TRANSVERSE VIBRATION OF NON-HOMOGENEOUS RECTANGULAR PLATES USING GDQ

RENU SAINI,ROSHAN LAL

Abstract: The free transverse vibration of non-homogeneous rectangular plates of uniform thickness has been analyzed using generalized differential quadrature (GDQ) method on the basis of Kirchhoff plate theory with the three boundary conditions: (i) fully clamped (ii) two opposite edges simply supported and other two edges clamped (iii) fully simply supported. The non-homogeneity of the plate material is assumed to arise due to linear variations in Young's modulus and density of the plate material with the in-plane Co-ordinates. The effect of various plate parameters has been investigated for the first three modes of vibration. A comparison of results with those available in literature has been presented.

Keywords: Rectangular, non-homogeneous, generalized differential quadrature (GDQ) method.

Introduction: Plates of various geometries are the key components in many technological situations. Of these, rectangular plates are extensively used as structural elements in various aspects of aerospace, mechanical, nuclear, marine and structural engineering. Although, a vast literature for the flexural vibrations of isotropic rectangular plates exists and reported in references [1]-[3], to mention a few, even the study of their dynamic behavior is continuing till today.

Non-homogeneous materials are of considerable interest to design engineers in various technological situations. Plywood, timber and fibre-reinforced plastic etc. are the examples of non-homogeneous materials. In many practical applications, particularly in aerospace industry, modern missile technology and microelectronics, plate type structural elements have to work under high temperature environment which causes non-homogeneity in the material. However, many structural components possess initial non-homogeneity due to the inclusion of foreign materials or imperfection or being composite materials. Various models for the non-homogeneity of the plate material have been proposed in the literature and a brief review is given in [4]. Recently, a number of papers are available in the literature analyzing the dynamic behaviour of non-homogeneous rectangular plates and reported in [5], [6]. In these papers, it is considered that non-homogeneity of the plate material arises due to change in only one space variable. Very recently, Lal and Kumar [7] obtained the numerical results for the transverse vibrations of non-homogeneous rectangular plates of uniform thickness using boundary characteristic orthogonal polynomials.

Keeping in view the above fact, a study dealing with the free transverse vibration of non-homogeneous rectangular plates of uniform thickness employing generalized differential quadrature (GDQ) method has been presented on the basis of Kirchhoff plate theory. Non-homogeneity of the plate material is assumed to arise due to linear variation in Young's modulus and density of the plate material with both the in-plane Co-ordinates. The Poisson ratio is assumed to remain constant. In the present work three boundary conditions has been considered: (i) CCCC-all edges are clamped (ii) SCSC- two opposite edges are simply supported and other two edges are clamped (iii) SSSS-all edges are simply supported. The effect of non-homogeneity parameters, density parameters and the aspect ratio on the natural frequencies has been investigated for the first two modes of vibration. A comparison of results has been presented.

2. Mathematical Formulation: Referred to a Cartesian co-ordinate (x, y, z), the configuration of a thin non-homogeneous isotropic rectangular plate of length a, breadth b, thickness h and density $\rho(x, y)$ is shown in fig.1. The x - and y-axes are taken along the edges of the plate, the axis of z is perpendicular to the xy-plane. The middle surface being $z = 0$ and origin is at the one of the corners of the plate. The differential equation governing the transverse vibration of such plates, is given by

$$D\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial y^4}\right)+2D\frac{\partial^4 w}{\partial x^2\partial y^2}+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)$$
$$+\frac{\partial D}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y}\right)+\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 D}{\partial x^2}+\nu\frac{\partial^2 D}{\partial y^2}\right) \quad (1)$$
$$+\frac{\partial^2 w}{\partial y^2}\left(\nu\frac{\partial^2 D}{\partial x^2}+\frac{\partial^2 D}{\partial y^2}\right)+2(1-\nu)\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\rho h\frac{\partial^2 w}{\partial t^2}=0$$

Where $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity, $w(x, y, t)$ is the transverse displacement at the point (x, y), $E(x, y)$ is the Young's modulus, ν is the Poisson ratio, $\rho(x, y)$ is the density and t is the time.Fig.1 Geometry of the rectangular plate.

For a harmonic solution, the displacement w is

(2)

assumed to be

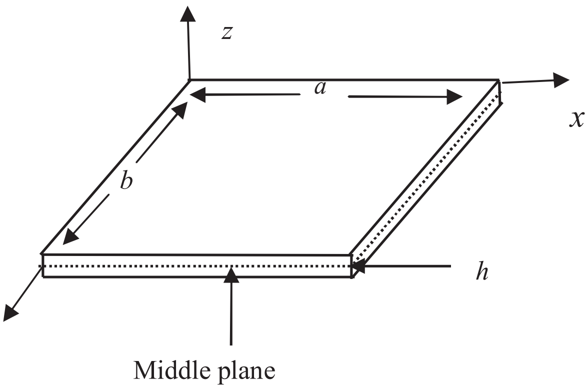

Fig:1 geometryof rectangular plane

$$w(x, y, t) = \overline{w}(x, y)\, e^{i\omega t}$$

Where ω is the circular frequency in radians and $\overline{w}(x,y)$ represent the maximum transverse displacement at the point (x, y). Using (2), (1) reduces to

$$D\left(\frac{\partial^4 \overline{w}}{\partial x^4} + \frac{\partial^4 \overline{w}}{\partial y^4}\right) + 2D\frac{\partial^4 \overline{w}}{\partial x^2 \partial y^2} + 2\frac{\partial D}{\partial x}\left(\frac{\partial^3 \overline{w}}{\partial x^3} + \frac{\partial^3 \overline{w}}{\partial x \partial y^2}\right)$$
$$+\frac{\partial D}{\partial y}\left(\frac{\partial^3}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}\right) + \frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 D}{\partial x^2} + \nu\frac{\partial^2 D}{\partial y^2}\right) \quad (3)$$
$$+\frac{\partial^2 w}{\partial y^2}\left(\nu\frac{\partial^2 D}{\partial x^2} + \frac{\partial^2 D}{\partial y^2}\right) + 2(1-\nu)\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \rho h\frac{\partial^2 w}{\partial t^2} = 0$$

Introducing the non-dimensional variables $X = x/a,\ Y = y/b,\ H = h/a,\ W = \overline{w}/a$ and assuming that Young's modulus and density of the plate material vary with the space co-ordinates by the functional values

$$E(X,Y) = E_0(1 + \alpha_1 X + \alpha_2 Y),$$
$$\rho(X, Y) = \rho_0(1 + \beta_1 X + \beta_2 Y) \quad (4)$$

where E_0, ρ_0 are the Young's modulus and density of the plate material at $X = Y = 0$, α_1, α_2 are non-homogeneity parameters and β_1, β_2 are the density parameters respectively. Equation (3) now, reduces to

$$(1 + \alpha_1 X + \alpha_2 Y)\left(\frac{\partial^4 W}{\partial X^4} + 2\lambda^2\frac{\partial^4 W}{\partial X^2 \partial Y^2} + \lambda^4\frac{\partial^4 W}{\partial Y^4}\right)$$
$$+2\alpha_1\left(\frac{\partial^3 W}{\partial X^3} + \lambda^2\frac{\partial^3 W}{\partial X \partial Y^2}\right) + 2\alpha_2\left(\lambda^4\frac{\partial^3 W}{\partial Y^3} + \lambda^2\frac{\partial^3 W}{\partial X^2 \partial Y}\right)$$
$$\Omega^2 (1 + \beta_1 X + \beta_2 Y)W = 0$$

where $\lambda = a/b$ and $\Omega^2 = 12\,\rho_0(1-\nu^2)\omega^2 a^2 / E_0 H^2$

Equation (5) is a fourth order partial differential equation of variable coefficients with respect to X and Y. It requires two boundary conditions at each edge. The combinations of following boundary conditions are considered in the present paper.

For clamped edge: $W = \frac{dW}{dX} = 0,\ W = \frac{dW}{dY} = 0$, at $X = 0$ or $X = 1$, and $Y = 0$ or $Y = 1$, respectively.

For simply supported edge: $W = \frac{d^2 W}{dX^2} = 0,$

$W = 0\frac{d^2 W}{dY^2} = 0$, at $X = 0$ or $X = 1$, and $Y = 0$ or $Y = 1$, respectively.

Generalized differential quadrature (GDQ) method: According to Generalized differential quadrature (GDQ) the derivative of a function, with respect to a space variable at a given grid point, is approximated as a weighted linear sum of the function values at all of the grid points in the computational domain of that variable [8] .

The computational domain of a rectangular plate is $0 \le X \le 1,\ 0 \le Y \le 1$. Let $X_1, X_2, \ldots, X_N$ and $Y_1, Y_2, \ldots, Y_M$ are grid points in X and Y directions respectively. In this method, the n^{th} and m^{th} order derivatives of $W(X,Y)$ with respect to X, Y and its mixed derivative with respect to X and Y are approximated as

$$\frac{\partial^n W(X_i, Y_j)}{\partial X^n} = \sum_{l=1}^{N} a_{il}^{(n)}\, W(X_l, Y_j)$$
$$\frac{\partial^m W(X_i, Y_j)}{\partial Y^m} = \sum_{l=1}^{M} b_{jl}^{(m)}\, W(X_i, Y_l) \quad (6)$$
$$\frac{\partial^{m+n} W(X_i, Y_j)}{\partial X^n\, \partial Y^m} = \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(n)}\, b_{jl_2}^{(m)}\, W(l_1, l_2)$$

For $i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, M;$ $n = 1, 2, \ldots, N-1;\ m = 1, 2, \ldots, M-1;$

Where $a_{il}^{(n)}$ and $b_{jl}^{(m)}$ are the weighting coefficients associated with n^{th} and m^{th} order derivatives with respect to X and Y respectively.

The weighting coefficient of first order derivative are determined as

$$a_{ij}^{(1)} = \begin{cases} \dfrac{P^{(1)}(X_i)}{(X_i - X_j)\, P^{(1)}(X_j)} & j \ne i, \\ -\displaystyle\sum_{j=1,\ j\ne i}^{N} a_{ij}^{(1)} & j = i, \end{cases} \quad (7)$$

for $i, j = 1, 2, \ldots, N$

Where $P^{(1)}(X_i) = \prod_{j=1,\ j\neq i}^{N} (X_i - X_j)$

Similarly, for the second and higher order derivatives the recurrence relationships are obtained as follows

$$a_{ij}^{(n)} = \begin{cases} n\left(a_{ii}^{(n-1)} a_{ij}^{(1)} - \dfrac{a_{ij}^{(n-1)}}{(X_i - X_j)}\right) & j \neq i, \\ -\displaystyle\sum_{j=1,\ j\neq i}^{N} a_{ij}^{(n)} & j = i, \end{cases} \tag{8}$$

for $i,\ j = 1,\ 2, \ldots, N, \quad n = 2,\ 3, \ldots, N-1$

The corresponding coefficients $b_{jl}^{(m)}$ associated with derivatives with respect to y required can be similarly determined [8].

Discretizing eq. (5) at the internal grid point (X_i, Y_j), with $3 \le i \le N-2$ and $3 \le j \le M-2$, it reduces to,

$$\begin{aligned} &(1+\alpha_1 X(i) + \alpha_2 Y(j)) \left(\sum_{l=1}^{N} a_{il}^{(4)} W_{l,j} + 2\lambda^2 \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(2)} b_{jl_2}^{(2)} W_{l_1,l_2} + \lambda^4 \sum_{l=1}^{M} b_{lj}^{(4)} W_{i,l} \right) \\ &+ 2\alpha_1 \left(\sum_{l=1}^{N} a_{il}^{(3)} W_{l,j} + \lambda^2 \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(1)} b_{jl_2}^{(2)} W_{l_1,l_2} \right) \\ &+ 2\alpha_2 \left(\lambda^4 \sum_{l=1}^{M} b_{lj}^{(3)} W_{i,l} + \lambda^2 \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(2)} b_{jl_2}^{(1)} W_{l_1,l_2} \right) \\ &= \Omega^2 (1+\beta_1 X(i) + \beta_2 Y(j)) W_{i,j} \end{aligned} \tag{9}$$

where N, M are the number of grid points in the X and Y directions and $a_{il}^{(n)}$, $b_{il}^{(m)}$ are the weighting coefficients in the X and Y directions, respectively. Similarly, the boundary conditions can be non-dimensionalized and then discretized by the GDQ as shown below.

For clamped edge:

$$\begin{aligned} W_{1,j} = W_{N,j} = \sum_{l=1}^{N} a_{1l}^{(1)} W_{l,j} = \sum_{l=1}^{N} a_{Nl}^{(1)} W_{l,j} = 0, \\ W_{i1} = W_{iM} = \sum_{l=1}^{M} b_{1l}^{(1)} W_{i,l} = \sum_{l=1}^{M} b_{Ml}^{(1)} W_{i,l} = 0 \end{aligned} \tag{10}$$

For simply supported edge:

$$\begin{aligned} W_{1,j} = W_{N,j} = \sum_{l=1}^{N} a_{1l}^{(2)} W_{l,j} = \sum_{l=1}^{N} a_{Nl}^{(2)} W_{l,j} = 0, \\ W_{i1} = W_{iM} = \sum_{l=1}^{M} b_{1l}^{(2)} W_{i,l} = \sum_{l=1}^{M} b_{Ml}^{(2)} W_{i,l} = 0 \end{aligned} \tag{11}$$

Here, the grid points chosen for collocation are the zeroes of shifted Chebyshev polynomial with orthogonality range [0, 1] given by

$$\begin{aligned} X_{i+1} &= \frac{1}{2}\left[1 + \cos\left(\frac{2i-1}{N-2}\frac{\pi}{2}\right)\right] & i = 1, 2, \ldots, N-2 \\ Y_{j+1} &= \frac{1}{2}\left[1 + \cos\left(\frac{2j-1}{M-2}\frac{\pi}{2}\right)\right] & j = 1, 2, \ldots, \ldots \end{aligned} \tag{12}$$

Numerical results and discussions: Equation (8) together with the boundary conditions form a standard eigenvalue problem [8], which has been solved to obtained the numerical values of frequency parameters Ω for various values of plate parameters. The lowest three eigenvalues have been considered as the first three natural frequencies corresponding to the different boundary conditions considered here. The values of various plate parameters in the present work are taken as follows: Non-homogeneity parameters α_1, α_2 = (-0.5(0.1)0.5), density parameters β_1, β_2 = (-0.5(0.1)0.5), aspect ratio a/b = (0.25(0.25)2.0) and Poisson's ratio ν =0.3.

To choose an appropriate number of grid points (N, M), convergence studies have been carried out for various set of plate parameters until the first six significant digits had converged. The convergence of frequency parameter Ω for the first three modes of vibration for a particular set i.e. $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0.5,\ a/b = 1$ is shown in Table 1. The values of both the grid points N and M have been fixed as 15 for all the three boundary conditions, since there was no further improvement in the values of frequency parameter Ω with the increasing values of grid points.

A comparison of frequency parameter Ω for homogeneous $(\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0)$ plate with different value of aspect ratio $(a/b = 0.4,\ 1.0,\ 2.0)$ with those results obtained by other methods has been presented in Table 2. A close agreement of results is obtained.

The effect of non-homogeneity parameter α_1 on the frequency parameter Ω for $\alpha_2 = \pm 0.5,\ \beta_1 = \pm 0.5,\ \beta_2 = 0.5$ and $a/b = 1$ for the first two modes of vibration has been shown in fig.2. It is observed that (i) frequency parameter Ω increases with the increasing values of α_1 (ii) frequency parameter Ω increases with the increasing values of α_2 and (iii) as β_1 increases, the value of Ω decreases for all the three boundary conditions. The rate of increase of Ω with α_1 decreases with the increasing values of α_2 as well as β_1 keeping all other parameters fixed. The rate of increase of Ω with α_1 is in the order of the boundary conditions CCCC>SCSC>SSSS for both the values of α_2 and β_1 for the first two modes of vibration whatever be the values of other parameters.

This rate is higher in the second mode as compared to the first mode for all the three boundary conditions.

Fig. 2 depicts the behavior of the frequency parameter Ω with the density parameter β_1 for $\alpha_1 = \pm 0.5,\ \beta_2 = \pm 0.5,\ \alpha_2 = 0.5$ and $a/b = 1$ for the first two modes of vibration. It is found that (i) frequency parameter Ω decreases with the increasing values of β_1 (ii) frequency parameter Ω increases with the increasing values of α_1 (iii) as β_2 increases, the value of Ω decreases for the fixed values of the other parameters. The rate of decrease of frequency parameter Ω with β_1 increases with the increasing values of α_1 while it is decreases with the increasing values of β_2. The rate of decrease of Ω with the increasing values of β_1 for CCCC plate is higher than that for SCSC and SSSS plates when α_1 and β_2 changes from -0.5 to 0.5. This rate is more pronounced in the second mode as compared to first mode.

Fig. 3 illustrate the behavior of frequency parameter Ω with the increasing values of a/b for $\alpha_2 = \beta_2 = \pm 0.5$, $\alpha_1 = 0.5$ and $\beta_1 = 0.5$ for the first two modes of vibration. It is clear that (i) frequency parameter Ω increases with the increasing values of aspect ratio a/b (ii) frequency parameter Ω decreases with the increasing values of α_2 and (iii) further it increases, with the increasing values of β_2 for the fixed values of other parameters. The rate of increase of Ω with a/b is in the order of the boundary conditions CCCC>SSSS>SCSC for the first mode of vibration and it becomes CCCC>SCSC>SSSS for the second mode of vibration keeping other parameters fixed. This rate of increase is much higher for $a/b > 1$ as compared to $a/b < 1$. In case of third mode of vibration the behavior of frequency parameter with other parameters remains almost same as that for first two modes except that the rate of increase/decrease with a specific parameter is higher (graphs are not given here).

Conclusion : The effect of non-homogeneity of the plate material arises due to the linear variations in Young's modulus and density of the plate material with in-plane co-ordinates x and y, on the natural frequencies of isotropic plate of uniform thickness has been studied using generalized differential quadrature method on the basis of Kirchhoff plate theory. It is observed that the values of frequency parameter Ω increases with the increasing values of non-homogeneity parameters α_1 and α_2 as the plate become stiffer and stiffer towards both the edges $x = 1$ and $y = 1$, while it decreases as the plate becomes more and more dense towards both the edges $x = 1$ and $y = 1$ due to an increase in the values of density parameter β_1 and β_2 for all the three boundary conditions keeping other plate parameters fixed. The frequency parameter Ω also increases with the increasing values of $a/b = 1$. The present analysis will be of great use to the design engineers in obtaining the desired frequency by varying one or more plate parameters considered here.

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Table 1. Convergence study for the first three frequencies for $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0.5, a/b = 1$

MODE ⟶	I	II	III	I	II	III	I	II	III
No. of grid points	CCCC			SCSC			SSSS		
$N = M = 8$	35.9157	72.6215	72.6535	28.8985	54.9363	68.5322	19.7143	49.5724	49.6016
$N = M = 10$	35.9098	73.2830	73.3153	28.8935	54.6951	69.2420	19.7077	49.2999	49.3345
$N = M = 12$	35.9097	73.2890	73.3213	28.8934	54.7059	69.2490	19.7078	49.3122	49.3464
$N = M = 13$	35.9097	73.2889	73.3213	28.8934	54.7057	69.2490	19.7078	49.3120	49.3462
$N = M = 14$	35.9097	73.2889	73.3213	28.8934	54.7056	69.2489	19.7078	49.3119	49.3461
$N = M = 15$	35.9097	73.2889	73.3213	28.8934	54.7056	69.2489	19.7078	49.3119	49.3461

Table 2. Comparison of frequency parameter Ω for homogeneous $\alpha_1 = \beta_1 = \alpha_2 = \beta_2 = 0$ and $\nu = 0.3$.

Boundary conditions	Ref.	Mode I	II	III	I	II	III	I	II	III
		0.4			1.0			2.5		
CCCC	[9]	23.648	27.817	35.446	35.992	73.413	73.413	147.80	173.85	221.54
	[10]	--	--	--	35.9855	73.395	73.395	--	--	--
	[11]	--	--	--	35.9855	73.3947	73.3947	--	--	--
	[12]	23.64	27.81	35.44	35.99	73.41	108.26	147.78	173.84	221.52
	[13]	23.64	27.81	35.44	35.99	73.39	73.39	147.78	173.80	221.50
	[14]	23.31	26.69	33.48	35.45	72.03	72.03	145.72	166.84	209.31
	[7]	--	--	--	35.9855	73.3954	73.3954	--	--	--
	present	23.6438	27.8069	35.4171	35.9852	73.3938	73.3938	147.774	173.793	221.357
SCSC	[9]	12.1347	18.3647	27.9657	28.9509	54.7431	69.3270	145.484	164.739	202.227
	[11]	--	--	--	28.9509	54.7432	69.3270	--	--	--
	[12]	12.13	18.37	28.00	28.95	54.88	69.34	145.48	164.80	202.43
	[13]	12.13	18.36	27.99	28.95	54.74	69.33	145.48	164.74	202.24
	[14]	--	--	--	--	--	--	--	--	--
	[7]	--	--	--	28.9509	54.7431	69.3270	--	--	--
	present	12.1347	18.3647	27.9657	28.9509	54.7431	69.327	145.484	164.739	202.227
SSSS	[9]	11.4487	16.1862	24.0818	19.7392	49.3480	49.3480	71.5564	101.1634	150.571
	[10]	--	--	--	19.739	49.348	49.348	71.555	101.164	150.991
	[11]	--	--	--	19.7392	49.3481	49.3481	--	--	--
	[12]	11.45	16.19	24.15	19.74	49.35	79.03	71.55	101.19	150.95
	[13]	11.45	16.19	24.08	19.74	49.35	49.35	71.00	101.16	150.53
	present	11.4487	16.1862	24.0818	19.7392	49.348	49.348	71.5546	101.1641	150.5115

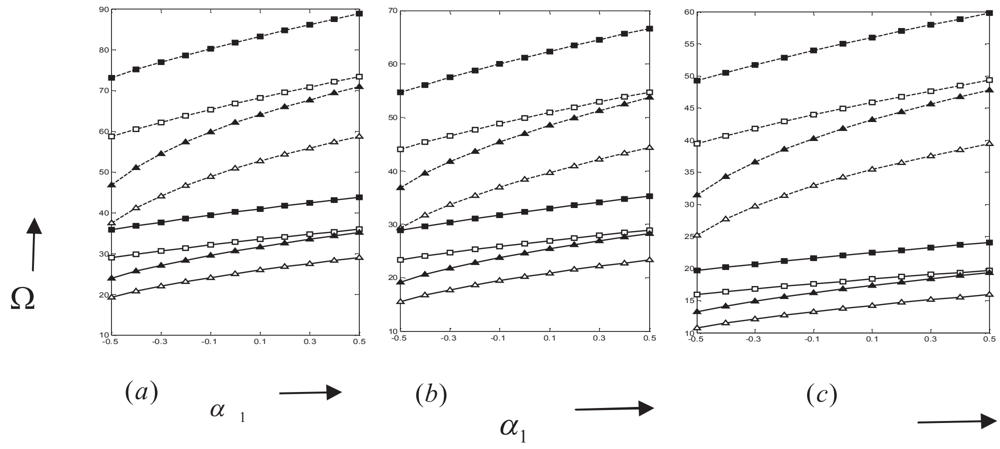

Fig. 1 Frequency parameter Ω for (a) CCCC, (b) SCSC, and (c) SSSS plate for $\beta_2 = 0.5$. First mode : ——; second mode: ……; Δ, $\alpha_2 = 0.5,\ \beta_1 = 0.5$; ▲, $\alpha_2 = 0.5,\ \beta_1 = -0.5$; □, $\alpha_2 = -0.5,\ \beta_1 = 0.5$; ■, $\alpha_2 = -0.5,\ \beta_1 = -0.5$.

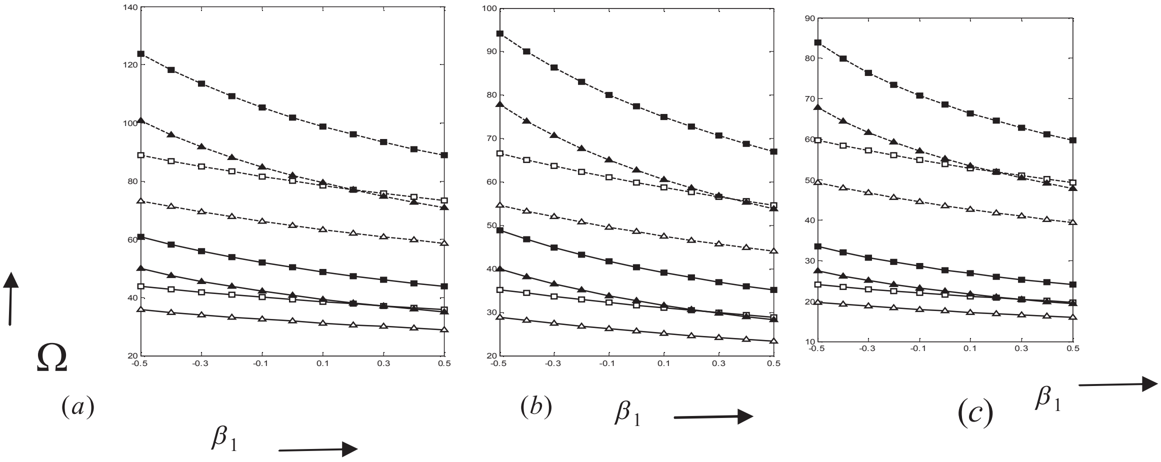

Fig. 2 Frequency parameter Ω for (a) CCCC, (b) SCSC, and (c) SSSS plate for $\alpha_2 = 0.5$. First mode : ——; second mode: ……; Δ, $\alpha_1 = 0.5,\ \beta_2 = 0.5$; ▲, $\alpha_1 = 0.5,\ \beta_2 = -0.5$; □, $\alpha_1 = -0.5,\ \beta_2 = 0.5$; ■, $\alpha_1 = -0.5,\ \beta_2 = -0.5$.

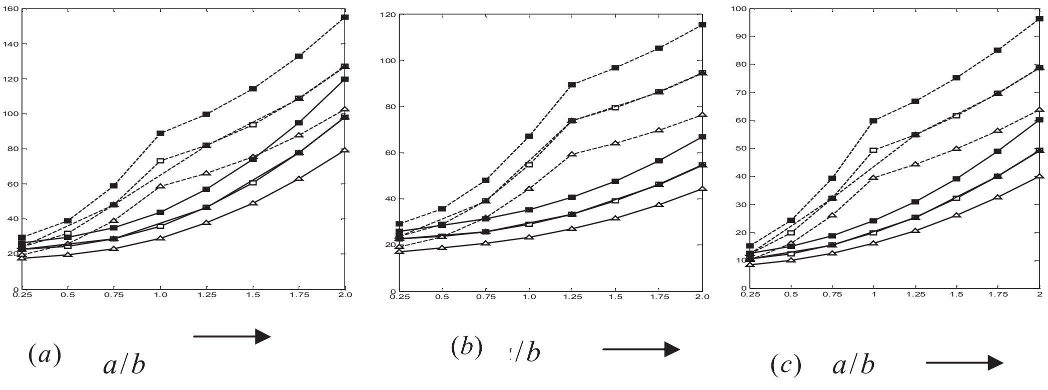

Fig. 3 Frequency parameter Ω for (a) CCCC, (b) SCSC, and (c) SSSS plate for $\alpha_1 = \beta_2 = 0.5$. First mode : ——; second mode: ……; Δ, $\alpha_2 = 0.5,\ \beta_2 = 0.5$; ▲, $\alpha_2 = 0.5,\ \beta_2 = -0.5$; □, $\alpha_2 = -0.5,\ \beta_2 = 0.5$; ■, $\alpha_2 = -0.5,\ \beta_2 = -0.5$.

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