STABILITY OF TWO SUPERPOSED POROUS ELASTICO-VISCOUS FLUID WITH FINE DUST AND DIFFERENT PERMEABILITY

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Abstract : The instability of plane interface between two superposed Walter's elastico-viscous fluids in porous medium has been studied to include the suspended (dust) particles effect. It is found that for potentially stable arrangement Walter's elastico-viscous fluid of different permeabilities in the presence of suspended particles in porous medium is found to be stable.

Keywords: elastico-viscous fluid,fine dust, incompressible, permeability.

Introduction: The influence of the viscosity on the stability of a plane interface separating two incompressible superposed conducting fluids of uniform densities, when the whole system is acted on by a uniform magnetic field, has been studied by Bhatia [1]. Chandra [1b] observed a contradiction between the theory for the onset of convection in fluids heated from below and his experiments. He performed the experiment in an air layer and found that the instability depended on the depth of layer. A Bénard-type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed, for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower temperature gradient than predicted, if the layer depth was less than 7mm and called this 'columnar instability'. He added an aerosol to mark the flow pattern. Scanlon and Segel [3] studied the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles.With growing importance of effect of suspended particles on the stability of superposed fluids in the field of industrial and chemical engineering, Sharma [6] has studied the thermal instability of a layer of Oldroydian viscoelastic fluid acted on by a uniform rotation whereas Sharma and Sharma [7] have studied the instability of the plane interface separating two Oldroydian viscoelastic superposed fluids of uniform densities.

With the growing importance of elastico-viscous fluids, suspended particles and porous medium, the present paper attempts to study the hydrodynamic stability of superposed Walter's elastico-viscous fluids permeated with suspended particles in porous medium.

Formulation Of The Problem: Let ρ, p and $\vec{q}$ (u, v, w) denote respectively the density, pressure and filter velocity of the pure fluid; $\vec{q}_d$ (x, t) and N(x, t) denote the velocity and number density of the suspended particles, $\vec{q}_d$ = (l, r, s), $\overline{x}$ = (x, y, z) and $\vec{\lambda}$ = (0, 0 ,1). respectively. K = 6πρνη, where η being the particle radius, is the Stokes's drag coefficient. Let ∈, k1, μ, μ/ and g stand for medium porosity, medium permeability, viscosity of fluid, viscoelasticity of fluid and acceleration due to gravity respectively. Then the equations of motion and continuity for the Walters' viscoelastic fluid permeated with suspended particles through porous medium, Scanlon and Segal [3], Sharma and Kumar [12] are

$$\frac{\rho}{\in}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}\cdot\nabla)\vec{q}\right]=-\nabla p-\rho g\vec{\lambda}+\frac{KN}{\in}(\vec{q}_d-\vec{q})-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\vec{q}, \quad (1)$$

$$\nabla\cdot\vec{q}=0. \quad (2)$$

$$mN\left[\frac{\partial \vec{q}_d}{\partial t}+\frac{1}{\in}(\vec{q}_d\cdot\nabla)\vec{q}_d\right]=KN(\vec{q}-\vec{q}_d), \quad (3)$$

$$\in\frac{\partial N}{\partial t}+\nabla\cdot(N\vec{q}_d)=0. \quad (4)$$

Since the density of a fluid particle moving with the fluid remains unchanged, we have

$$\in\frac{\partial \rho}{\partial t}+(\vec{q}\cdot\nabla)\rho=0. \quad (5)$$

Let δρ, δp, $\vec{q}$ (u,v,w) and $\vec{q}_d$ (l,r,s) denote respectively the perturbations in density ρ, pressure p, fluid velocity (0,0,0) and particle velocity (0,0,0). Then the linearized perturbation equations of the Walters' fluid-particle layer are

$$\frac{\rho}{\in}\frac{\partial \vec{q}}{\partial t}=-\nabla\delta p-g\delta\rho\vec{\lambda}+\frac{KN}{\in}(\vec{q}_d-\vec{q})-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\vec{q}, \quad (6)$$

$$\nabla\cdot\vec{q}=0, \quad (7)$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\vec{q}_d=\vec{q}, \quad (8)$$

$$\in \frac{\partial}{\partial t}\delta\rho = -w(D\rho), \quad (9)$$

$$\frac{\partial M}{\partial t} + \nabla \cdot \vec{q}_d = 0, \quad (10)$$

where M = $\frac{\in N}{N_0}$ and No, N stand for initial uniform number density and perturbation in number density respectively and D = $\frac{d}{dz}$.

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is given by

exp (ikxx+ikyy +nt), (11)

where n is, in general, a complex constant. kx, ky are wave numbers along x- and y- directions and k2 = k2x+ k2y.

For perturbations of the form (11); equations (6)- (9), after elimination $\vec{q}_d$, and further eliminating δp yield

$$n\left[D\left(\frac{\rho}{\in}Dw\right) - \frac{k^2}{\in}\rho w\right] + \left[\begin{array}{c} D\left\{\left(\frac{\mu}{k_1} - \frac{\mu^{/}}{k_1}n\right)Dw\right\} \\ -k^2\left(\frac{\mu}{k_1} - \frac{\mu^{/}}{k_1}n\right)w \end{array}\right]$$

$$+\frac{n}{\in(\tau n+1)}\left\{D[mNDw] - k^2 mNw\right\} = -\frac{gk^2}{\in n}(D\rho)w \quad (12)$$

Two Uniform Fluids Separated By A Horizontal Boundary: Consider the case of two uniform Walters' viscoelastic fluids of densities, viscosities, viscoelasticities, suspended particles number densities as ρ2, μ2, μ/2, N2 and ρ1, μ1, μ1/, N1 separated by a horizontal boundary at z =0. The subscripts 1 and 2 distinguish the lower and the upper fluids respectively. The medium porosity ∈ is assumed to be the same in both the regions. Let the medium permeabilities of upper (z>0) and lower (z<0) media be k12 and k11 respectively.

Then in each region of constant ρ, constant μ, constant μ/, constant N and constant K1 equation (12) reduces to

(D2- k2) =0 (13).

The general solution of equation (13) is

w = A e+kz + B e-kz, (14)

where A and B are arbitrary constants. Then the boundary conditions to be satisfied in the present problem are

The velocity w → 0 when z → ∞ (for the upper fluid) and z → – ∞ (for the lower fluid).

w(z) is continuous at z =0.

The jump condition at the interface z = 0 between the fluids. is obtained by integrating equation (12) across interface z = 0 and is

$$\frac{n}{\in}\left(\rho_2 Dw_2 - \rho_1 Dw_1\right)_{z=0} + \left[\begin{array}{c} \left(\frac{\mu_2}{k_{12}} - \frac{\mu_2^{/}}{k_{12}}n\right)Dw_2 \\ -\left(\frac{\mu_1}{k_{11}} - \frac{\mu_1^{/}}{k_{11}}n\right)Dw_1 \end{array}\right]_{z=0}$$

$$+\frac{mn}{\in(\tau n+1)}\left[N_2 Dw_2 - N_1 Dw_1\right]_{z=0}$$

$$= -g\frac{k^2}{\in n}(\rho_2 - \rho_1)w_0,$$

(15)

where wo is the common value of w at z = 0.

Applying boundary conditions (i) and (ii), we can write

w1 = Aekz, (z < 0), (16)

w2 = Ae-kz, (z > 0), (17)

where the same constant A has been considered to ensure the continuity of w at z = 0. Applying condition (15) to the solutions (16) and (17), we obtain

$$\tau\left[\frac{\alpha_2+\alpha_1}{\varepsilon} - \left(\frac{\alpha_2\upsilon_2^{/}}{k_{12}} + \frac{\alpha_1\upsilon_1^{/}}{k_{11}}\right)\right]n^3 +$$

$$\left[\begin{array}{c} \frac{1}{\varepsilon} + \frac{m}{\varepsilon}\left(\frac{N_2+N_1}{\rho_2+\rho_1}\right) + \tau\left(\frac{\alpha_2\upsilon_2}{k_{12}} + \frac{\alpha_1\upsilon_1}{k_{11}}\right) - \\ \left(\frac{\alpha_2\upsilon_2^{/}}{k_{12}} + \frac{\alpha_1\upsilon_1^{/}}{k_{11}}\right) \end{array}\right]n^2$$

$$+\left[\left(\frac{\alpha_2\upsilon_2}{k_{12}} + \frac{\alpha_1\upsilon_1}{k_{11}}\right) - \frac{gk\tau}{\varepsilon}(\alpha_2 - \alpha_1)\right]n - \frac{gk}{\varepsilon}(\alpha_2 - \alpha_1) = 0, \quad (18)$$

where $\upsilon_{1,2} = \frac{\mu_{1,2}}{\rho_{1,2}}$, $\upsilon_{1,2}^{/} = \frac{\mu_{1,2}^{/}}{\rho_{1,2}^{/}}$, and

$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}$. So α1 + α2 = 1.

(a) Stable case $(v_2^{'} < \frac{k_{12}}{\varepsilon})$

For the potentially stable arrangement $\left(v_1^{'} < \frac{k_{11}}{\varepsilon}\right)$

for potential stable arrangement, there is no change in the coefficients of equation (18). Therefore, all the three roots of equation (18) are either real, negative or there is one real, negative root and the other two complex conjugates with negative real parts. The system is, therefore, stable in each case.

(b) Unstable case ($\alpha_2 > \alpha_1$)

For the potentially unstable arrangement

$$\frac{\alpha_2 \nu_2'}{k_{12}} + \frac{\alpha_1 \nu_1'}{k_{11}} > \frac{1}{\varepsilon} + \frac{m}{\varepsilon}\left(\frac{N_2 + N_1}{\rho_2 + \rho_1}\right) + \tau\left(\frac{\alpha_2 \nu_2}{k_{12}} + \frac{\alpha_1 \nu_1}{k_{11}}\right)$$

and $\frac{gk\tau}{\varepsilon}(\alpha_2 - \alpha_1) > \frac{\alpha_2 \nu_2}{k_{12}} + \frac{\alpha_1 \nu_1}{k_{11}}$ are real and negative . The constant term in equation (18) is negative. Equation (18) has a change of sign and hence allows one positive root. The occurrence of positive root implies instability of the system.

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