MULTI DECISION MAKING IN GENERALIZED SOFT-ROUGH MATRICES

S. VIJAYABALAJI

Abstract: Inspired by the theory of soft-rough matrices, we re-define the notion of soft-rough matrices by generalizing it. We also provide a multi-decision making theory in soft-rough matrices which generalizes the existing multi-decision theory.

Keywords: Soft sets, soft-rough sets, soft matrix, rough matrix, soft-rough matrix, generalized soft-rough matrix.

Introduction: Uncertainty occurs in our day to day life. Till 1980 there are several theories that dealt with uncertainties namely the theory of probability, fuzzy sets, intuitionistic fuzzy sets, vague sets and interval mathematics. In 1982 Z.Pawlak [8] introduced an interesting theory to deal with uncertainty namely rough sets. The significant of rough set theory is that the vagueness is represented by means of boundary region of a set instead of membership function. Using the operator int P.Pagliani and M.K.Chakraborty [7], Y.Y.Yao [18], I.Du¨ntsch and G.Gegida[3] generalized the theory of rough sets. S.Vijayabalaji and P.Balaji [15] constructed rough matrices using rough membership function and provided a decision theory on it. In 1999, D.A.Molodstov [6] stated the difficulties that arise in all the theories on uncertainty, including rough sets and gave a remedy by introducing the notion of soft sets. The speciality of this soft set is that it answers the difficulties that arises due to inadequency of the parameterization tool of the above theories. It is evident that any convenient parameterization such as real numbers, functions, words and so on can be used in soft sets.

Later P.K.Maji and A.R.Roy[5] applied this soft set theory to decision making problems. R.Wille[17] and G.Sambin[9] described about property systems and its relation with data analysis. It is evident to note that property system almost coincides with soft approximation space. Considering G to be the set of objects, M as a set of properties, D.Vakarelov [13] represented property system as a Boolean matrix G×M. N.Cagman [1] gave a uni-int decision making method and in [2] N.Cagman defined about soft matrices which are representation of soft sets. There are several advantages of this method, as it is easy to store and manipulate matrices using computer.S.Vijayabalaji and A.Ramesh[14] introduced the notion of product soft matrices and provided a decision theory on it. Feng Feng[4] discussed about the various hybrid models that emerges from soft sets and rough sets namely the theory of soft-rough sets, rough- soft sets and so on. Motivated by the theory of soft-rough sets, S.Vijayabalaji [16] introduced the notion of soft-rough matrices and provided a decision theory on it. Analytic Hierarchy Process (AHP) was introduced and developed by T.L.Saaty[10,11]. This AHP is used as an interesting structure to deal with complex decision making situations. AHP can be applied to find the critical weights of decision maker under group decision environment. It is interesting to note that P.K.Maji and A.R.Roy[5] used this AHP in assigning weights to solve group decision problems on soft matrices. S.A.Razak[10] improved their work and applied this AHP technique effectively by providing a decision theory on house selection problem.Inspired by the above theories, in this paper we generalize the notion of soft-rough matrices [16]. Further we provide a multi decision theory on this generalized soft-rough matrices using AHP technique by assigning weights to each parameter.

Preliminaries: This section recalls some interesting concepts which will be needed in the sequel. D.A. Molodtsov [6] defined soft sets in the following way.

Definition 2.1 [6] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $x \in A, F(x)$ may be considered as the set of x-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

N. Cagman entered into the new notion of soft matrix following the theory of soft sets as follows:

Definition 2.2 [2]. Let (F, A) be a soft set over U. Then, a subset of $U \times E$ is uniquely defined by $R_A = \{(u, e): e \in A, u \in f_A(e)\}$ which is called a relation form of (F, A). The characteristic function of R_A is written by $\chi_{R_A}: U \times E \rightarrow \{0,1\}$,

$$\chi_{R_A}(u, e) = \begin{cases} 1, (u, e) \in R_A \\ 0, (u, e) \notin R_A \end{cases}$$

If $U = \{u_1, u_2, ..., u_m\}$ and $E = \{e_1, e_2, ..., e_n\}$ and $A \subseteq E$, then the R_A can be presented by a table as in the following form:

R_A	e_1	e_2	$\cdots$	e_n
d_1	$\chi_{R_A}(u_1, e_1)$	$\chi_{R_A}(u_1, e_2)$	$\cdots$	$\chi_{R_A}(u_1, e_n)$
d_2	$\chi_{R_A}(u_2, e_1)$	$\chi_{R_A}(u_2, e_2)$	$\cdots$	$\chi_{R_A}(u_2, e_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$

$d_n \quad \chi_{R_A}(u_m, e_1) \quad \chi_{R_A}(u_m, e_2) \quad \cdots \quad \chi_{R_A}(u_m, e_n)$

If $a_{ij} = \chi_{R_A}(u_i, e_j)$, we define a matrix

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

which is called an $m \times n$ soft matrix of the soft set (F, A) over U.

Definition 2.3 [4]. Let $\Im = (F, A)$ be a soft set over U. Then the pair $P = (U, \Im)$ is called a soft approximation space. Based on P we define the following two operations:

$$\underline{apr_P}(X) = \left\{ \begin{matrix} u \in U : \exists a \in A, (u \in f(a), \\ f(a) \subseteq X \end{matrix} \right\},$$

$$\overline{apr_P}(X) = \left\{ \begin{matrix} u \in U : \exists a \in A, (u \in f(a), \\ f(a) \cap X \neq \phi \end{matrix} \right\},$$

assigning to every subset $X \subseteq U$ two sets $\underline{apr_P}(X)$ and $\overline{apr_P}(X)$ called the lower and upper soft rough approximations of X in P, respectively. Moreover,

$Pos_P(X) = \underline{apr_P}(X)$,

$Neg_P(X) = U - \overline{apr_P}(X)$,

$Bnd_P(X) = \overline{apr_P}(X) - \underline{apr_P}(X)$ are called the soft positive, soft negative and soft boundary regions of X, respectively. If $\overline{apr_P}(X) = \underline{apr_P}(X)$, X is said to be soft definable, otherwise X is called a soft rough set.

By definition, we immediately have that $X \subseteq U$is a soft definable set if $Bnd_P(X) = \emptyset$. Also it is clear that $\overline{apr_P}(X) \subseteq X$ and $\underline{apr_P}(X) \subseteq \overline{apr_P}(X)$ for all $X \subseteq U$. Nevertheless, it is worth nothing that $X \subseteq \overline{apr_P}(X)$does not hold in general. Let us recall the concepts of rough sets and rough member ship function by Z.Pawlak as follows:

Definition 2.4 [8]. The set X can be divided according to the basic sets of R, namely a lower approximation set and upper approximation set. Approximation is used to represent the roughness of the knowledge. Suppose a set $X \subseteq U$ represents a vague concept, the R-lower and R-upper approximations of X are defined by the equations

R-lower approximation of X:

$$R_*(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}$$

R-upper approximation of X:

$$R^*(X) = \bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}$$

R-boundary region of X:

$RN_R(X) = R^*(X) - R_*(X)$.

Also we can define

R-positive of X : $POS_R(X) = R_*(X)$,

R-negative of : $NEG_R(X) = U - R^*(X)$.

Definition 2.5 [8]. Rough Membership function defined by Pawlak is given by

$\mu_X^R: U \to [0,1]$, where $\mu_X^R(x) = \frac{|X \cap R(X)|}{|R(X)|}$.

Definition 2.6 [15]. Let (U, A) be a rough set over any approximations. Consider a subset of $U \times A$, uniquely defined by

$R_A = \{(u, a): u \in R_A(a), a \in A\}$

which is called indiscernibility relation form of (U, A).

We define a function

$\emptyset_{R_A}: U \times A \to [0,1]$ by

$$\emptyset_{R_A} = \begin{cases} 1; & if\ x \in R_*(X) \\ (0,1); & if\ x \in RN_R(X) \\ 0; & if\ x \in U - R_*(X) \end{cases}$$

Then, $R_M = \emptyset_{R_A}(u_i, a_i)$

$= [r_{ij}] = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix}$ is called an $m \times n$ rough matrix over approximation.

The notion of soft-rough matrices was introduced in [16] as follows:

Definition 2.7 [16]. Let $\Im = (F, A)$ be a soft set over U with the soft approximation space $P = (U, \Im)$. $\underline{apr_P}(X)$ and $\overline{apr_P}(X)$ be the two operations as in Definition 2.3. Let X be a soft-rough set with the soft positive ($Pos_P(X)$), soft negative ($Neg_P(X)$) and soft boundary $(Bnd_P(X))$ regions of X respectively. We now define a special function $C_{SR}: U \to \{0, 0.5, 1\}$ in this soft-rough set as follows:

$$C_{SR} = \begin{cases} 1, & if\ u \in Pos_P(X) \\ 0, & if\ u \in Neg_P(X) \\ 0.5, & if\ u \in Bnd_P(X) \end{cases}$$

A soft-rough matrix is a matrix whose elements are in C_{SR}.

That is $C_{SR} = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{bmatrix}$, where $c_{ij} \in C_{SR}$.

Definition 2.8 [16]. Let $[a_{ij}]$ and $[b_{ij}] \in C_{SR}$. Then the product of $[a_{ij}]$ and $[b_{ij}]$ is defined by $[a_{ij}] \wedge [b_{ij}] = Min\{a_{ij}, b_{ij}\}$.

Definition 2.9 [16]. Let $[a_{ij}]$ and $[b_{ij}] \in C_{SR}$. Then the sum of $[a_{ij}]$ and $[b_{ij}]$ is defined by $[a_{ij}] \vee [b_{ij}] = Max\{a_{ij}, b_{ij}\}$.

Definition 2.10[16]. The choice value of an object $C_k \in U$ is defined by

$C_k = Max\left\{Min\left(c(u_i, e_j)\right)\right\}$, where $c(u_i, e_j)$ are the entries of $[c_{ij}]$.

Definition 2.11[16].

Let $U = \{u_1, u_2, \dots \dots, u_m\}$ be an initial universe and $Max\{Min(d(h, e))\} = [u_{i1}]$. Then a subset of U can be obtained by using $[u_{i1}]$ as in the following way $opt_{[u_{i1}]} = \{u_i: u_i \in U, u_{i1} = \max(1\ or\ 0.5\ or\ 0)\}$, which is called an optimum set of U.

Generalized Soft-Rough Matrices

Inspired by the theory of soft rough matrices, we generalize Definition 2.7. as follows:

Definition 3.1. Let $\Im = (F, A)$ be a soft set over U with the soft approximation space $P = (U, \Im)$.

$\underline{apr}_P(X)$ and $\overline{apr_P}(X)$be the two operations as in Definition 2.3. Let X be a soft-rough set with the soft positive $(Pos_P(X))$, soft negative $(Neg_P(X))$ and soft boundary $(Bnd_P(X))$ regions of X respectively. We now define a special function $C_{SR}: U \to [0,1]$ in this soft-rough set as follows:

$$C_{SR} = \begin{cases} 1, \ if \ u \in Pos_P(X) \\ 0, \ if \ u \in Neg_P(X) \\ a, if \ u \in Bnd_P(X), a \in (0,1) \end{cases}$$

A generalized soft-rough matrix is a matrix whose elements are in C_{SR}.

That is $C_{SR} = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{bmatrix}$, where $c_{ij} \in C_{SR}$.

Remark 3.2. It is evident to note that each column of the matrix C_{SR} can be defined as elements from $Pos_P(X)$, $Neg_P(X)$, and $Bnd_P(X)$ of X respectively. When $a = 0.5$, this definition reduces to Definition 2.7.

To strengthen the above definition we present the following example.

Example 3.3. Consider the example of soft-rough set defined by Feng Feng [4] as follows.

Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ and $A = \{e_1, e_2, e_3, e_4\} \subset E$. Let $\mathfrak{I} = (F, A)$ be a soft set over U with the following table:

	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	0	0	0	0	1
e_2	0	0	1	0	0	0
e_3	0	0	0	0	0	0
e_4	1	1	0	0	1	0

For $X = \{u_3, u_4, u_5\} \subset U$, we have $\underline{apr}_P(X) = \{u_3\}$ and $\overline{apr_P}(X) = \{u_1, u_2, u_3, u_5\}$

Further $Pos_P(X) = \{u_3\}$, $Neg_P(X) = \{u_4, u_6\}$and $Bnd_P(X) = \{u_1, u_2, u_5\}$.

For this soft-rough set with our new function

$$C_{SR} = \begin{pmatrix} a & a & 1 & 0 & a & 0 \\ a & a & 1 & 0 & a & 0 \\ a & a & 1 & 0 & a & 0 \\ a & a & 1 & 0 & a & 0 \end{pmatrix}.$$

Hence $a \in (0,1)$. It is important to note that the first, second and fifth column elements of this matrix are the value derived from $Bnd_P(X)$, fourth and sixth row elements are from $Neg_P(X)$, third row elements are from $Pos_P(X)$.

Assume that a set of alternative and a set of parameters are given. We now try to proceed to our decision making theory in soft-rough matrices by constructing the following soft -rough Max-Min decision making algorithm which generalizes [16]:

Step 1: Constructing the comparison matrices in AHP

Step 2: Choose feasible subsets of the set of parameters and soft sets.

Step 3: Construct the $\underline{apr}_P(X_i)$ and $\overline{apr_P}(X_i)$.

Step 4: Find the $Pos_P(X_i)$, $Neg_P(X_i)$ and $Bnd_P(X_i)$.

Step 5: Find the soft-rough matrices.

Step 6: Calculating weight of criteria by every decision makers using AHP procedure.

Step 7: Input the criteria weight w_k and compute the values for each alternative and the construct the soft-rough matrices.

Step 8: Compute the Max-Min decision matrix of the product.

Step 9: Find an optimum set of U.

A Max-min Decision Making technique is applied to the above algorithm and is explained below by means of an example:

Example 3.4 [**Max-Min Decision Making**].

Suppose that U be the set of computer soft wares, say, $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$. Let E be a set of parameters, given by

$$E = \{\text{correctness, usability, efficiency, reliability, security, portability, maintainability}\}. = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}.$$

Step 1: To evaluate matrix for each criterion according to decision makers A, B, and C are constructed via pairwise comparison using T.L.Saaty nine point scale and are given as follows:$A =$

$$\begin{pmatrix} 1 & 1/5 & 4 & 1/7 & 1/9 & 4 & 1/6 \\ 5 & 1 & 5 & 1/7 & 1/9 & 5 & 1/6 \\ 1/4 & 1/5 & 1 & 1/7 & 1/9 & 1/3 & 1/6 \\ 7 & 7 & 7 & 1 & 1/9 & 7 & 7 \\ 9 & 9 & 9 & 9 & 1 & 9 & 9 \\ 1/4 & 1/5 & 3 & 1/7 & 1/9 & 1 & 1/6 \\ 6 & 6 & 6 & 1/7 & 1/9 & 6 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 3 & 1/4 & 1/6 & 1/7 & 3 & 1/5 \\ 1/3 & 1 & 1/4 & 1/6 & 1/7 & 2 & 1/5 \\ 4 & 4 & 1 & 1/6 & 1/7 & 4 & 1/5 \\ 6 & 6 & 6 & 1 & 1/7 & 4 & 1/5 \\ 7 & 7 & 7 & 7 & 1 & 7 & 7 \\ 1/3 & 1/2 & 1/4 & 1/6 & 1/7 & 1 & 1/5 \\ 5 & 5 & 5 & 1/6 & 1/7 & 5 & 1 \end{pmatrix}$$

and $C = \begin{pmatrix} 1 & 4 & 4 & 1/5 & 1/8 & 4 & 1/6 \\ 1/4 & 1 & 3 & 1/5 & 1/8 & 3 & 1/6 \\ 1/4 & 1/3 & 1 & 1/5 & 1/8 & 2 & 1/6 \\ 5 & 5 & 5 & 1 & 1/8 & 5 & 1/6 \\ 8 & 8 & 8 & 8 & 1 & 8 & 8 \\ 1/4 & 1/3 & 1/2 & 1/5 & 1/8 & 1 & 1/6 \\ 6 & 6 & 6 & 6 & 1/8 & 6 & 1 \end{pmatrix}$

Step 2: Suppose that the three friends Mr. F, Mr. G, and Mr. H together wants to buy a software according to their choice parameters $A = \{e_1, e_2, e_4, e_5\} \subseteq E$, $B = \{e_1, e_4, e_5, e_6\} \subseteq E$, and $C = \{e_1, e_3, e_5, e_6, e_7\} \subseteq E$. Let the soft set $(F, A), (G, B)$, and (H, C) be the evaluations of Mr. F, Mr. G, and Mr. H, respectively. Let X be the set of soft wares Mr. F is interested in, Y be the set of soft wares Mr. G is interested in, and Z be the set of soft wares Mr. H is interested in. Our aim is to find the most optimum choice of Mr. F, Mr.

G, and Mr. H to buy a software in a combined manner.

Tabular representation of the soft set (F,A)							
	u_1	u_2	u_3	u_4	u_5	u_6	u_7
e_1	1	0	1	0	0	1	0
e_2	0	0	1	0	0	0	0
e_3	0	0	0	0	0	0	0
e_4	1	1	1	0	0	0	0
e_5	0	0	1	1	0	1	1
e_6	0	0	0	0	0	0	0
e_7	0	0	0	0	0	0	0

Tabular representation of the soft set (G,B)							
	u_1	u_2	u_3	u_4	u_5	u_6	u_7
e_1	1	0	1	0	0	0	0
e_2	0	0	0	0	0	0	0
e_3	0	0	0	0	0	0	0
e_4	1	0	1	1	0	0	0
e_5	0	1	0	1	0	1	0
e_6	1	0	0	0	0	0	1
e_7	0	0	0	0	0	0	0

Tabular representation of the soft set (H,C)							
	u_1	u_2	u_3	u_4	u_5	u_6	u_7
e_1	0	1	0	1	0	1	0
e_2	0	0	0	0	0	0	0
e_3	1	0	1	0	1	0	0
e_4	0	0	0	0	0	0	0
e_5	1	0	1	1	1	0	0
e_6	0	1	1	0	0	0	0
e_7	0	0	1	1	0	0	0

Step 3: For $X = \{u_1, u_3, u_5, u_7\} \subset U$, we have $\underline{apr_P}(X) = \{u_3\}$ and $\overline{apr_P}(X) = \{u_1, u_2, u_3, u_6, u_7\}$. Thus, $\underline{apr_P}(X) \neq \overline{apr_P}(X)$ and (F,A) is a soft rough set. It is clear that $X = \{u_1, u_3, u_5, u_7\} \nsubseteq \overline{apr_P}(X) = \{u_1, u_2, u_3, u_6, u_7\}$.
For $Y = \{u_1, u_3, u_5, u_6\} \subset U$, we have $\underline{apr_P}(Y) = \{u_1, u_3\}$ and $\overline{apr_P}(Y) = \{u_1, u_2, u_3, u_4, u_6, u_7\}$. Thus, $\underline{apr_P}(Y) \neq \overline{apr_P}(Y)$ and (G,B) is a soft rough set. It is clear that $Y = \{u_1, u_3, u_5, u_6\} \nsubseteq \overline{apr_P}(Y) = \{u_1, u_2, u_3, u_4, u_6, u_7\}$.
For $Z = \{u_2, u_3, u_5, u_7\} \subset U$, we have $\underline{apr_P}(Z) = \{u_2, u_3\}$ and $\overline{apr_P}(Z) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. Thus, $\underline{apr_P}(Z) \neq \overline{apr_P}(Z)$ and (H,C) is a soft rough set. It is clear that $Z = \{u_2, u_3, u_5, u_7\} \nsubseteq \overline{apr_P}(Z) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.

Step 4: Moreover, we have $Pos_P(X) = \{u_3\}$, $Neg_P(X) = \{u_4, u_5\}$ and $Bnd_P(X) = \{u_1, u_2, u_6, u_7\}$. Similarly we have $Pos_P(Y) = \{u_1, u_3\}$, $Neg_P(Y) = \{u_5\}$ and $Bnd_P(Y) = \{u_2, u_4, u_6, u_7\}$.
$Pos_P(Z) = \{u_2, u_3\}$, $Neg_P(Z) = \{u_7\}$ and $Bnd_P(Z) = \{u_1, u_4, u_5, u_6\}$.

Step 5: The soft-rough matrices for (F,A), (G,B), and $(H.C)$are

$$[a_{ij}] = \begin{pmatrix} a & a & 1 & 0 & 0 & a & a \\ a & a & 1 & 0 & 0 & a & a \\ a & a & 1 & 0 & 0 & a & a \\ a & a & 1 & 0 & 0 & a & a \\ a & a & 1 & 0 & 0 & a & a \\ a & a & 1 & 0 & 0 & a & a \\ a & a & 1 & 0 & 0 & a & a \end{pmatrix},$$

$$[b_{ij}] = \begin{pmatrix} 1 & a & 1 & a & 0 & a & a \\ 1 & a & 1 & a & 0 & a & a \\ 1 & a & 1 & a & 0 & a & a \\ 1 & a & 1 & a & 0 & a & a \\ 1 & a & 1 & a & 0 & a & a \\ 1 & a & 1 & a & 0 & a & a \\ 1 & a & 1 & a & 0 & a & a \end{pmatrix}, \text{ and}$$

$$[c_{ij}] = \begin{pmatrix} a & 1 & 1 & a & a & a & 0 \\ a & 1 & 1 & a & a & a & 0 \\ a & 1 & 1 & a & a & a & 0 \\ a & 1 & 1 & a & a & a & 0 \\ a & 1 & 1 & a & a & a & 0 \\ a & 1 & 1 & a & a & a & 0 \\ a & 1 & 1 & a & a & a & 0 \end{pmatrix}$$

Step 6: Weight of each criterion is calculated using AHP and are obtained as:

$$[W_A] = \{e_1 = 0.0389, e_2 = 0.0658, e_3 = 0.0184, e_4 = 0.2262, e_5 = 0.5068, e_6 = 0.0252, e_7 = 0.1187\}^T$$

$$[W_B] = \{e_1 = 0.0454, e_2 = 0.0313, e_3 = 0.0733, e_4 = 0.2364, e_5 = 0.4602, e_6 = 0.0257, e_7 = 0.1277\}^T$$

$$[W_C] = \{e_1 = 0.0687, e_2 = 0.0426, e_3 = 0.0294, e_4 = 0.1198, e_5 = 0.4935, e_6 = 0.0242, e_7 = 0.2218\}^T$$

Step 7: Multiply each parameter with weight of criteria for each decision makers. We obtain:

$$\left[[a_{ij}] \times [W_A]\right] = [A_{ij}] = \begin{pmatrix} 0.0389a & 0.0389a & 0.0389 & 0 & 0 & 0.0389a & 0.0389a \\ 0.0658a & 0.0658a & 0.0658 & 0 & 0 & 0.0658a & 0.0658a \\ 0.0184a & 0.0184a & 0.0184 & 0 & 0 & 0.0184a & 0.0184a \\ 0.2262a & 0.2262a & 0.2262 & 0 & 0 & 0.2262a & 0.2262a \\ 0.5068a & 0.5068a & 0.5068 & 0 & 0 & 0.5068a & 0.5068a \\ 0.0252a & 0.0252a & 0.0252 & 0 & 0 & 0.0252a & 0.0252a \\ 0.1187a & 0.1187a & 0.1187 & 0 & 0 & 0.1187a & 0.1187a \end{pmatrix}$$

$$\left[[b_{ij}]\times[W_B]\right]=[B_{ij}]=\begin{pmatrix}0.0454 & 0.0454a & 0.0454 & 0.0454a & 0 & 0.0454a & 0.0454a\\ 0.0313 & 0.0313a & 0.0313 & 0.0313a & 0 & 0.0313a & 0.0313a\\ 0.0733 & 0.0733a & 0.0733 & 0.0733a & 0 & 0.0733a & 0.0733a\\ 0.2364 & 0.2364a & 0.2364 & 0.2364a & 0 & 0.2364a & 0.2364a\\ 0.4602 & 0.4602a & 0.4602 & 0.4602a & 0 & 0.4602a & 0.4602a\\ 0.0257 & 0.0257a & 0.0257 & 0.0257a & 0 & 0.0257a & 0.0257a\\ 0.1277 & 0.1277a & 0.1277 & 0.1277a & 0 & 0.1277a & 0.1277a\end{pmatrix}$$

And

$$\left[[c_{ij}]\times[W_C]\right]=[C_{ij}]=\begin{pmatrix}0.0687a & 0.0687 & 0.0687 & 0.0687a & 0.0687a & 0.0687a & 0\\ 0.0426a & 0.0426 & 0.0426 & 0.0426a & 0.0426a & 0.0426a & 0\\ 0.0294a & 0.0294 & 0.0294 & 0.0294a & 0.0294a & 0.0294a & 0\\ 0.1198a & 0.1198 & 0.1198 & 0.1198a & 0.1198a & 0.1198a & 0\\ 0.4935a & 0.4935 & 0.4935 & 0.4935a & 0.4935a & 0.4935a & 0\\ 0.0242a & 0.0242 & 0.0242 & 0.0242a & 0.0242a & 0.0242a & 0\\ 0.2218a & 0.2218 & 0.2218 & 0.2218a & 0.2218a & 0.2218a & 0\end{pmatrix}$$

Step 8: The entire decision theory is based on choosing the value of "a". It is the key factor in deciding the output.

Consider $a=0.5$. Then

$$[A_{ij}]\wedge[B_{ij}]\wedge[C_{ij}]=Min\{[A_{ij}],[B_{ij}],[C_{ij}]\}=\begin{pmatrix}0.0195 & 0.0195 & 0.0389 & 0 & 0 & 0.0195 & 0\\ 0.0213 & 0.0157 & 0.0314 & 0 & 0 & 0.0157 & 0\\ 0.0092 & 0.0092 & 0.0184 & 0 & 0 & 0.0147 & 0\\ 0.0599 & 0.1131 & 0.1198 & 0 & 0 & 0.0599 & 0\\ 0.2468 & 0.2301 & 0.4602 & 0 & 0 & 0.2301 & 0\\ 0.0121 & 0.0126 & 0.0242 & 0 & 0 & 0.0121 & 0\\ 0.0594 & 0.0594 & 0.1187 & 0 & 0 & 0.0594 & 0\end{pmatrix}$$

So, $Max\left\{Min\{[A_{ij}],[B_{ij}],[C_{ij}]\}\right\}$

$$=\left\{\{u_3\},\{u_3\},\{u_3\},\{u_3\},\{u_3\},\{u_3\},\{u_3\}\right\}$$

$=\{u_3\}$.

(The choice of $\{u_3\}$ is based on choosing maximum element in each row of the matrix $\{[A_{ij}],[B_{ij}],[C_{ij}]\}$. The final value $\{u_3\}$ is based on the repetation of the values of $u_i, i=1,2,\ldots\ldots,7$).

Step 9: Hence the optimum choice of Mr. F, Mr. G, and Mr. H jointly to buy the Computer software is u_3.

Conclusion: A new decision making technique is provided in generalized soft-rough matrices by taking AHP technique as a tool. This generalizes the existing technique of decision theory on soft-rough matrices that was proposed in [16].

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Department of Mathematics, University College of Engineering Panruti,
Panruti-607106, Tamilnadu, India.
E-mail: balaji1977harshini@gmail.com