GENERALIZED REGULAR FUZZY LOCALLY CLOSED SETS AND SOME GENERALIZATIONS OF FUZZY LC -CONTINUOUS FUNCTIONS

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Abstract: The aim of this paper is to introduce fuzzy locally closed sets, generalized fuzzy locally closed sets, generalized regular fuzzy locally closed sets and studies their properties. Here we also introduce fuzzy locally closed continuous functions, generalized fuzzy locally closed continuous functions, and generalized regular fuzzy locally closed continuous functions. More over some properties of said functions are also discussed and then interrelation is established.

Keywords: Fuzzy locally closed set, generalized fuzzy locally closed set, generalized regular fuzzy locally closed set, fuzzy lc-continuity**.**

Introduction: The notion of locally closed sets in a topological space was introduced by N.Bourbaki [1]. Ganster and Reilly[2] further studied the properties of locally closed sets and defined the LC-continuity and LC-irresoluteness. K.Balachandran, P.Sundaram and H.Maki [3] introduced the concept of generalized locally closed sets and generalized locally closed continuous functions and investigated some of their properties. Later on I. Arockiarani, K.Balachandran and M.Ganster [4] introduced regular generalized locally closed sets and RGL-continuous functions. G.Balasubramanian and P.Sundaram [5] defined generalized fuzzy closed set. Recently authors defined the concept of generalized regular fuzzy closed sets. A fuzzy set A in a fts X is called generalized regular fuzzy closed set (in short, grf-closed) if $Rcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open. The aim this paper is to continue the study of generalization of locally closed sets and investigate the classes of generalized regular fuzzy locally closed continuous functions in fuzzy topological space.

2. Fuzzy locally closed sets:

2.1. Definition

Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is called fuzzy locally closed set if $\lambda = \mu \wedge \eta$ where μ is fuzzy open and η is fuzzy closed in (X,τ).

We denote the collection of all fuzzy locally closed subset of (X,τ) by $FLC(X,\tau)$.

2.2. Theorem

(i) A fuzzy set λ of a fts (X,τ) is fuzzy locally closed iff $1-\lambda$ is the union of a fuzzy open set and fuzzy closed set.

(ii) Every fuzzy open subset of (X,τ) is fuzzy locally closed.

(iii) Finite intersection of fuzzy locally closed sets is fuzzy locally closed but finite union of fuzzy locally closed sets need not be a fuzzy locally closed.

(iv) A dense subset is fuzzy open iff it is fuzzy locally closed set.

2.3. Theorem : For a fuzzy set λ of (X,τ) the followings are equivalent:

(i) λ is fuzzy locally closed;

(ii) $\lambda = \mu \wedge cl(\lambda)$ for some fuzzy open set μ;

(iii) $cl(\lambda) - \lambda$ is fuzzy closed;

(iv) $\lambda \vee (X- cl(\lambda))$ is fuzzy open.

2.4. Theorem: A fuzzy set λ is fuzzy closed iff λ is generalized fuzzy closed and fuzzy locally closed.

Proof: It is obvious.

2.5. Theorem: If $\lambda \in FLC(X,\tau)$ and μ is fuzzy open or fuzzy closed then $\lambda \wedge \mu \in FLC(X,\tau)$.

Proof: It is straightforward.

2.6. Theorem: Let $\lambda \in FLC(X,\tau)$ and $\mu \in FLC(X,\tau)$. If λ and μ are separated i.e $\lambda \wedge cl(\mu) = \mu \wedge cl(\lambda) = 0_x$ then $\lambda \vee \mu \in FLC(X,\tau)$.

2.7. Theorem: If λ is fuzzy closed and fuzzy pre-open set then $1-\lambda$ is generalized regular fuzzy closed set.

Proof: Let λ is fuzzy closed as well as fuzzy pre-open set. Then $\lambda = \mu \wedge \eta = \mu \wedge cl(\lambda)$ where μ is fuzzy open and η is fuzzy closed in (X,τ). Again $\lambda \leq int(cl(\lambda))$.Therefore $\lambda = \mu \wedge cl(\lambda) = int(cl(\lambda)) \wedge cl(\lambda) = int(cl(\lambda))$.This implies λ is regular open. Hence $1-\lambda$ is generalized regular fuzzy closed set.

3. Generalized fuzzy locally closed sets :

3.1. Definition

Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is called generalized fuzzy locally closed set (in short gflc) if $\lambda = \mu \wedge \eta$,where μ is generalized fuzzy open and η is generalized fuzzy closed in (X,τ).

We denote the collection of all generalized fuzzy locally closed subset of (X,τ) by $GFLC(X,\tau)$.

3.2. Definition: Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is $gflc^*$ if $\lambda = \mu \wedge \eta$, where μ is generalized fuzzy open and η is fuzzy closed in (X,τ).

We denote the collection of all $gflc^*$ subset of (X,τ) by $GFLC^*(X,\tau)$.

3.3. Definition: Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is called $gflc^{**}$ if $\lambda = \mu \wedge \eta$, where μ is fuzzy open and η is generalized fuzzy closed in (X,τ).

We denote the collection of all gflc** subset of (X,τ) by GFLC**(X,τ).

3.4. Remarks : Equivalent relations of (Theorem 2.3) is also exists for generalized fuzzy locally closed sets.

However from the above definitions we have the following results:

3.5. Theorem : (i) Every fuzzy locally closed set is gflc.
(ii) Every fuzzy locally closed set is gflc* and gflc**.
(iii) Every gflc* or gflc** is gflc.

Proof: It is obvious from the definition 2.1, 3.1,3.2 and 3.3.

Though the converses of the above theorem may not be true in general.

3.6. Theorem: If λ is gflc (gflc*, gflc**) then λ is generalized fuzzy closed set.

Proof: It is obvious.

3.7. Theorem: Let λ and μ be two fuzzy set of (X,τ).
(i) If $\lambda \in$ GFLC(X,τ) and μ is generalized fuzzy open or fuzzy closed then $\lambda \wedge \mu \in$ GFLC(X,τ).
(ii) If $\lambda \in$ GFLC*(X,τ) and $\mu \in$ GFLC*(X,τ) then $\lambda \wedge \mu \in$ GFLC*(X,τ).
(iii) If $\lambda \in$ GFLC**(X,τ) and μ is fuzzy open or fuzzy closed then $\lambda \wedge \mu \in$ GFLC**(X,τ).

3.8. Remarks : Finite union and finite intersection of gflc sets is not gflc.

4. Generalized regular fuzzy locally closed sets :

4.1. Definition : Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is called regular fuzzy locally closed set (in short rflc) if $\lambda = \mu \wedge \eta$ where μ is fuzzy regular open and η is fuzzy regular closed in (X,τ).

We denote the collection of all regular fuzzy locally closed subset of (X,τ) by RFLC(X,τ).

4.2. Definition : Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is called generalized regular fuzzy locally closed set (in short grflc) if $\lambda = \mu \wedge \eta$ where μ is generalized regular fuzzy open and η is generalized regular fuzzy closed in (X,τ).

We denote the collection of all generalized regular fuzzy locally closed subsets of (X,τ) by GRFLC(X,τ).

4.3. Definition: Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is grflc* if $\lambda = \mu \wedge \eta$, where μ is generalized regular fuzzy open and η is fuzzy regular closed in (X,τ).

We denote the collection of all grflc*subsets of (X,τ) by GRFLC*(X,τ).

4.4. Definition: Let (X,τ) be a fuzzy topological space. A fuzzy set λ in (X,τ) is called grflc** if $\lambda = \mu \wedge \eta$ where μ is fuzzy regular open and η is generalized fuzzy regular closed in (X,τ).

We denote the collection of all grflc**subsets of (X,τ) by GRFLC**(X,τ).

From the above definitions we have the following results:

4.5. Theorem: (i) Every rflc set is flc, grflc, grflc* and grflc**.
(ii) Every grflc* or grflc**set is grflc.

Proof: It is obvious.

However the converse of the above theorem may not be true in general.

4.6. Theorem: Let λ and μ be two fuzzy set of (X,τ).
(i) If $\lambda \in$ GRFLC(X,τ) and μ is generalized fuzzy open or fuzzy regular closed then $\lambda \wedge \mu \in$ GRFLC(X,τ).
(ii) If $\lambda \in$ GRFLC*(X,τ) and $\mu \in$ GRFLC*(X,τ) then $\lambda \wedge \mu \in$ GRFLC*(X,τ).
(iii) If $\lambda \in$ GRFLC**(X,τ) and μ is fuzzy regular open or fuzzy regular closed then $\lambda \wedge \mu \in$ GRFLC**(X,τ).

Proof:
(i) Proof is obvious.
(ii) Let λ and $\mu \in$ GRFLC*(X,τ). Then there exist generalized regular fuzzy open sets η_1 and η_2 such that $\lambda = \eta_1 \wedge Rcl(\lambda)$ and $\mu = \eta_2 \wedge Rcl(\mu)$. Therefore $\lambda \wedge \mu = \eta_1 \wedge Rcl(\lambda) \wedge \eta_2 \wedge Rcl(\mu) = \eta_1 \wedge \eta_2 \wedge Rcl(\lambda) \wedge Rcl(\mu)$ where $\eta_1 \wedge \eta_2$is generalized regular fuzzy open and $Rcl(\lambda) \wedge Rcl(\mu)$ is regular closed. Therefore $\lambda \wedge \mu \in$ GRFLC*(X,τ).
(iii) Proof is similar to (ii).

5. Generalized regular fuzzy lc-continuity :.

5.1. Definition: A function f : (X,τ) → (Y,σ) is called fuzzy LC-continuous if $f^{-1}(\lambda) \in$FLC(X,τ), for every $\lambda \in \sigma$.

5.2. Definition: A function f : (X,τ) → (Y,σ) is called generalized fuzzy LC-continuous (respectively generalized fuzzy LC*-continuous, generalized fuzzy LC**-continuous) if $f^{-1}(\lambda) \in$GFLC(X,τ), (respectively $f^{-1}(\lambda) \in$GFLC*(X,τ), $f^{-1}(\lambda) \in$ GFLC**(X,τ)), for every $\lambda \in \sigma$.

5.3. Definition: A function f : (X,τ) → (Y,σ) is called regular fuzzy LC-continuous if $f^{-1}(\lambda) \in$RFLC(X,τ) for every $\lambda \in \sigma$.

5.4. Definition: A function f : (X,τ) → (Y,σ) is called generalized regular fuzzy LC-continuous (respectively generalized regular fuzzy LC*-continuous, generalized regular fuzzy LC**-continuous) if $f^{-1}(\lambda) \in$GRFLC(X,τ) (respectively$f^{-1}(\lambda) \in$GRFLC*(X,τ)) $f^{-1}(\lambda) \in$GRFLC**(X,τ)) for every $\lambda \in \sigma$.

5.5. Definition: A function f : (X,τ) → (Y,σ) is called generalized fuzzy LC-irresolute (respectively generalized fuzzy LC*-irresolute, generalized fuzzy LC**- irresolute if $f^{-1}(\lambda) \in$GFLC(X,τ), (respectively $f^{-1}(\lambda) \in$GFLC*(X,τ), $f^{-1}(\lambda) \in$GFLC**(X,τ)),for every $\lambda \in$ GFLC (Y,σ) (respectively $\lambda \in$ GFLC* (Y,σ), $\lambda \in$ GFLC**

(Y,σ)).

5.6. Definition: A function f : (X,τ) → (Y,σ) is called generalized regular fuzzy LC-irresolute (generalized regular fuzzy LC *- irresolute, generalized regular fuzzy LC **- irresolute) if $f^{-1}(\lambda)\in$GRFLC(X,τ), (respectively $f^{-1}(\lambda)\in$GRFLC*(X,τ), $f^{-1}(\lambda)\in$GRFLC**(X,τ)),for every λ∈ GRFLC (Y,σ) (respectively λ∈ GRFLC* (Y,σ), λ∈ GRFLC** (Y,σ)).

5.7.Theorem: Let f : (X,τ) → (Y,σ) be a function.
(i) If f is fuzzy LC -continuous then it is generalized fuzzy LC –continuous (generalized fuzzy LC *-continuous, generalized fuzzy LC**-continuous).
(ii) If f is regular fuzzy LC -continuous then it is generalized regular fuzzy LC –continuous (respectively generalized regular fuzzy LC *-continuous, generalized regular fuzzy LC **-continuous).
(iii) If f is generalized regular fuzzy LC *-continuous (respectively generalized regular fuzzy LC **-continuous) then it is generalized regular fuzzy LC -continuous.
(iv) If f is regular fuzzy LC -continuous then it is fuzzy LC –continuous (respectively generalized fuzzy LC - continuous, generalized fuzzy LC *-continuous, generalized fuzzy LC **-continuous).

Proof: It is straight forward from definition 5.1, 5.2, 5.3, 5.4, 5.5 and 5.6.
However the converse of the each statement of the theorem may not be true in general.

5.8. Theorem: Let f : X→Y and g: Y→Z be function.
(i) If f is fuzzy LC -continuous (respectively generalized fuzzy LC -continuous, generalized regular fuzzy LC -continuous) and g is fuzzy continuous then gof is fuzzy LC-continuous (respectively generalized fuzzy LC–continuous, generalized regular fuzzy LC –continuous).
(ii) If f is generalized regular fuzzy LC -irresolute and g is generalized regular fuzzy LC -continuous then gof is generalized regular fuzzy LC -continuous.
(iii) If f is generalized regular fuzzy LC -irresolute and g is regular fuzzy LC -continuous then gof is generalized regular fuzzy LC -continuous.
(iv) If f is generalized regular fuzzy LC -irresolute and g is generalized regular fuzzy LC - irresolute then gof is generalized regular fuzzy LC - irresolute.

Proof: (i) Let λ is open in Z. Then $g^{-1}(\lambda)$ is open in Y, since g is fuzzy continuous. Fuzzy LC –continuity (respectively generalized fuzzy LC-continuity, generalized regular fuzzy LC-continuity) of f implies $f^{-1}(g^{-1}(\lambda))$ is fuzzy locally closed set (respectively generalized fuzzy locally closed set, generalized regular fuzzy locally closed set) in X. That is $(gof)^{-1}(\lambda)$ is fuzzy locally closed set (respectively generalized fuzzy locally closed set, generalized regular fuzzy locally closed set) in X. Then gof is fuzzy LC-continuous (respectively generalized fuzzy LC–continuous, generalized regular fuzzy LC –continuous).
(ii) Let λ is open in Z. Then $g^{-1}(\lambda)$ is generalized regular fuzzy locally closed set in Y, since g is generalized regular fuzzy LC -continuous. Generalized regular fuzzy LC –irresoluteness of f implies $f^{-1}(g^{-1}(\lambda))$ is generalized regular fuzzy locally closed set in X. Then gof is generalized regular fuzzy LC -continuous.
(iii) Proof is very much similar to (ii)
(iv) Proof is very much similar to (ii)

Conclusion: The concept of locally closed set is introduced in fuzzy topological space. Also it is proved that generalized regular fuzzy locally closed set is generalized fuzzy locally closed set.

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