KALUZA-KLEIN ANISOTROPIC UNIVERSE WITHOUT BIG SMASH DRIVEN BY LAW OF VARIATION OF HUBBLE'S PARAMETER

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Abstract: Kaluza-Klein universe has been studied with ω<-1 without Big Smash driven by Hubble's law of variation of parameter. It is investigated that if cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$, (p and ρ being pressure and energy density respectively) as well as generalized Chaplygin gas simultaneously, Big Rip OR Big Smash problem does not arise even for equation of state parameter $\omega < -1$. Also the scale factor for the future universe found to be regular for all time.

Keywords: Big Smash, Dark Energy , Kaluza-Klein Universe.

Introduction: The possibility that the world may have more than four dimensions is due to Kaluza [1] and Klien [2], who used one extra dimension to unify Maxwell's theory of electromagnetism and Einstein's gravitational theory by adding fifth dimension. Due to its potential function to unify the fundamental interaction, Kaluza-Klein theory has been regarded as a candidate of fundamental theory. In recent years, the theory has been revived in modern physics such as in string theory [3], in super gravity theory [4] and in super string theory [5]. This idea has been worked by several ansatzs who have found models for various phenomenon in practical physics and cosmology using five or more dimensions [6-10]. Overduin and Wesson [11] have presented an excellent review of Kaluza-Klein theory and higher dimensional unified theories in which the cosmological and astrophysical implications of extra dimensions have been discussed. Wesson and Liu [12] have investigated the cosmological constant problem in Kaluza-Klein cosmology. Adhav *et al.* [13] have studied the string cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model [14]-[18]. Recent observations indicate that our universe is currently expanding with an acceleration [19]. The main responsible candidate for this acceleration is exotic type of dark energy with negative pressure having equation of state (EoS) parameter ω close to -1. When $\omega < -1$, the dark energy model is phantom and when $-1 < \omega < \frac{-1}{3}$, it is quintessence. The dark energy models in which EoS parameter ω cross the phantom divide $\omega = -1$ both sides then termed as quintom. "Phantom Cosmologies" (ω could be slightly less than -1) often have the property that they end in Big Smash, a final singularity in which the universe is destroyed in a finite proper time by excessive expansion. Recently new scenario to avoid the future singularity has been proposed by Framptom *et al.* [20]. In this scenario, ω is less than -1 so that the dark energy density increases with time, but ω approaches asymptotically and sufficiently rapidly that the singularity is avoided [21], [22]. Caldwell [21], [22] proposed the phantom dark energy model exhibiting cosmic doomsday of the future universe, cosmologist started making efforts to avoid this problem using $\omega < -1$, [23]. Sahni and Shtanov [24] has obtained well behaved expansion of universe without Big Rip problem with $\omega < -1$ they have that acceleration is transient phenomenon in current universe and future universe will reenter matter dominated deceleration phase. Shrivastava [25] have obtained future universe with $\omega < -1$ without Big Smash. He considered FRW model with Chaplygin gas as good source of dark energy for having negative pressure given by

$$p = \frac{-A}{\rho}, \quad (1)$$

where p and ρ being pressure and energy density respectively in co-moving frame of reference such that $A > 0$ and $\rho > 0$. Bertolami *et al.* [26] have found that generalized Chaplygin gas (GCG) is better fit for latest supernova observations . The equation of state is modified in case of GCG as

$$p = \frac{-A}{\rho^{\frac{1}{\alpha}}}, \quad (2)$$

where $1 \le \omega < \infty$.

In this paper, Kaluza-Klein space time is considered where the dark energy behaves like generalized Chaplygin gas obeying equation of state is $\omega(t) = -\frac{A}{\rho^{\frac{1+\alpha}{\alpha}}}$. In Section 2, the model and field equations have been presented. In Section 3, the law of variation of Hubble's parameter have been used to find the solution. In last section 4, concluding remarks have been expressed.

2. Metric and Field Equations:

We consider the Kaluza-Klein universe given by ,

$$ds^2 = -dt^2 + a^2(t)\,(dx^2 + dy^2 + dz^2) + b^2(t)\,du^2 \quad (3)$$

where $a(t)$, $b(t)$ are scale factors and are functions of t only, m is a constant and t is a cosmic time.

The energy momentum tensor is given by ,

$$T_j^i = (\rho + p)u^i u_j + pg_j^i, \quad (4)$$

where p is isotropic pressure, ρ is energy density and g_j^i is metric tensor.

With the help of above equations, matter tensor is given by

$$T_j^i = (-\rho, p, p, p, p). \quad (5)$$

The Einstein's field equations are,

$$R_j^i - \frac{1}{2} g_j^i R = -T_j^i, \quad (6)$$

where R_j^i is Ricci tensor , R is Ricci scalar, T_j^i is energy momentum tensor for bulk viscous cosmology.

From equations (3), (4), (5), Einstein's field equation (6) gives

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -p, \quad (7)$$

$$\frac{\dot{a}\dot{b}}{ab} + 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p, \quad (8)$$

$$3\frac{\dot{a}^2}{a^2} + 3\frac{\dot{a}\dot{b}}{ab} = \rho. \quad (9)$$

Here $\left(\bullet\right)$ dot represents the differentiation with respect to t.

The energy conservation equation is given by

$$T_{;j}^{ij} = 0, \quad (10)$$

where $T_{;j}^{ij} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^j}[T^{ij}\sqrt{-g}] + T^{ik}\Gamma_{jk}^i$,

which simplifies to

$$\dot{\rho} + (p + \rho)\left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) = 0, \quad (11)$$

where $\dot{\rho}$ is differentiation of ρ with respect to t.

Define average scale factor as ,

$$B = (a^3 b)^{\frac{1}{4}} \quad (12)$$

so that we have ,

$$4\frac{\dot{B}}{B} = \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right). \quad (13)$$

Using equation (13), equation (11) can be written as

$$\dot{\rho} = -4\frac{\dot{B}}{B}(p + \rho). \quad (14)$$

Using equation (2), equation (14) can be written as

$$\dot{\rho} = -4\frac{\dot{B}}{B}\left(\frac{\rho^{\frac{1+\alpha}{\alpha}} - A}{\rho^{\frac{1}{\alpha}}}\right), \quad (15)$$

where A is a constant.

Integrating equation (15) we get

$$\rho^{\frac{1+\alpha}{\alpha}} = A + [\rho_0^{\frac{1+\alpha}{\alpha}} - A]\left(\frac{B_0}{B}\right)^{\frac{4(1+\alpha)}{\alpha}}. \quad (16)$$

In present model, the equation state parameter for dark energy is ratio of pressure p and energy density ρ given by

$$\omega = \frac{p}{\rho}. \quad (17)$$

We assumed that the dark energy behaves like generalized Chaplygin gas obeying equation of state as in equation (2).

Equations (2) and (17) leads to

$$\omega(t) = -\frac{A}{\rho^{\frac{1+\alpha}{\alpha}}}. \quad (18)$$

At present time $t = t_0$ $\rho(t_0) = \rho_0$ and $\omega(t_0) = \omega_0$, equation (18) can be put as

$$A = -\omega_0 \rho_0^{\frac{1+\alpha}{\alpha}}. \quad (19)$$

Using equation (19) in equation (16), we get

$$\rho = \rho_0 \left[-\omega_0 + (1+\omega_0)\left(\frac{B_0}{B}\right)^{\frac{4(1+\alpha)}{\alpha}}\right]^{\frac{\alpha}{(1+\alpha)}} \text{,with}$$

$$\omega < -1. \quad (20)$$

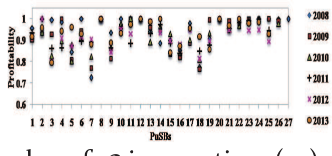
In homogeneous model of universe, a scalar field $\phi(t)$ with potential $v(\phi)$ has energy density

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + v(\phi). \quad (21)$$

and pressure

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - v(\phi). \quad (22)$$

Using equations (2), (19) (21)and (22), we get



. (23)

Putting value of ρ in equation (23), we get

$$\dot{\phi}^2(t) = \frac{\rho_0(1+\omega_0)\left(\frac{B_0}{B}\right)^{\frac{4(1+\alpha)}{\alpha}}}{\left[-\omega_0 + (1+\omega_0)\left(\frac{B_0}{B}\right)^{\frac{4(1+\alpha)}{\alpha}}\right]^{\frac{1}{(1+\alpha)}}}. \tag{24}$$

From equation (24), it is clear that if $\omega_0 > -1$, $\dot{\phi}^2(t) > 0$ giving positive kinetic energy and if $\omega_0 < -1$, $\dot{\phi}^2(t) < 0$ giving negative kinetic energy. It is important to note that similar results are obtained by Hoyle and Naralikar for C-field with negative kinetic energy for steady state theory of the Universe [27[-[28].

3. Law of variation of Hubble's Parameter

The Hubble's parameter and deceleration parameter plays important roll to show the physical relevancy of the model in cosmology. The first sets the present time scale of expansion while second gives that the present stage is speeding up instead of slowing down.

We define, the generalized mean Hubble's parameter H as

$$H = \frac{1}{4}\left(H_1 + H_2 + H_3 + H_4\right), \tag{25}$$

where $H_1 = H_2 = H_3 = \frac{\dot{a}}{a}$, $H_4 = \frac{\dot{b}}{b}$ are directional Hubble's factor in the directions of x, y, z and u-axes respectively.

Using equations (12) and (25), we get

$$H = \frac{1}{4}\left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right). \tag{26}$$

Since the line element is completely characterized by Hubble's parameter H, therefore let us consider that mean Hubble's parameter H is related to average scale factor B by following relation as stated by Berman[29] for FRW model by

$$H = DB^{-n}, \tag{27}$$

where D and n are positive constants..

The deceleration parameter is given by

$$q = -\frac{B\ddot{B}}{\dot{B}^2}. \tag{28}$$

From equations (26) and (27), we get

$$\dot{B} = DB^{-n+1} \tag{29}$$

$$\ddot{B} = -D^2((n-1)B^{-2n+1}. \tag{30}$$

From equations (28), (29), (30), we get

$$q = (n-1). \tag{31}$$

Equation (31) shows that deceleration parameter q is constant. If $n > 1$, q is positive gives standard model and if $0 \le n < 1$, $-1 \le q < 0$ which shows inflation. However the current observations of SNeIa and CMBR favor accelerating model, i.e. $q < 0$.

From equations (26) and (27), we have

$$\frac{\dot{B}}{B} = DB^{-n}. \tag{32}$$

On integrating equation (24), we get

$$B = (nDt + c_0)^{\frac{1}{n}}, \tag{33}$$

where c_0 is a constant of integration.

From equation (33), it is clear that $B(t) \to \infty$ as $t \to \infty$ shows universe is accelerating which support observational evidences of Ia Supernova [30, 31] and WMAP [32, 33]. Also this model is free from finite time future singularity.

In this model Hubble's distance is given by

$$H^{-1} = (nt + D_0), \tag{34}$$

where $D_0 = \frac{c_0}{D}$ is constant.

From equation (34) we have, if $t \to 0$ then $H^{-1} \to D_0$ and if $t \to \infty$ then $H^{-1} \to \infty$. Thus in present case galaxies will not disappear when $t \to \infty$ unlike the phantom models with future singularity, where galaxies are expected to vanish near future singularity.

The horizon distance is obtained as

$$d_H = B(t)\int_0^t \frac{dt}{B(t)}. \tag{35}$$

From equations (33) and (35), we get

$$d_H = \frac{1}{D(n-1)}\left[(nDt + c_0) - D'(nDt + c_0)^{\frac{1}{n}}\right], \tag{36}$$

which gives

$$d_H > B(t), \text{ for } t > \frac{(1+D')^{\frac{n}{(n-1)}} - c_0}{nD}.$$

That is horizon grows more rapidly than scale factor which implies colder and darker universe. It is like flat universe dominated by dark energy.

From equations (20) and (33), the energy density is given by

$$\rho = \rho_0\left[-\omega_0 + (1+\omega_0)\left(\frac{B_0}{(nDt + c_0)^{\frac{1}{n}}}\right)^{\frac{4(1+\alpha)}{\alpha}}\right]^{\frac{\alpha}{(1+\alpha)}}$$

. (37)

Conclusion: Kaluza-Klein anisotropic universe have studied by the law of variation of Hubble's parameter stated by Berman [29] . It is found that when cosmic dark energy behaves like fluid as well as generalized Chaplygin gas simultaneously then Big Smash problem does not arise unlike other phantom models. It is conclude that $\dot{\phi}^2(t) > 0$ representing positive kinetic energy if $\omega_0 > -1$ and $\dot{\phi}^2(t) < 0$ representing negative kinetic energy if $\omega_0 < -1$ for the quintessence and phantom fluid dominated universe respectively. The results are analogous with Hoyle and Narlikar C-field cosmology with negative kinetic energy for steady state universe. Further the model is free from singularity as $B(t) \to \infty$ for $t \to \infty$. Also the horizon distance H^{-1} tends to finite value and diverge for large values of t. Therefore at present time galaxies will not disappear when $t \to \infty$ avoiding Big Smash singularity. The energy density increases with time indicating that the model has not finite future singularity. The results in our model as analogous with the results obtained by Shrivastav [34] and Yadav [35].

Referenes :

1. T. Kaluza, Sitz. Preuss, "Zum Unitätsproblem in der Physik". *Acad. Wiss. Phys. Mth., KI*. 966, (1921).
2. O. Klein, "Quantentheorie und fünfdimensionale Relativitäts theorie", *Phys.*, 37 (12), 895, (1926).
3. J. Polchinski, "String Theory", Vol. I and II (*Cambridge Univ. Press*, 1998).
1. 4. M. J. Duff, B. E. W. Nilsson, C. N. Pope, "Kaluza-Klein supergravity" *phys. Rep*. 130, 1 (1986).
4. M. B. Green, J. H. Schwarz, E. Witten, "Super String Theory" (*Cambridge Uni. Press*, 1987).
5. L. K. Chi, " New cosmological models in the five-dimensional space-time-mass gravity theory" *Gen. Rel. Grav*., 22, 1347 (1990).
6. A. A. Coley, " Higher dimensional vacuum cosmologies" *Astro. Phys. J*., Vol. 427, No.2, pp 585-602, (1994).
7. T. Fukui, " 5-D geometrical property and 4-D property of matter " *Gen. Rel. Grav*., 25, pp 931-938, (1993).
8. H. Liu, P. S. Wesson, "Cosmological solutions and their effective properties of matter in Kaluza-Klein theory". Int. J. Mod. Phys. D, 3, pp 627-637, (1994).
9. J. Ponce de Leon, "Cosmological Models in a Kaluza-Klein Theory With Variable Rest Mass" *Gen. Rel. Grav*., 20, pp 539-550,(1988).
10. J. M. Overduin, P. S. Wesson, "Kaluza Klein gravity" *Phys. Rep*., Vol., 283, pp 303-378, (1997).
11. H. Liu, P. S. Wesson, "The Cosmological Constant Problem and Kalu Klein Theory" *Int. J. Mod. Phys. D* 10 , 905-912, (2001).
12. K. S. Adhav, A. S. Nimkar, M. V. Dawande, "String cloud and domain walls with quark matter in n-dimensioanl Kaluza-Klein cosmological model" *Int. J. Theo. Phys*., 47, pp. 2002-2010, (2008).
13. G. S. Khadekar, *et al*., "Domain wall with strength quark matter in Kaluza-Klien type cosmological model" *Int. J. Theo. Phy.*, 48, 2550-2557 (2009).
14. K. S. Adhav, A. S. Bansod, R. P. Wankhade, H. G. Ajimire, "Kaluza-Klein cosmological model with anisotrophic dark energy" *Mod. Phy. Lett.A*, 26, 10, 739-750 (2001).
15. D. R. K. Reddy, R. L. Naidu, B. Sattanarayana, "Kaluza-Klein cosmological model in f(R,T) Gravity" *Int. J. Theo. Phy.,* doi 10.1007/s 10773.012-1203-x.
16. Shri Ram, Priyanka Kumari, "Some cosmological constant models in f(R,T) Gravity theory", *Astro. Spa. Sci*. 2013 ; Doi.10.1007-s10509-013-1517-z.
17. H. R. Ghate, A. S. Patil, ";Kaluza-Klein type dissipative future universe without Big Rip" *Prespacetime Journal* , Vol. 4, No 5 (2013).
18. H. R. Ghate, A. S. Patil, "Higher dimentional dissipative future universe without Big Rip", IJSER, Vol. 4, Issue 12,(2013).
19. P. H. Framtom, K. J. Ludwick, R. J. Schemer, "Models for literary Dark Energy" *Phys. Rev. D* 84, 063 003 (2011) ; arxiv,1106-4996/Astro-ph.co.
20. R. R. Caldwell, "A phantom menace ? cosmological consequences of Dark Energy component with super negative equation of state". ,*Phys. Lett,. B* 545, 23-29, (2002).
21. R. R. Caldwell, M. Kamionkowski, N. N. Weinberg, "Phantom energy and cosmic doomsday" *Phys. Rev. Lett*. 91 (2003), 071301.
22. B. Mclnnes, "The dS/CFT Correspondence and the Big Smash" *JHEP* 0208, 029 (2002).
23. V. Sahni, YUV Shtanov, "Braneworld models of dark energy," *JCAP* 0311, 014 (2003).; arXiv:astro-ph/0202346.
24. S. K. Shrivastav, " Future Universe with $\omega < -1$ without Big Smash" *Phys. Lett. B* 619,1-4(2005).
25. O. Bertolami,Anjan Ananda Sen, S.Sen,P.T.Silva."Latest supernova data in the framework of the generalized Chaplygin gas model" *Mon. Not. R. Astron. Soc*. 35 , 329 (2004));

astro- ph/0402387
26. F. Hoyle, J. V. Naralikar, "A New Model for the Expanding Universe," *MNRAS* 108, 372-82 (1948).
28. M. S. Berman, "A special law of variation for Hubble's parameter," Nuovo Cimento B, 74, 2, 82, (1986), .Doi:10.1007/BF02721676.
29. A. G. Riess, *et al*., " Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant" .*Astron. J.*, 116 , 1009 (1998).
30. S. Perlmutter, *et al*. "Measurements of Ω and Λ from 42 High-Redshift Supernovae *Astrophys. J.* 517, 565, (1999).
31. D. N. Spergel, *et al*, "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters" *Astrophys. J. Suppl*. 148, 175, (2003).
27. F. Hoyle, J. V. Narlikar, "A Different approach to cosmology" *Cambridge University Press*, Cambridge, (2000).
32. L. Page, *et al.*, "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Beam Profiles and Window Functions" *ApJS*, 148, 39, (2003).
33. S. K. Shrivastav, " Future Universe with $\omega < -1$ without Big Smash" *Phys. Lett. B* 619,1-4, (2005).
34. A. K. Yadav, "Bianchi type-I anisotropic universe without Big Smash driven by law of variation of Hubble's parameter", Rom. J. Phys., 56, 609, (2011).; arXiv:1005.0537v1

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