EFFECTS OF HEAT TRANSFER ON THE PERISTALTIC MHD FLOW OF A BINGHAM FLUID THROUGH A POROUS MEDIUM IN AN INCLINED CHANNEL

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Abstract: In this investigation, effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channel is discussed. Analytical solution is carried out for long wavelength and low Reynolds number considerations. The flow is investigated in a wave frame of reference moving with velocity of the wave. Closed form expressions have been obtained for the axial velocity component in fixed frame by using Adomian decomposition method. The expressions for pressure rise and frictional force are obtained. Lastly, the physical features of various parameters have been discussed through graphs.

Keywords: Adomian decomposition method, Bingham fluid, Peristaltic flow, and heat transfer analysis.

Introduction: Peristaltic transport is a well known process of a fluid transport which is induced by a progressive wave of area contraction or expansion along the length of distensible tube containing the fluid. It is used by many systems in the living body to propel or to mix the contents of a tube. The mechanics of peristalsis has been examined by a number of investigators. Latham [1] was probably the first to investigate the mechanism of peristalsis in relation to mechanical pumping. Peristaltic flow of non-Newtonian fluids in a tube was first studied by Raju and Devanathan [2]. Effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid when the tube wall is provided with non-erodible porous lining is made by Rathod and Channakote [32]

The study of fluid flows through and past porous medium has attracted much attention recently. This is primarily because of numerous applications of flow through porous medium, such as storage of radioactive nuclear waste materials transfer, separation processes in chemical industries, filtration, transpiration cooling, transport processes in aquifers and ground water pollution. Flow through a porous medium has been studied by A.E. Scheidegger [3]. Some studies about this point have been made by Varshney [4] and EL-Dave and EL-Mohendis [5]. Elshehawey et al., [6] studied peristaltic motion of a generalized Newtonian fluid through a porous medium. Ramireddy et al. [7] studied peristaltic transport of a conducting fluid in an inclined asymmetric channel. Peristaltic motion of a generalized Newtonian fluid under the effect of a transverse magnetic field is studied by Elshehawey et al.,[8]. Rathod and Asha [9] studied Peristaltic Transport of Couple Stress Fluids in a Uniform and Non-Uniform Annulus through Porous Media. Satyanarayana et al.,[14] studied Hall current effect on magnetohydro dynamics Free-convection flow past a semi-infinite vertical porous plate with mass transfer. A study of ureteral peristalsis in cylindrical tube through porous medium is made by Rathod and Channakote [10]. Rathod and Pallavi [36] studied The effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall.

Srinivas and Pushparaj [22] have investigated the peristaltic transport of MHD flow of a viscous incompressible fluid in a two dimensional asymmetric inclined channel. However, the interaction of peristalsis and heat transfer has not received much attention, which may become highly relevant and significant in several industrial processes. Also, thermodynamic aspects of blood may become significant in processes like oxygenation and hemodialysis [23]-[27] when blood is drawn out of the body. Slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel is made by Rathod and Channakote [12]. Lately, the combined effects of magneto hydrodynamics and heat transfer on the peristaltic transport of viscous fluid in a channel with compliant walls have been discussed by Mekheimer and Abdelmaboud and co-workers [28]-[30].

The subject of Adomian Decomposition Method (ADM) is a relatively new approach, which provides an analytic approximation to linear and non-linear problems. The method is quantitative rather than qualitative. It is analytic and requires neither linearization nor perturbation. It is also continuous with no resort to discretization. The method provides the solution as an infinite series in which each term can be determined. Rathod and Laxmi [11] have investigated Slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method.

In view of these, effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channelunder the assumptions of long wavelength and low Reynolds number has been studied. The expressions for the velocity field and pressure gradient are obtained analytically. The effects of various emerging parameters on the pumping characteristics are discussed through graphs in detail.

2. Mathematical formulation: Consider the peristaltic pumping of a conducting Bingham fluid in a channel of half-width a .The channel is inclined at an angle Φ with the horizontal. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity, we restrict our discussion to the half-width of the channel as shown in the figure. The region between $y=0$ and $y=y_0$ is called plug flow region. In the plug flow region, $\left|\tau_{yx}\right| \le \tau_0$. In the region between $y=y_0$ and $y=H$, $\left|\tau_{yx}\right| > \tau_0$. Fig. 1 shows the physical model of the problem. The wall deformation is given by

$$H(X,t) = a + b\sin\frac{2\pi}{\lambda}(x-ct) \qquad (2.1)$$

where b is the amplitude, λ the wavelength and c is the wave speed.

Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X,Y) .The transformation between these two frames is given by

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$$x = X - ct, y = Y, u(x,y) = U(X-ct,Y)\, v(x,y) = V(X-ct,Y) \qquad (2.2)$$

Where U and V are velocity components in the laboratory frame and u and v are velocity components in the wave frame.

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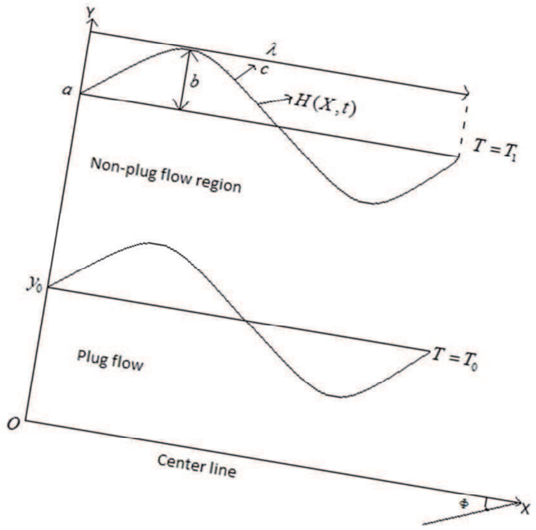

Fig. 1The Physical model

The equations governing the flow in wave frame are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.3)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p^1}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}-\sigma B_0^{\,2}(u+c)-\frac{\mu}{k}(u+c)+\rho g\alpha(T-T_0)-\rho g sin(\Phi) \quad (2.4)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p^1}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}-\frac{\mu}{k}v-g\rho\cos(\Phi) \quad (2.5)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+Q_0 \quad (2.6)$$

Introducing the non-dimensional quantities

$$x=\frac{x}{\lambda}, y=\frac{y}{a}, u=\frac{u}{c}, v=\frac{v}{c\delta}, \delta=\frac{a}{\lambda},$$

$$p^1=\frac{p^1a^2}{\mu c\lambda}, t=\frac{ct}{\lambda}, h=\frac{H}{a}, \phi=\frac{b}{a},$$

$$\tau_y=\frac{a\tau_y}{\lambda}, y_0=\frac{y_0}{a}, \theta=\frac{T-T_0}{T_1-T_0},$$

$$\beta=\frac{Q_0a^2}{k(T_1-T_0)}, G_r=\frac{\rho\alpha g a^2(T_1-T_0)}{\mu c},$$

$$p_r=\frac{\mu c_p}{k}, M=\sqrt{\frac{\sigma}{\mu}}aB_0, \mathrm{Re}=\frac{\rho ca}{\mu}, Da=\frac{k}{a^2}$$

Into equations (2.3) to (2.6), we get (dropping the bars)

Where $\eta=\frac{a^2g}{\gamma c}$, $\eta_1=\frac{a^2g}{\gamma c\lambda}$, g is the acceleration due to gravity, $\gamma=\frac{\mu}{\rho}$ is the kinematic viscosity of the fluid. Under the assumptions of long wavelength and low Reynolds number, the equations reduce to

$$\frac{d^2u}{dy^2}-N^2u=\frac{\partial p}{\partial x}+N^2+G_r\theta-\eta\sin(\Phi) \quad (2.7)$$

$$\frac{\partial p}{\partial y}=0 \quad (2.8)$$

$$\frac{\partial^2\theta}{\partial y^2}+\beta=0 \quad (2.9)$$

Here τ_y is the yield stress.

Here equation (2.8) indicates that p is independent of y and depends only upon x. Therefore, Eq. (2.7), can be rewritten as

$$\frac{d^2u}{dy^2}-N^2u=\frac{dp}{dx}+N^2+G_r\theta-\eta\sin(\Phi) \quad (2.10)$$

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y}=\tau_y \text{ at } y=0 \quad (2.11)$$

$$u=-1 \text{ at } y=h \quad (2.12)$$

$$\frac{\partial\theta}{\partial y}=0 \text{ at } y=0 \text{ and} \quad (2.13a)$$

$$\theta=1 \text{ at } y=h \quad (2.13b)$$

The volume flux q through each cross section in the wave frame is given by

$$q=\int_0^{y_0}u_p dy+\int_{y_0}^{h}u dy \quad (2.14)$$

The average volume flow rate $\overline{Q}$ over one wave period $T=\frac{\lambda}{c}$ of the peristaltic wave is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Q(X,t)dt=q+1 \quad (2.15)$$

3. Solution Of The Problem

The temperature Eq. (2.9) with the boundary conditions (2.13) gives

$$\theta=1+\frac{\beta}{2}\left(h^2-y^2\right) \quad (3.1)$$

For the solution of Eq. (2.10), we use Adomian decomposition method, we write in operator form

$$L_{yy}u-N^2u=\frac{dp}{dx}+N^2+G_r\theta-\eta\sin(\Phi) \quad (3.2)$$

Putting Eq. (3.1) in Eq. (3.2), we get

$$L_{yy}u-N^2u=\frac{dp}{dx}+N^2+G_r\left(1+\frac{\beta}{2}\left(h^2-y^2\right)\right)-\eta\sin(\Phi) \quad (3.3)$$

Where $L=\frac{d^2}{dy^2}$ Since L is a second-order differential operator, L^{-1} is a second-fold integration operator defined by:

$$L^{-1}(.)=\int_0^y\int_0^y(.)\,dydy\,. \qquad (3.4)$$

Operating with L^{-1}, Eq. (3.3) becomes

$$u=c_1+c_2y+L^{-1}\left(\frac{dp}{dx}+N^2+G_r\left(1+\frac{\beta}{2}\left(h^2-y^2\right)\right)-\eta\sin(\Phi)\right)+L^{-1}\left(N^2u\right) \qquad (3.5)$$

By the standard Adomian decomposition method, one can write: $u=\sum_{n=0}^{\infty}u_n$

From eq. (3.5)

$$u_0=c_1+c_2y+\left(\frac{dp}{dx}+N^2+G_r\frac{\beta h^2}{2}+G_r-\eta\sin(\Phi)\right)\frac{y^2}{2!}-G_r\beta\frac{y^4}{4!}$$

$$u_{n+1}=N^2L^{-1}\left(u_n\right),\ n\geq 0\,. \qquad (3.6)$$

$$u=\frac{1}{N^2}\left(\frac{dp}{dx}+G_r-\eta\sin(\Phi)\right)\left(\frac{\cosh(Ny)}{\cosh(Nh)}-1\right)-\frac{\tau_y}{N}\tanh(Nh)\cosh(Ny)+\frac{\tau_y}{N}\sinh(Ny)-1-G_r\beta\left(\frac{1}{N^4}\frac{\cosh(Ny)}{\cosh(Nh)}+\frac{h^2}{2N^2}-\frac{1}{N^4}-\frac{y^2}{2N^2}\right) \qquad (3.7)$$

We find the upper limit of plug flow region using the boundary condition $\frac{\partial u}{\partial y}=0$ at $y=y_0$. So, we have

$$\tau_y=-\frac{1}{N}\left(G_r+\frac{dp}{dx}-\eta\sin(\Phi)\right)\left(\frac{\sinh(Ny_0)}{\cosh N(h-y_0)}\right)+\frac{G_r\beta}{N^3}\left(\frac{-N\ y_0+\sinh(Ny_0)}{\cosh N(h-y_0)}\right) \qquad (3.8)$$

Taking $y=y_0$ in Eq. (3.7), we get the velocity in plug flow region as

$$u_p=\frac{1}{N^2}\left(\frac{dp}{dx}+G_r-\eta\sin(\Phi)\right)\left(\frac{1-\cosh(N(h-y_0))}{\cosh(N(h-y_0))}\right)-1-\frac{G_r\beta}{N^4}\left(\frac{\cosh(Nh)-Ny_0\tanh(N(h-y_0))}{\cosh(Nh)\cosh(N(h-y_0))}\right)-G_r\beta\left(\frac{h^2}{2N^2}-\frac{1}{N^2}-\frac{y_0{}^2}{2}\right) \qquad (3.9)$$

The volume flux q through each cross section in the wave frame is given by

$$q=\int_0^{y_0}u_pdy+\int_{y_0}^{h}udy=-h+\frac{h\eta_1\sin(\Phi)}{N^2}-\frac{\eta_1\sin(\Phi)y_0}{N^2}-A_1+A_2-A_3-\beta G_ry_0\left(-\frac{1}{N^4}+\frac{h^2}{2N^2}-\frac{y_0{}^2}{2N^2}\right)+\frac{1}{12N^5}\left(A_4G_r\left(A_5-A_6+A_7+3N\left(A_8\right)y_0-A_9\right)\right)-A_{10}+\frac{dp}{dx}A_{11} \qquad (3.10)$$

Where $A_1=\frac{\left(\begin{matrix}\eta_1(1-\cosh(N(h-y_0)))\\ \sec h(N(h-y_0))\sin(\Phi)y_0\end{matrix}\right)}{N^2}$

$$A_2=\frac{\left(\begin{matrix}(1-\cosh(N(h-y_0)))\\ \sec h(N(h-y_0))G_ry_0\end{matrix}\right)}{N^2}$$

$$A_3=\frac{1}{n^4}\left(\begin{matrix}\beta\sec h(Nh)\sec h(N(h-y_0))\\ G_ry_0\left(\begin{matrix}\cosh(hN)-\\ N\sinh(N(h-y_0))y_0\end{matrix}\right)\end{matrix}\right)$$

$$A_4=\sec h(hN)\sec h(N(h-y_0))$$

$$A_5=-2hN(-3\beta+N^2(3+h^2\beta))\cosh(N(2h-y_0))$$

$$A_6=2hN(-3\beta+N^2(3+h^2\beta))\cosh(Ny_0)$$

$$A_7=6(N^2-\beta)\left(\begin{matrix}\sinh(N(2h-y_0))-\\ \sinh(Ny_0)\end{matrix}\right)$$

$$A_8 = -4\beta + 4\beta\cosh(N(h-y_0)) + (-2\beta + N^2(2+h^2\beta))\left(\cosh(N(2h-y_0)) + \cosh(Ny_0)\right)$$

$$A_9 = N^3\beta\left(\cosh(N(2h-y_0)) + \cosh(Ny_0)\right)y_0^3$$

$$A_{10} = \frac{\eta_1 \sin(\Phi)\tanh(N(h-y_0))}{N^3}$$

$$A_{11} = \left(-\frac{h}{N^2} + \frac{y_0}{N^2} + \frac{\left(\begin{array}{c}(1-\cosh(N(h-y_0)))\\ \sec h(N(h-y_0))y_0\end{array}\right)}{N^2} + \frac{\tanh(N(h-y_0))}{N^3} \right)$$

The pressure rise and frictional force over one wavelength of the peristaltic are given by

$$\Delta p = \int_0^1 \frac{dp}{dx}dx \quad (3.11)$$

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right)dx \quad (3.12)$$

The above integrals numerically evaluated using the MATHEMATICA software.

4. Results and discussion: To see the variations in pressure rise, frictional force and axial pressure gradient caused by the amplitude ratio ϕ, Darcy number Da, Hartmann number M, half width of the plug flow region y_0, Grashof number G_r, heat Source/Sink parameter β, and angel of inclination Φ, we have plotted the graphs. Notice that for fixed values of all other parameters, the axial pressure gradient increases with increase in ϕ, M, β, G_r, Φ, Further, it is seen that axial pressure gradient decreases with increase in Da. (Figs. 8-13) shows the variation of pressure rise against flow rate $\overline{Q}$ for different parameters of interest. We observe that the increase in the values of ϕ, M, Φ, β and G_r causes the increase in pressure rise while with the increase in Da causes the decrease in pressure rise. The variations of frictional force with the flow rate $\overline{Q}$ for various values of amplitude ratio ϕ, Darcy number Da, Hartmann number M, half width of the plug flow region y_0, Grashof number G_r angel of inclination Φ and heat Source/Sink parameter β can be analyzed through (Figs. 14-19). It can be seen that frictional forces have opposite behavior as compared to the pressure rise.

To see the effects of temperature field for different values of β, we have plotted (Fig. 20) It is observed that with the increase of β, the temperature field increases. Further the temperature field is maximum at $y = 0$.

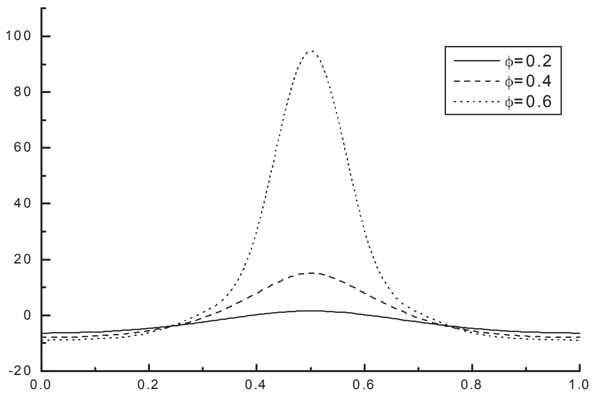

Fig. 2The variation of axial pressure gradient with ϕ for $G_r = 3, \beta = 0.1, M = 2, y_0 = 0.2$ $\eta = 0.5\,\Phi = \frac{\pi}{6}$ *and* $Da = 0.08$

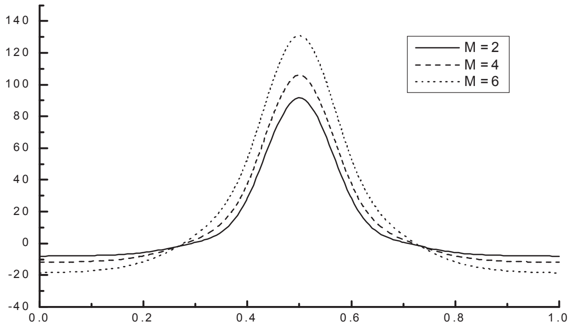

Fig. 3The variation of axial pressure gradient with M for

$$G_r = 3, \beta = 0.1, \phi = 0.6, y_0 = 0.2,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \text{ and } Da = 0.1$$

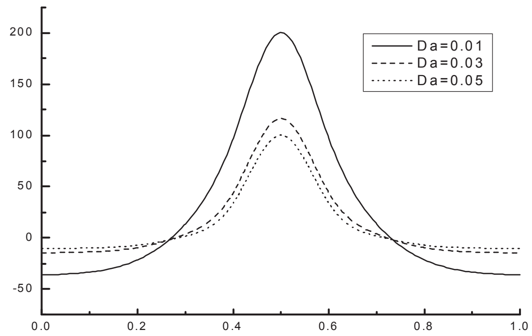

Fig.4.The variation of axial pressure gradient with Da for

$$G_r = 3, \beta = 0.1, \phi = 0.6, y_0 = 0.2,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \text{ and } M = 1$$

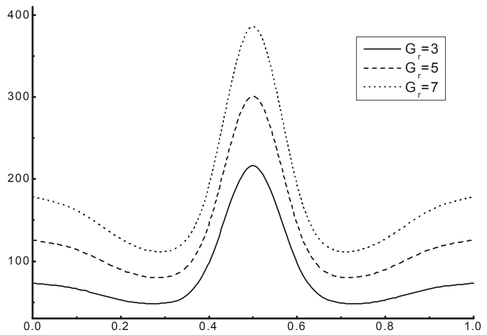

Fig.6.The variation of axial pressure gradient with G_r for

$$\beta = 5, M = 1, \phi = 0.6, y_0 = 0.2,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6}, \text{ and } Da = 0.1$$

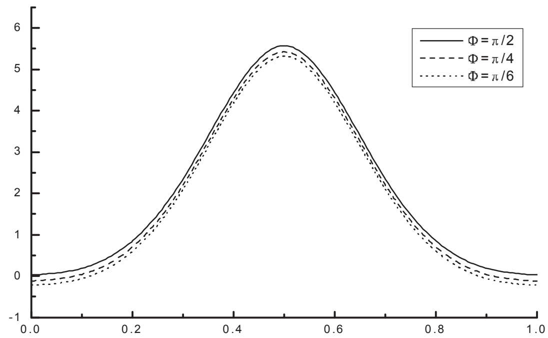

Fig.7.The variation of axial pressure gradient with Φ for

$$\beta = 0.5, M = 1, \phi = 0.2, y_0 = 0.2,$$

$$\eta = 0.5, G_r = 3 \text{ and } Da = 0.08$$

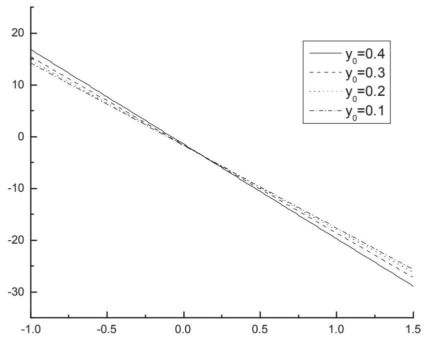

Fig.8.The variation of pressure rise with time-averaged flux for different values of y_0 for

$$\beta = 0.1, M = 1, \phi = 0.6, G_r = 3,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \text{ and } Da = 0.1$$

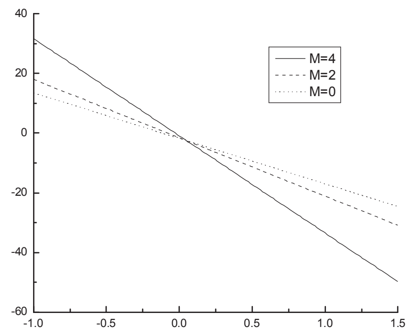

Fig.9.The variation of pressure rise with time-averaged flux for different values of M for

$$\beta = 0.1, y_0 = 0.2, \phi = 0.6, G_r = 3,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \text{ and } Da = 0.1$$

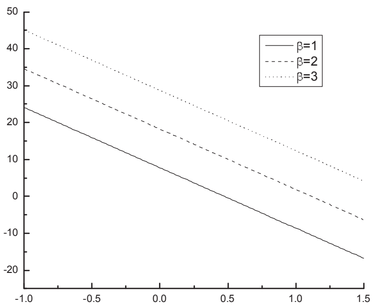

Fig.11.The variation of pressure rise with time-averaged flux for different values of β for

$$M = 1, y_0 = 0.2, \phi = 0.2, G_r = 3,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; Da = 0.1$$

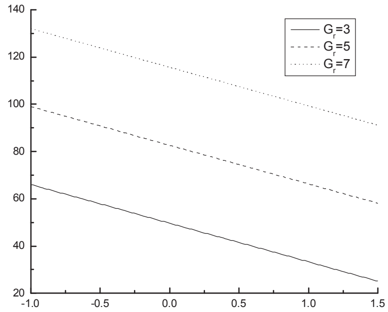

Fig.12.The variation of pressure rise with time-averaged flux for different values of G_r for

$$M = 1, y_0 = 0.2, \phi = 0.6, \beta = 5,$$

$$\eta = 0.5. \Phi = \frac{\pi}{6} \; and \; Da = 0.1$$

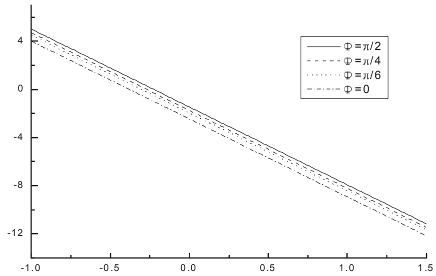

Fig.13.The variation of pressure rise with time-averaged flux for different values of Φ for

$$M = 1, y_0 = 0.2, \phi = 0.6, \beta = 0.1,$$

$$\eta = 0.5, G_r = 3 \; and \; Da = 1$$

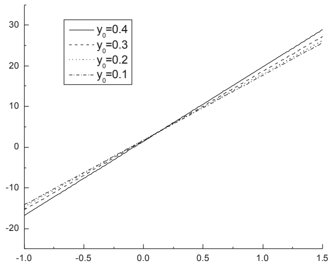

Fig.14.The variation of frictional force with time-averaged flux for different valuesof y_0 for

$$M = 1, G_r = 3, \phi = 0.6, \beta = 0.1,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; Da = 0.1$$

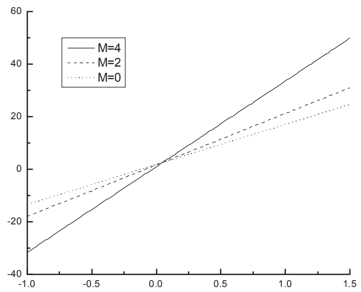

Fig.15.The variation of frictional force with time-averaged flux for different values of M for

$$\phi = 0.6, G_r = 3, y_0 = 0.2, \beta = 0.1,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; Da = 0.1$$

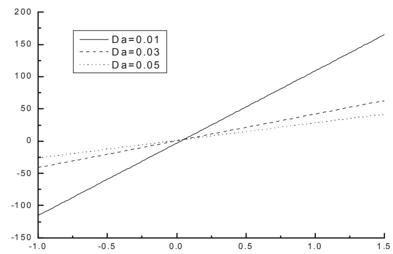

Fig.16.The variation of frictional force with time-averaged flux for different values of Da for

$$\phi = 0.6, G_r = 3, y_0 = 0.2, \beta = 0.1,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; M = 1$$

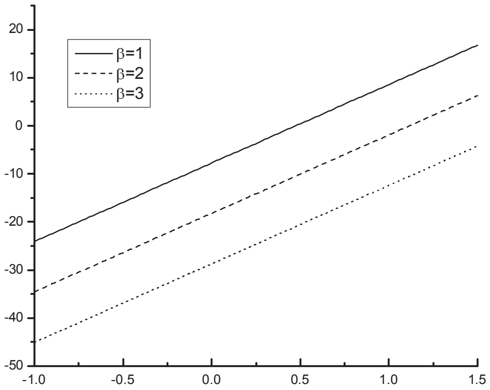

Fig.17.The variation of frictional force with time-averaged flux for different values of β for

$$\phi = 0.6, G_r = 3, y_0 = 0.2, M = 1,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; Da = 0.1$$

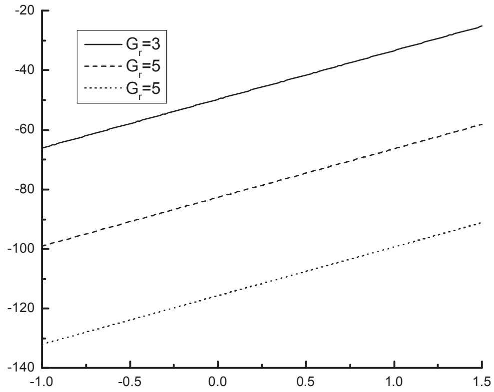

Fig.18.The variation of frictional force with time-averaged flux for different values of G_r for

$$\phi = 0.6, \beta = 5, y_0 = 0.2, M = 1,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; Da = 0.1$$

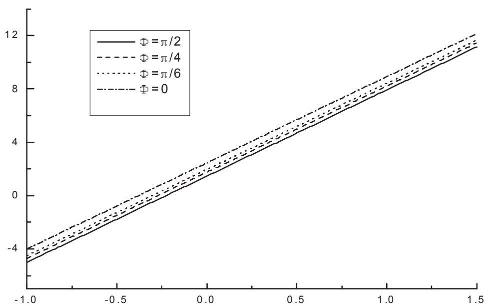

Fig. 19.The variation of frictional force with time-averaged flux for different values of Φ for

$$\phi = 0.6, \beta = 5, y_0 = 0.2, M = 1,$$

$$\eta = 0.5 \, G_r = 3 \; and \; Da = 1$$

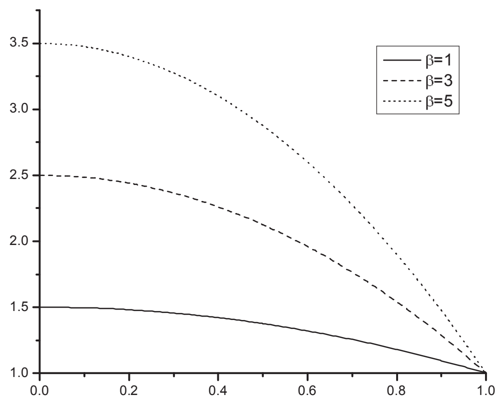

Fig.20.The effect of temperature field for different values of β the other parameters are

$$\phi = 0.6, G_r = 3, y_0 = 0.2, M = 1,$$

$$\eta = 0.5, \Phi = \frac{\pi}{6} \; and \; Da = 0.08$$

Acknowledgment: This work is supported by UGC, Research Fellowship in Science for Meritorious Students (RFSMS) [void UGC Ltr. No.F.No. 7-72/2007(BSR) date 10.01.2012]. Ms. LaxmiDevindrappa author, acknowledges UGC for awarding the Research Fellowship.

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