{(1-w)}{w(w+2)}\right) - 4(1-w)\left(\frac{1}{w} - \frac{t_b}{t(w+2)}\right) \quad (7)$$

The probability that the i^{th} node has degree smallerthan k, $P(k_i(t) < k)$ can also be written as,

$$P(k_i(t) < k) = P(t_{0i} > t_b) \quad (8)$$

We also have $P(t_{0i}) = 1/t$ and

$$P(t_{0i} > t_b) = 1 - \frac{t_b}{t} \quad (9)$$

. The degree distribution P(k) can be calculated using,

$$P(k) = \frac{\partial P(k_i < k)}{\partial k} = -\frac{\partial}{\partial k}\frac{t_b}{t} \quad (10)$$

From the defining equation of t_b we find,

$$P(k) = \frac{-1}{-\frac{w}{2}\left(\frac{t}{t_b}\right)^{\frac{w}{2}+1}\left(1 + 8\,\frac{(1-w)}{w(w+2)}\right) + \frac{4(1-w)}{w+2}} \quad (11)$$

In the following section we give the comparison between thedegree distributions found numerically and the distributions givenby the equation above for various values of theweight factor w.

Numerical results : In all computations we start with an initial threenode network with the adjacency matrix given by,

$$a(1,2) = a(2,1) = a(2,3) = a(3,2) = 1$$

and rest of the elements are zero. We consider only one connection at each time step. The starting time is t=4 and $t_{01} = 0$, $t_{02} = 1$ and t_{03}= 2. (the zero subscript indicates the time of introduction so t_{01} is the time-step as which node 1 was introduced). The graph in Fig. 1 shows the degree distributionfor a network with the number of nodes N=3000 for various values of w. Three values of the parameter w = .9, .8, .7 have been considered. It is found that that the degree distribution for all three w values is the same at low k but differs in the large k regime.

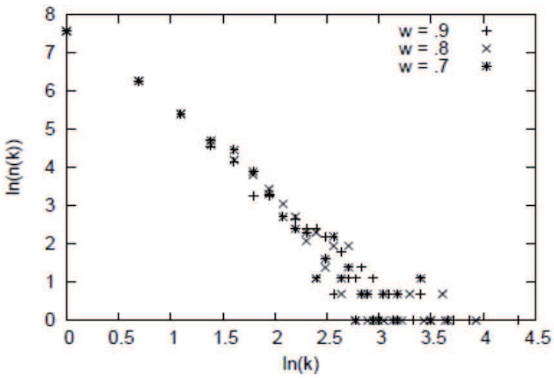

Fig.1: degree distribution for a 3000 node network with different weight factors

It is interesting to note that all the graphs are linearto a good approximation, meaning even with the new growth rule the network structure is scale-free to a good approximation. The graphs differ for high k values, which shows that the hubs have different distribution for different w.

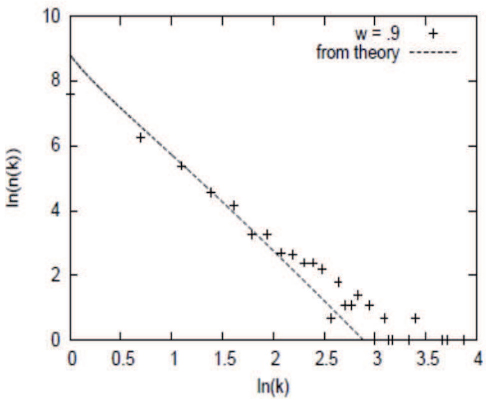

Fig.2: Comparision of theoretical and numerical degree distribution for w = .9.

Fig.2, Fig.3 and Fig.4 compare the analytical degree distributions with the numericalones. Our analytical results are in good agreement with the numericalones but they are not exact. We believe that the analytical valuesfail to be exact due to two main reasons, (1) we have taken a continuumapproximation for a problem which is essentially discrete, (2) At the startof calculations, in time evolution equation for k_i the term $t(t+1)$in the denominator has been replaced by t^2. We would also like topoint out that it has been shown in[8] thatfor all the networks that exhibit scale-free behavior in the large size limit,the power law has behavior has a cut-off value of $k = k_{cut}$ due to finite size effects. Beyond k_{cut}scale-free behavior is not seen.

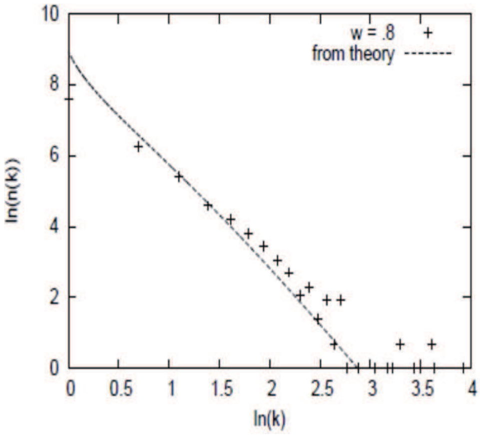

Fig.3:Comparison of theoretical and numerical degree distribution for w = .8

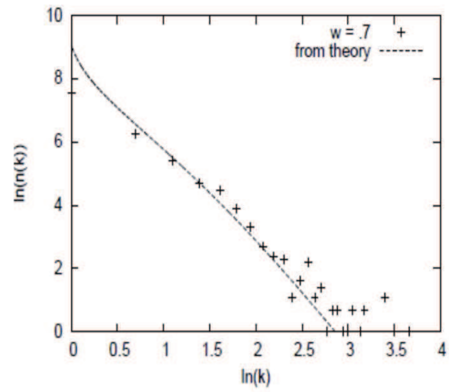

Fig.4: Comparison of theoretical and numerical degree distribution for w = .7

Conclusion : We have considered a model for evolving networks where the connectionprobability depends on the degree as well as life time of the node. We showfor three different values of the weight parameter w that the resultingdistribution is differs only slightly from the scale-free distribution for low k values. Howeverthe tails of the distributions are quite different fromthe usual scale-free network. Analytical formula is derivedfor the degree distribution and is shown to be in good agreementwith the numerical calculations.

Fault tolerance is an important aspectthat we have not yet studied. While it is possible to use the masterequation approach in [4], which would provide a better match between theory and numerics,the equations involved are likely to be very cumbersome. We would like to note that age dependent preferentialattachment has been discussed earlier in [9] and [10] where aging was considered to reduce the preference of the node. While the motivation for those authors toinclude age was to get more realistic degree distribution.Braha and Bar-Yam [11] have studied a time dependent complex networks based onempirical data of time dependence of network structures and found that the roles of nodes change in time. Effect of network topologyon transport efficiency has been recently studied in [12].It would be of interest to understand the distribution of high k nodes,i.e. hubs for our network model to see how various dynamical properties like transport, fault tolerance and synchronization get affected.

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