CHARACTERISTIC FUNCTION IN SEMI NEAR-RINGS

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Abstract: Semi near-ring is a generalization of both a semiring and a near-ring. In this paper s-ideal of semi near-ring and a characteristic function is considered and it is proved that the characteristic function is a fuzzy s-ideal of semi near-ring. If μ is a fuzzy s-ideal of a semi near-ring S, then it is proved that the level set μ_t, $t \leq \mu(0)$ is an s-ideal of S. If μ is a fuzzy subset of a semi near-ring S, and μ_t is an s-k-ideal for each $t \in [0, 1]$, $t \leq \mu(0)$, then it is proved that μ is a fuzzy s-k-ideal of S.

Keywords: Characteristic function, Semi near-ring, fuzzy s-ideal, fuzzy s-k-ideal.

Introduction: A seminear-ring S is an algebraic system with two binary operations: usual addition and usual multiplication such that S forms a semigroupwith respect to both the operations, and satisfies theright distributive law.For preliminary definitions and results on semi near-rings we refer to [1],[2,3],[8], [9]

Theorem 2.1: Let I be an s- ideal of a seminear-ring S. Then the characteristic function λ_I defined by

$$\lambda_I(x) = \begin{cases} 1 \text{ if } & x \in I \\ 0 & \text{otherwise} \end{cases}$$

is a fuzzy s-ideal of S. Further, if I is an s-k-ideal, then λ_I is a fuzzy s-k-ideal of S.

Proof: Suppose I is an s-ideal of S. First we show that λ_I is a fuzzy s-ideal of S.

Take x, y $\in$ S. If x, y $\in$ I, then x + y $\in$ I (since I is an s-ideal of S). Now,

$$\lambda_I(x+y) = 1 \geq \min\{1, 1\}$$
$$= \min\{\lambda_I(x), \lambda_I(y)\}$$

Suppose $x \notin I$ and $y \in I$. If $x + y \in I$ then it is clear from above.

$$\text{If } x + y \notin I, \text{ then } \lambda_I(x+y) = 0 = \min\{0,1\}$$
$$= \{\lambda_I(x), \lambda_I(y)\}$$

If $x \notin I$, $y \notin I$, then $x + y \notin I$ or $x + y \in I$.

Now, from each of these cases we get $\lambda_I(x+y) \geq \min\{\lambda_I(x), \lambda_I(y)\}$

Also, take $x \in I$, $y \in S \Rightarrow xy \in I$. Now

$$\lambda_I(xy) = 1$$
$$= \max\{1, 0\}$$
$$= \max\{\lambda_I(x), \lambda_I(y)\}$$

Suppose I is an s-k-ideal of S. We show that λ_I is a fuzzy s-k-ideal of S.

Take $y \in I$ and every $x \in S$, $x + y \in I$, since I is an s-k-ideal, we have $x \in I$, Further,

$$\lambda_I(x) = 1$$
$$\geq \min\{1, 1\}$$
$$= \min\{\mu(y), \mu(x+y)\}$$

Therefore, λ_I is a fuzzy s-k- ideal of S.

Theorem 2.2: Let μ be a fuzzy s-ideal of a seminear-ring S. Then the level set μ_t, $t \leq \mu(0)$ is an s-ideal of S.

Proof. Take x, y $\in \mu_t$, This implies $\mu(x) \geq t$ and $\mu(y) \geq t$.

Now, $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$ (since μ is a fuzzy s-ideal)

$\geq \{t, t\} = t$.

This implies $x + y \in \mu_t$

For any $s \in S$, we have $\mu(sx) \geq \mu(x)$ (Since μ is a fuzzy s-ideal)

$= t$.

This implies that $sx \in \mu_t$. Similarly, $\mu(xs) \geq \mu(x) \geq t$. This implies that $xs \in \mu_t$.

Therefore, μ_t is an s-ideal of S.

Theorem 2.3: Let μ be a fuzzy subset of a seminear-ring S. If μ_t is an s-k-ideal for each $t \in [0, 1]$, $t \leq \mu(0)$, then μ is a fuzzy s-k-ideal of S.

Proof: In view of Theorem 2.2, it is enough to show that $\mu(x + y) = \mu(0)$,
$\mu(y) = \mu(0)$ implies that $\mu(x) = \mu(0)$.

Write $\mu_o = \{x \in S \mid \mu(x) = \mu(0)\}$.

Suppose $\mu(x + y) = \mu(0)$, $\mu(y) = \mu(0)$. This implies $x + y \in \mu_o$, $y \in \mu_o$.

Since μ_o is an s-k-ideal we have $x \in \mu_o$ implies that $\mu(x) = \mu(0)$.

Converse is not true:

Let $S = \mathbb{Z}^+ \cup \{0\}$. Then S is a seminear-ring with usual addition and multiplication.

Define $\mu: S \to [0,1]$ by

$$\mu(x) = \begin{cases} 1 \quad \text{if} \quad x \in \langle 2 \rangle \\ \frac{1}{2} \quad \text{if} \quad x \in \langle \{2, 3\} \rangle - \langle 2 \rangle \\ 0 \quad \text{elsewhere} \end{cases}$$

Now for any x, y ∈ S, $\mu(x + y) = \mu(0)$, $\mu(y) = \mu(0)$ $\Rightarrow \mu(x) = \mu(0)$.

Therefore, μ is an s-k- ideal of S.

Take t = ½. Consider μ_t. Observe that $\mu(0) = 1$.
Then $\mu_{1/2} = <\{2, 3\}>$. Now $r = 3 \in \mu_{1/2}$, $1 \in S$. We have $r = 3 + 1 \in \mu_{1/2}$, (since $\mu(4) = 1 = \mu(0)$)
Here $\mu(4) = 1 > 1/2 \Rightarrow 4 \in \mu_{1/2}$. But $1 \notin \mu_{1/2}$ (since $\mu(1) = 0 \neq \mu(0)$)
Therefore, $\mu_{1/2} = <\{2, 3\}>$ is not an s-k-ideal of S.

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