# STEADY STATE THERMAL STRESSES IN THIN CIRCULAR DISC BY DEFORMATION WITH VARIABLE LOAD FOR DIFFERENT MATERIALS BY CONCEPT OF SETH'S TRANSITION THEORY

**PANKAJ THAKUR, ANUPAM SEMWAL**

**Abstract**: Seth's transition theory is applied to the problems steady state thermal stresses in a thin circular disc by deformation with load. Neither the yield criterion nor the associated flow rule is assumed here the result obtained here are applicable to compressible material as well as incompressible material. If the additional condition of incompressibility is imposed then the expression for stresses corresponds to those arising from Tresca yield condition. It has observed that for incompressible material (*i.e.* rubber) required higher angular speed for initial yielding as compare to compressible materials( *i.e.* Copper or Lead). With the introduction of thermal effect rotating disc with loading edge much required to lesser angular speed as compare to rotating disc without thermal effect. Thermal effect it decrease the value of circumferential stresses at the internal surface and radial stresses is maximum at the external surface for incompressible (*i.e* rubber) as compare to compressible materials (*i.e* copper or Lead).

**Keywords:** Thermal, transitional, stresses, disc, edge, yielding.

**Introduction:** Rotating disks are important components in various mechanical applications such as circular saws, disk brakes, hard disks, optical discs, flywheel, rotors, compressors and gas turbines. These are often subject to loading or excitation in the transverse (out-of-plane) direction. Analysis of rotating disks has been studied by a number of researchers. The analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier [1] in the elastic range and by Chakrabarty [2] and Heyman [3] for the plastic range. Their solution for the problem of fully plastic state does not involve the plane stress condition, that is to say, we can obtain the same stresses and angular velocity required by the disc to become fully plastic without using the plane stress condition (i.e. $T_{zz}=0$). Gupta *etl.* [9] obtained a different solution for the fully plastic state by using Seth's transition theory and plane stress condition. Seth's transition theory [5] does not required any assumptions like an yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems [9-12, 14-39].

Seth [4] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right],$$

$(i = 1,2,3)$ (1)

where $n$ is the measure and $\overset{A}{e}_{ii}$ is the Almansi finite strain components [5]. For $n$ = -2, -1, 0, 1, 2 it gives Cauchy, Green Hencky, Swainger and Almansi measures respectively.In this paper, we investigate the problem of steady state thermal stresses in circular disc by deformation with variable load for different materials was investigated by concept of Seth's transition theory. Results have been discussed and presented graphically.

**Mathematical Models:** We consider circular annular disc of inner radius $a$ and outer radius $b$ rotating with an angular speed $\omega$ about an axis perpendicular to its plane and passed through the center as shown in fig.1. The circular annular disc, produced of material of constant density, is mounted on a edge loading. The thickness of disc is assumed to be constant and is taken sufficiently small so that the disc is effectively in a state of plane stress, that is, the axial stress $T_{zz}$ is zero. The temperature at the central bore of the disc is $T$ .

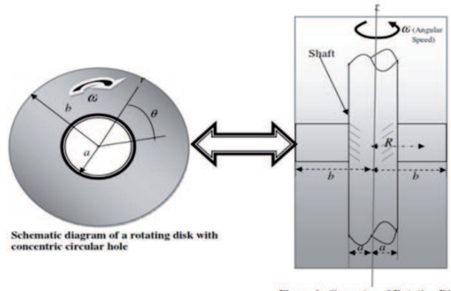



Figure 1. Geometry of Rotating Disc.

**Boundary Conditions:** The disk consider in the present study is subjected to a temperature gradient field. The inner surface of the disk is assume to be fixed to a shaft so that isothermal condition can be prevailed on it. The outer surface of the disc is free from any mechanical load and maintained uniform temperature gradient. So that boundary condition of the problem are given by:

$\tau_{rr} = 0$ at $r = a$, $\tau_{rr} = L_0$ at $r = b$ (2)

where $\tau_{rr}$ are the radial stresses and $L_0$ load at the external surface.

**Governing Equation:**The displacement components in cylindrical polar co-ordinate are given by [5]:

$u = r(1-\beta)$, $v = 0$, $w = dz$ (3)

where $\beta$ is position function, depending on $r = \sqrt{x^2+y^2}$ only, and $d$ is a constant.

The finite strain components are given by Seth [5] as:

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1-(r\beta'+\beta)^2\right], \overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right],$$

$$\overset{A}{e}_{zz} = \frac{1}{2}\left[1-(1-d)^2\right],$$

$$\overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0 \quad (4)$$

where $\beta' = d\beta/dr$ and meaning of superscripts "$A$" is Almansi.

By substituting eq. (4) in eq. (1), the generalized components of strain become:

$$e_{rr} = \frac{1}{n}\left[1-(r\beta'+\beta)^n\right], e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right],$$

$$e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right],\ e_{r\theta} = e_{\theta z} = e_{zr} = 0 \quad (5)$$

The stress–strain relations for isotropic material are given by [13]:

$\tau_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi T\delta_{ij}$, $(i, j = 1, 2, 3$ (6)

where $\tau_{ij}$ is the stress components, $\lambda$ and $\mu$ are Lame's constants and $I_1 = e_{kk}$ is the first strain invariant, $\delta_{ij}$ is the Kronecker's delta and $\xi = \alpha(3\lambda + 2\mu)$, $\alpha$ being the coefficient of thermal expansion and $T$ is the temperature Further, $T$ has to satisfy:

$\nabla^2 T = 0$.

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{d^2T}{dr^2} \equiv \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0,$$

or $\dfrac{dT}{dr} = \dfrac{K}{r}$.

which has solutions:

$T = K_1(\log r + k_2)$ (7)

where $K_1$ and $K_2$ are constant of integration and can determined from the boundary condition.

Equ.(6) for this problem become:

$$\tau_{rr} = \frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{rr} - \frac{2\mu\xi T}{(\lambda+2\mu)},$$

$$\tau_{\theta\theta} = \frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2e_{\theta\theta} - \frac{2\mu\xi T}{(\lambda+2\mu)},$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0. \quad (8)$$

where $E$ is the Young's modulus and $C$ is compressibility factor of the material in term of Lame's constant, there are given by $E = \dfrac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}$ and $C = \dfrac{2\mu}{\lambda+2\mu}$.

Substituting equation (5) in equation (8), we get

$$\tau_{rr} = \frac{2\mu}{n}\left[3-2C-\beta^n\left\{1-C+(2-C)(P+1)^n + \frac{nC\xi T}{2\mu\beta^n}\right\}\right],$$

$$\tau_{\theta\theta} = \frac{2\mu}{n}\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n + \frac{nC\xi T}{2\mu\beta^n}\right\}\right],$$

$$\tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = \tau_{zz} = 0, \quad (9)$$

where $C$ is compressibility factor of the material in term of Lame's constant, there are given by $C = \dfrac{2\mu}{\lambda+2\mu}$.Equations of equilibrium are all satisfied except:

$$\frac{d}{dr}(r\tau_{rr}) - \tau_{\theta\theta} + \rho\omega^2 r^2 = 0, \quad (10)$$

where $\rho$ is the density of the material of the rotating disc. The temperature satisfying Laplace equation (7) with boundary condition: $T = T_0$ at $r = a$, $T = 0$ at $r = b$, where $T_0$ is constant, given by [13]: $k_1 = \dfrac{T}{\log(a/b)}$ and $k_2 = -\log b$.

Substituting $k_1$ and $k_2$ form equation (7), we get

$$T = \frac{T_0\log(r/b)}{\log(a/b)}. \quad (11)$$

Using equations (9) and (11) in equation (10), we get a non- linear differential equation in $\beta$ as

$$(2-C)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta}=\frac{n\rho\omega^2 r^2}{2\mu}$$
$$-\frac{nC\xi\overline{T}_0}{2\mu}+\beta^n\left[1-(P+1)^n-nP\left\{1-C+(2-C)(P+1)^n\right\}\right], \qquad (12)$$

where $\overline{T}_0=\dfrac{T_0}{\log(a/b)}$ and $r\beta'=\beta P$ ($P$ is function of $\beta$ and $\beta$ is function of $r$).

From equation (12), the turning points of $\beta$ are $P=-1$ and $\pm\infty$.

## 5. Solution through the Principal Stresses

For finding the plastic stress, the transition function is taken through the principal stress (see Seth [4-7], Gupta *etl.* [9-12], Pankaj Thakur [14 – 39]) at the transition point $P\to\pm\infty$. We take the transition function $\Re$ is defined as:

$$\Re=\frac{n}{2\mu}\left[T_{\theta\theta}-C\xi\Theta\right]=\left[\begin{matrix}(3-2C)\\ -\beta^n\left\{\begin{matrix}2-C\\ +(1-C)(P+1)^n\end{matrix}\right\}-\dfrac{nC\xi T}{\mu}\end{matrix}\right] \qquad (13)$$

Taking the logarithmic differentiating of eq. (13) with respect to r, one gets:

$$\frac{d(\log\Re)}{dr}=-\left(\frac{n\beta^n P}{r}\right)\left[\frac{\begin{matrix}2-C+(1-C)(P+1)^{n-1}\\ \left\{(P+1)+\beta.dp/d\beta\right\}+(2-C)nP\beta^n\end{matrix}}{r\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}-\dfrac{nC\xi T}{2\mu}\right]}\right]$$

(14)

By substituting the value of $dP/d\beta$ from eq. (12) into eq. (14) we have

$$\frac{d(\log\Re)}{dr}=-\left[\frac{\begin{matrix}\beta^n\left(\dfrac{1-C}{2-C}\right)\\ \left[\begin{matrix}1-(P+1)^n-n(1-C)P\\ +\dfrac{n\rho\omega^2r^2}{2\mu\beta^n}+\dfrac{nC\xi\overline{T}_o(3-2C)}{\mu(4-2C)\beta^n}\end{matrix}\right]\\ +(2-C)nP\beta^n\end{matrix}}{r\left[\begin{matrix}3-2C\\ -\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\\ -\dfrac{nC\xi T}{2\mu}\end{matrix}\right]}\right] \qquad (15)$$

Taking the asymptotic value $P\to\pm\infty$ from equation (15), one get

$$\frac{d(\log\Re)}{dr}=-\frac{1}{(2-C)r} \qquad (16)$$

Integrating equation (16), one gets:

$$\Re=K_1r^{-1/(2-C)} \qquad (17)$$

where $K_1$ is a constant of integration, which can be determine by boundary condition. From equation (13) and (17), we have

$$\tau_{\theta\theta}=\left(\frac{2\mu}{n}\right)k_1r^{-1/(2-C)}+\frac{C\xi T_0\log(r/b)}{\log(a/b)}. \qquad (18)$$

Substituting equation (18) in equation (10) and integrating, we get:

$$\tau_{rr}=\left(\begin{matrix}\left\{\dfrac{2\mu(2-C)}{n(1-C)}\right\}K_1r^{-1/(2-C)}+\dfrac{C\xi T_0\log(r/b)}{\log(a/b)}\\ -\dfrac{C\xi T_0}{\log(a/b)}-\dfrac{\rho\omega^2r^2}{3}+\dfrac{k_2}{r}\end{matrix}\right) \qquad (19)$$

where $k_2$ is a constant of integration, which can be determine by boundary condition.

By applying boundary condition (2) in eq. (19), we gets:

$$k_1=\frac{n(1-C)}{2(2-C)\mu}\left[\frac{bL_0+\dfrac{\rho\omega^2}{3}\left[b^3-a^3\right]+\dfrac{C\xi T_0}{\ln(a/b)}\left[a\ln(a/b)-a+b\right]}{b^{\frac{1-C}{2-C}}-a^{\frac{1-C}{2-C}}}\right];$$

$$k_1=-a^{\frac{1-C}{2-C}}\left[\frac{\begin{matrix}bL_0+\dfrac{\rho\omega^2}{3}\left[b^3-a^3\right]\\ +\dfrac{C\xi T_0}{\ln(a/b)}\left[a\ln(a/b)-a+b\right]\end{matrix}}{b^{\frac{1-C}{2-C}}-a^{\frac{1-C}{2-C}}}\right]-C\xi T_0a\left[\frac{\ln(a/b)-1}{\ln(a/b)}\right]+\frac{\rho\omega^2a^3}{3}$$

By substituting the value of $k_1$ and $k_2$ into equations (18) and (19), we gets:

$$\tau_{rr}=\left[\begin{matrix}\dfrac{r^{\frac{1-C}{2-C}}-a^{\frac{1-C}{2-C}}}{r\left(b^{\frac{1-C}{2-C}}-a^{\frac{1-C}{2-C}}\right)}\left[\begin{matrix}bL_0+\dfrac{\rho\omega^2}{3}\left[b^3-a^3\right]\\ +\dfrac{\alpha E(2-C)T_0}{\ln(a/b)}\\ \left[a\ln(a/b)-a+b\right]\end{matrix}\right]+\dfrac{\rho\omega^2}{3r}\left[a^3-r^3\right]\\ +\dfrac{\alpha E(2-C)T_0}{\log(a/b)}\left[\left\{\ln(r/b)-1\right\}-\dfrac{a}{r}\left\{\ln(a/b)-1\right\}\right]\end{matrix}\right] \qquad (20)$$

$$\tau_{\theta\theta} = \frac{r^{-\frac{1}{2-C}}(1-C)}{(2-C)}\left[\frac{bL_0 + \frac{\rho\omega^2}{3}\left(b^3 - a^3\right) + \frac{\alpha E(2-C)T_0}{\ln(a/b)}\left(a\ln(a/b) - a + b\right)}{b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}}}\right] + \frac{\alpha E(2-C)T_0 \log(r/b)}{\log(a/b)}$$

(21)

It is seen from equ. (21) that $\tau_{\theta\theta}$ is maximum at the internal surface, therefore, yielding will take place at the internal surface and equ. (21) become:

$$\left|\tau_{\theta\theta}\right|_{r=a} = \frac{a^{-\frac{1}{2-C}}(1-C)}{(2-C)}\left[\frac{bL_0 + \frac{\rho\omega^2}{3}\left[b^3 - a^3\right] + \frac{\alpha E(2-C)T_0}{\ln(a/b)}\left[a\ln(a/b) - a + b\right]}{b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}}}\right] + \alpha E(2-C)T_0 \equiv Y(say)$$

and angular speed $\omega_i$ necessary for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \frac{3b^2}{\left(b^3 - a^3\right)}\left[\frac{\left(b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}}\right)(2-C)}{a^{-\frac{1}{2-C}}(1-C)} - b\sigma_0 - \frac{\alpha ET_0(2-C)}{Y}\left\{a - \frac{a}{\ln(a/b)} + \frac{b}{\ln(a/b)} + \frac{(2-C)\left(b^{\frac{1-C}{2-C}} - a^{\frac{1-C}{2-C}}\right)}{(1-C)a^{-\frac{1}{2-C}}}\right\}\right]$$

(22)

where $\sigma_0 = L_0/Y$ and $\omega_i = \frac{1}{b}\Omega_i(Y/\rho)^{1/2}$. We introduce the following non-dimensional components as: $R = r/b$, $R_0 = a/b$, $\Omega^2 = \rho\omega^2 b^2/Y$, $\sigma_r = \tau_{rr}/Y$, $\alpha ET_0/Y = T_1$. Eqs. (20), (21) and (22) become:

$$\sigma_r = \left[\left(\frac{R^{\frac{1-C}{2-C}} - R_0^{\frac{1-C}{2-C}}}{R\left(1 - R_0^{\frac{1-C}{2-C}}\right)}\right)\left[\sigma_0 + \frac{\Omega_i^2}{3}\left(1 - R_0^3\right) + \frac{T_1(2-C)}{\ln(R_0)}\left\{R_0\ln(R_0) - R_0 + 1\right\}\right] - \frac{\Omega_i^2}{3R}\left(R^3 - R_0^3\right) + \frac{T_1(2-C)\{\ln(R_0) - 1\}}{R\ln(R_0)}\left[R - R_0\right]\right]$$

(23)

$$\sigma_\theta = \frac{(1-C)R^{-\frac{1}{2-C}}}{(2-C)\left(1 - R_0^{\frac{1-C}{2-C}}\right)}\left[\sigma_0 + \frac{\Omega_i^2}{3}\left(1 - R_0^3\right) + \frac{T_1(2-C)}{\ln(R_0)}\left\{R_0\ln(R_0) - R_0 + 1\right\}\right] + \frac{T_1(2-C)\ln R}{\ln(R_0)}$$

(24)

$$\Omega_i^2 = \frac{3}{\left(1 - R_0^3\right)}\left[\frac{\left(1 - R_0^{\frac{1-C}{2-C}}\right)(2-C)}{R_0^{-\frac{1}{2-C}}(1-C)} - \sigma_0 - T_1(2-C)\left\{R_0 - \frac{R_0}{\ln R_0} + \frac{1}{\ln R_0} + \frac{(2-C)\left(1 - R_0^{\frac{1-C}{2-C}}\right)}{(1-C)R_0^{-\frac{1}{2-C}}}\right\}\right]$$

(25)

Eqs. (23) (24) and (25) gives thermo elastic-plastic transitional stresses and angular speed for thin circular disc with loading edge. Stresses and angular speed given by eqn. (23) and (24) for fully plasticity *i.e.* $C = 0$ become:

$$\sigma_r = \left[\left(\frac{R^{\frac{1}{2}} - R_0^{\frac{1}{2}}}{R\left(1 - R_0^{\frac{1}{2}}\right)}\right)\left[\sigma_0 + \frac{\Omega_i^2}{3}\left(1 - R_0^3\right) + \frac{2\Theta_1}{\ln(R_0)}\left\{R_0\ln(R_0) - R_0 + 1\right\}\right] - \frac{\Omega_i^2}{3R}\left(R^3 - R_0^3\right) + \frac{2T_1\{\ln(R_0) - 1\}}{R\ln(R_0)}\left[R - R_0\right]\right]$$

(26)

$$\sigma_\theta = \frac{(1-C)R^{-\frac{1}{2}}}{(2-C)\left(1 - R_0^{\frac{1}{2}}\right)}\left[\sigma_0 + \frac{\Omega_i^2}{3}\left(1 - R_0^3\right) + \frac{2T_1}{\ln(R_0)}\left\{R_0\ln(R_0) - R_0 + 1\right\}\right] + \frac{2T_1\ln R}{\ln(R_0)}$$

(27)

From eqn. (21) the angular speed $\omega_f > \omega_i$ for which the disc becomes fully plastic $\nu = 0.5$ or C = 0 at the external surface is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y} = \frac{3}{\left(1-R_0^3\right)}\left[\begin{array}{l} 2\left(1-\sqrt{R_0}\right)-\sigma_0 \\ -\dfrac{2T_1}{\ln R_0}\left[R_0 \ln R_0 - R_0 + 1\right] \end{array}\right]$$

(28)

where $\omega_f = \frac{1}{b}\Omega_f \left(Y/\rho\right)^{\frac{1}{2}}$.

**Numerical Illustration and Discussion:** To see the effect of temperature on a rotating circular disc with loading edge following values have been taken: $C$ = 0 (Incompressible material *i.e.* rubber), 0.25 (Compressible material *i.e. Lead*), 0.5 (Compressible material *i.e. Copper*); $\sigma_0$ =0, 0.1, 0.3 , $T_1$ =0, 0.125, 0.29. In figure 2, curve have been drawn between angular speed $\Omega_i^2$ required for initial yielding at the internal surface of the rotating disc having loading edge $\sigma_0$ = 0, 0.1, 0.3 at different values of temperature along the radii ratio Ro = a/b. It has observed that for incompressible material (*i.e.* rubber) required higher angular speed for initial yielding as compare to compressible material (*i.e.* Lead or Copper). With the effect of loading edge, lesser angular speed required for initial yielding as compare to without edge loading. With the introduction of thermal effect rotating disc with loading edge much required to lesser angular speed as compare to rotating disc without thermal effect. Figures 3(a)-3(c), 4, curves have been drawn for stresses distribution at elastic-plastic transitional state and fully plastic state of a disc with edge loading with temperature and without temperature and radius R = r/b. It has been seen that the circumferential stresses has maximum value at the internal surface and radial stresses is maximum at the external surface of the rotating disc made of incompressible material(*i.e.* rubber) as compare to compressible material(*i.e.* Lead or Copper). With the effect of edge loading stresses must be decreased with increase values of edge load. It has been seen that temperature has a quit effect on circumferential stresses *i.e.* with the introduction of thermal effect it decrease the value of circumferential stresses at the internal surface and radial stresses is maximum at the external surface for incompressible (*i.e.* rubber) as compare to compressible material (*i.e.* Lead or Copper). Whereas from fig. 4, it can be seen that thermal effect increase the value of circumferential and radial stresses at the internal and external surface for fully-plastic state.

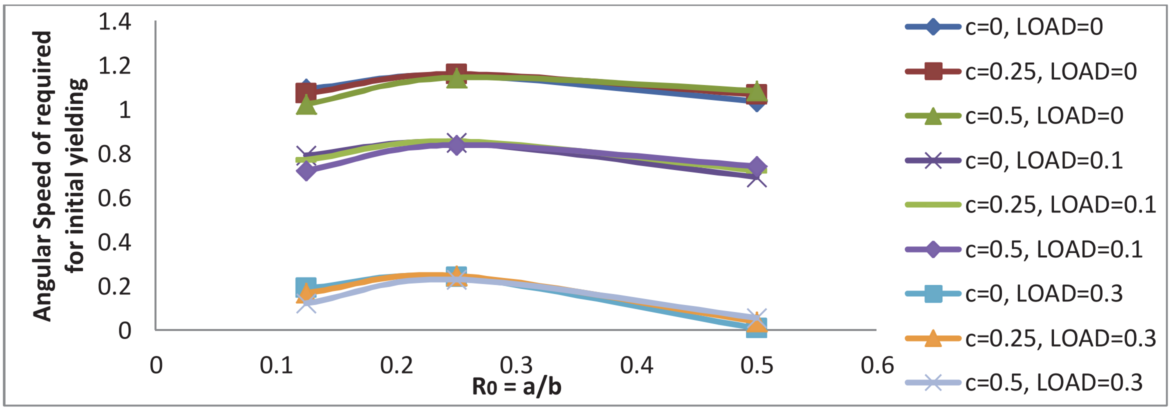
**Conclusion:** It has observed that for incompressible material (*i.e.* rubber) required higher angular speed for initial yielding as compare to compressible material(*i.e.* Lead or Copper). With the effect of loading edge, lesser angular speed required for initial yielding as compare to without edge loading. With the introduction of thermal effect rotating disc with loading edge much required to lesser angular speed as compare to rotating disc without thermal effect. With the introduction of thermal effect it decrease the value of circumferential stresses at the internal surface and radial stresses is maximum at the external surface for incompressible (*i.e.* rubber) as compare to compressible material(*i.e.* Lead or Copper).

**References:**

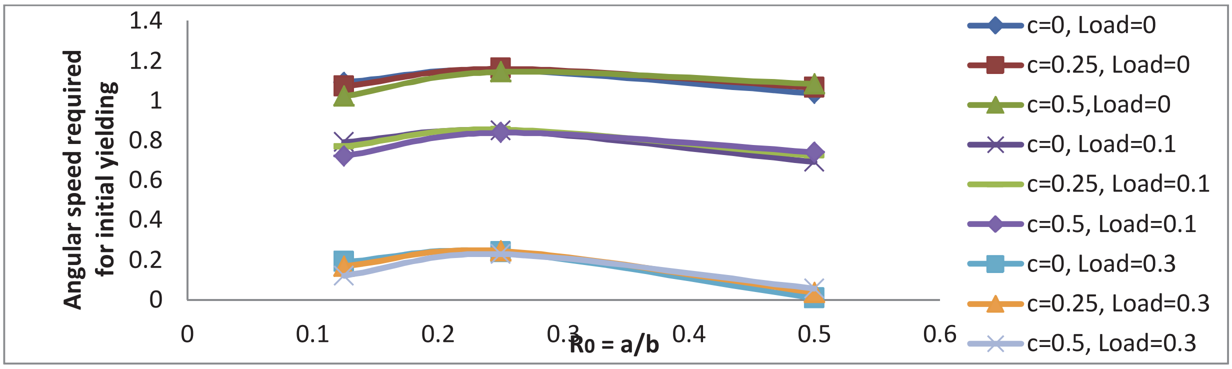

1. S.P. Timoshenko, J.N. Goodier Theory of Elasticity ", 3rd Edition, New York, McGraw-Hill Book Coy., London,1951, Ch. 2.
2. J. Chakrabarty (1987). Theory of Plasticity, New York: McGraw-Hill Book Coy., 1987, Ch. 3.
3. J. Heyman (1958). Plastic Design of Rotating Discs, Proc. Inst. Mech. Engrs., 172, 1958, pp. 531-546.
4. B.R. Seth,Transition theory of Elastic-plastic Deformation, Creep and Relaxation", Nature, 195, 1962, pp.896-897.
5. B.R. Seth, Measure Concept in Mechanics, Int. J. Non-linear Mech., 1, 1966, pp. 35-40.
6. B.R. Seth, Creep Transition, J. Math. Phys. Sci., 8, 1972, pp.1-2.
7. B.R. Seth, Elastic-plastic transition in shells and tubes under pressure, *ZAMM*, 43, 1963, pp. 345-346.
8. S. Hulsurkar (1966). Transition theory of creep shells under uniform pressure, ZAMM, 46(1): 431-437.
9. S.K. Gupta, S.K., Shukla, R.K., Elastic-plastic Transition in a Thin Rotating Disc", Ganita, 45, 1994, pp. 78-85.
10. S. K. Gupta and Pankaj, Creep Transition in an isotropic disc having variable thickness subjected to internal pressure, Proc. Nat. Acad. Sci. India, Sect. A, 78(I), 2008, pp. 57-66.
11. S.K. Gupta and Pankaj, Thermo elastic - plastic transition in a thin rotating disc with inclusion, Thermal Science, 11, 2007, pp. 103-118.
12. S.K. Gupta and Pankaj, Creep transition in a thin rotating disc with rigid inclusion, Defence Science Journal, vol. 57, 2007, pp. 185-195.
13. Parkus, H., Thermo-Elasticity, Springer-Verlag, Wien, New York, 1976, Ch. 2.
14. Pankaj, Some Problems in Elastic-plastic and Creep Transition, Ph.D. Thesis, Department of Mathematics, H.P.U. Shimla, 2006,India, Ch. 3.
15. Pankaj Thakur, Elastic - Plastic Transition Stresses in a transversely isotropic Thick -walled Cylinder

subjected to internal Pressure and steady - state Temperature, *Thermal Science*, Vol. 13, 2009, pp. 107-118.
16. Pankaj Thakur, Elastic-plastic transition stresses in a thin rotating disc with rigid inclusion by infinitesimal deformation under steady state Temperature, *Thermal Science*, Vol. 14,2012, pp. 209-219.
17. Pankaj Thakur, Creep Transition Stresses in a thin rotating disc with shaft by finite deformation under steady state temperature, *Thermal Science*, Vol. 14, 2010, pp. 425-436.
18. Pankaj Thakur, Elastic - Plastic Transition Stresses In Rotating Cylinder By Finite Deformation Under Steady- State Temperature, *Thermal Science*, Vol. 15, 2011, pp. 537-543.
19. Pankaj Thakur, Creep Transition stresses of a Thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure, *Thermal Science*, Belgrade, Serbia and Montenegro, Vol. 15, 2011, pp. S157-S165.
20. Pankaj Thakur, Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature, *Thermal Science,* Vol. 18, 2013(in press)
21. Pankaj Thakur, Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature, *Thermal Science* , Vol. 18, 2013 (in press).
22. Pankaj Thakur, Stresses in a thin rotating disc of variable thickness with rigid shaft, *Journal for Technology of Plasticity*, Serbia, Vol. 37, 2012, pp. 1-14.
23. Pankaj, Elastic-plastic transition stresses in an isotropic disc having variable thickness subjected to internal pressure, *International journal of Physical Science*, African Journal, Vol. 4,2009, pp. 336-342.
24. Pankaj Thakur and Gaurav Sharma (2009). Creep transition stresses in thick walled rotating cylinder by finitesimal deformation under steady state temperature, *International Journal of Mechanics and Solids*, Vol. 4, 2009, pp. 39-44.
25. Pankaj Thakur, Elastic - Plastic Transition in a Thin Rotating Disc having variable density with Inclusion, *Structural Integrity and life, Serbia* , Vol. 9, 2009, pp.171-179.
26. Pankaj Thakur, Elastic-Plastic Transitional Stresses in a Thin Rotating Disc with Loading Edge, Proceeding of International conference on Advances in Modeling, optimization and Computing (AMOC-2011), Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee, Dec. 5-7, 2011, pp. 318-326.
27. Dr Pankaj (2011). Stresses in a spherical shell by using Lebesgue measure concept, International Journal of the Physical Sciences, Vol. 6, , 2011, pp. 6537 – 6540.
28. Pankaj Thakur (2011) Effect of transition stresses in a disc having variable thickness and Poisson's ratio subjected to internal pressure, WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS, Vol. 6, 2011, pp. 147-159.
29. Pankaj Thakur (2011). Creep transition stresses in a spherical shell under internal pressure by using lebesgue measure concept, *International journal Applied Mechanics and Engineering*, Poland, Vol. 16, 2011, pp. 83-87.
30. Pankaj Thakur, Deformation in a thin rotating disc having variable thickness and edge load with inclusion at the elastic-plastic transitional stresses, *Structural Integrity and life,* Vol.12,2012, pp.65–70.
31. Pankaj Thakur, Thermo Creep Transition Stresses in a Thick-Walled Cylinder Subjected to Internal Pressure by finitesimal deformation, *Structural Integrity and Life*, Vol.12,2012, pp. 165-173.
32. Pankaj Thakur, S.B. Singh, Jatinder Kaur, Effect of stresses in a Thin rotating disk with loading edge for different materials, *Journal for Technology of Plasticity*, Serbia, Vol. 38, 2013, pp. 30-41.
33. Pankaj Thakur, S.B. Singh, Jatinder Kaur (2013), Thickness variation parameter in a thin rotating disc by finite deformation accepted for publication *FME Transaction*, Serbia, Vol. 41,2013,pp. 96-102.
34. Pankaj Thakur, Analysis of Stresses in a Thin Rotating Disc With Inclusion and Edge Loading, Scientific Technical Review, Serbia, Vol. 63, 2013, pp. 9-16.
35. Pankaj Thakur, Stresses in a thick-walled circular cylinder having Pressure by using concept of generalized strain measure, Kragujevac Journal of Science, Serbia, Vol. 35, 2013, pp. 41-48.
36. Pankaj Thakur and Sandeep Kumar, Analysis of stresses in a Transversely Isotropic Thin Rotating Disc with rigid inclusion having variable density parameter, JSTP Open e-journal, Japan, Vol. 9, 2013, pp. 1-13.
37. Pankaj Thakur, S. B. Singh, Jatinder Kaur, Steady Thermal stresses in a rotating disk with shaft having density variation parameter subjected to thermal load , Structural Integrity and Life, Vol. 13, 2013, pp. 109-116.
38. Pankaj Thakur, Creep transitional stresses of Orthotropic Thick-Walled cylinder under combined axial Load under internal Pressure, *Facta Universities* Series: Mechanical engineering, Vol. 11, 2013, pp. 13-18.
39. Pankaj Thakur, Finitesimale deformation in a transversely isotropic Thin rotating disc with

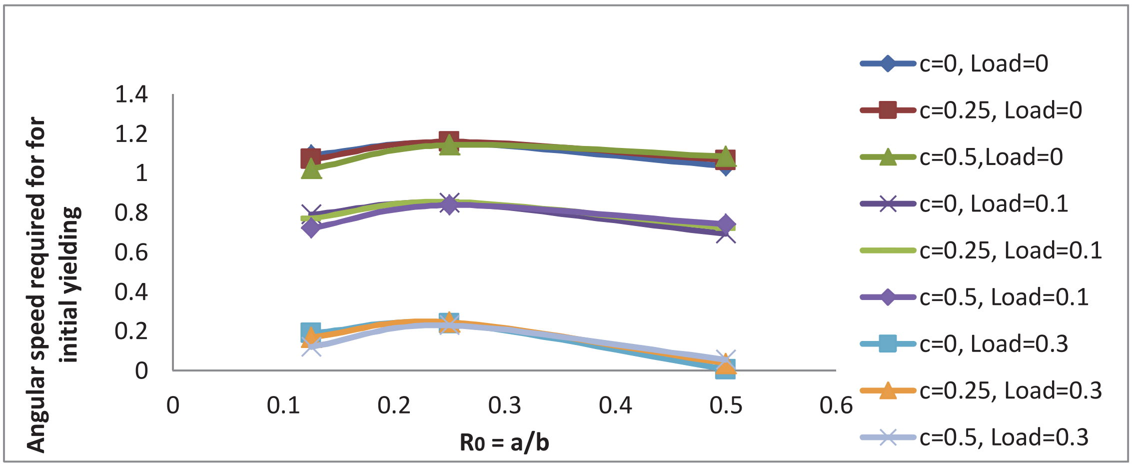
rigid shaft, Vol. 38, 2013, pp. 143-156.




$T_1 = 0.125$




$T_1 = 0.29$




$T_1 = 0$

**Figure 2**

Angular speed required for initial yielding at the internal surface of the rotating disc having loading edge $\sigma_0$ = 0, 0.1, 0.3 at different values of temperature along the radii ratio $R0$ = a/b.

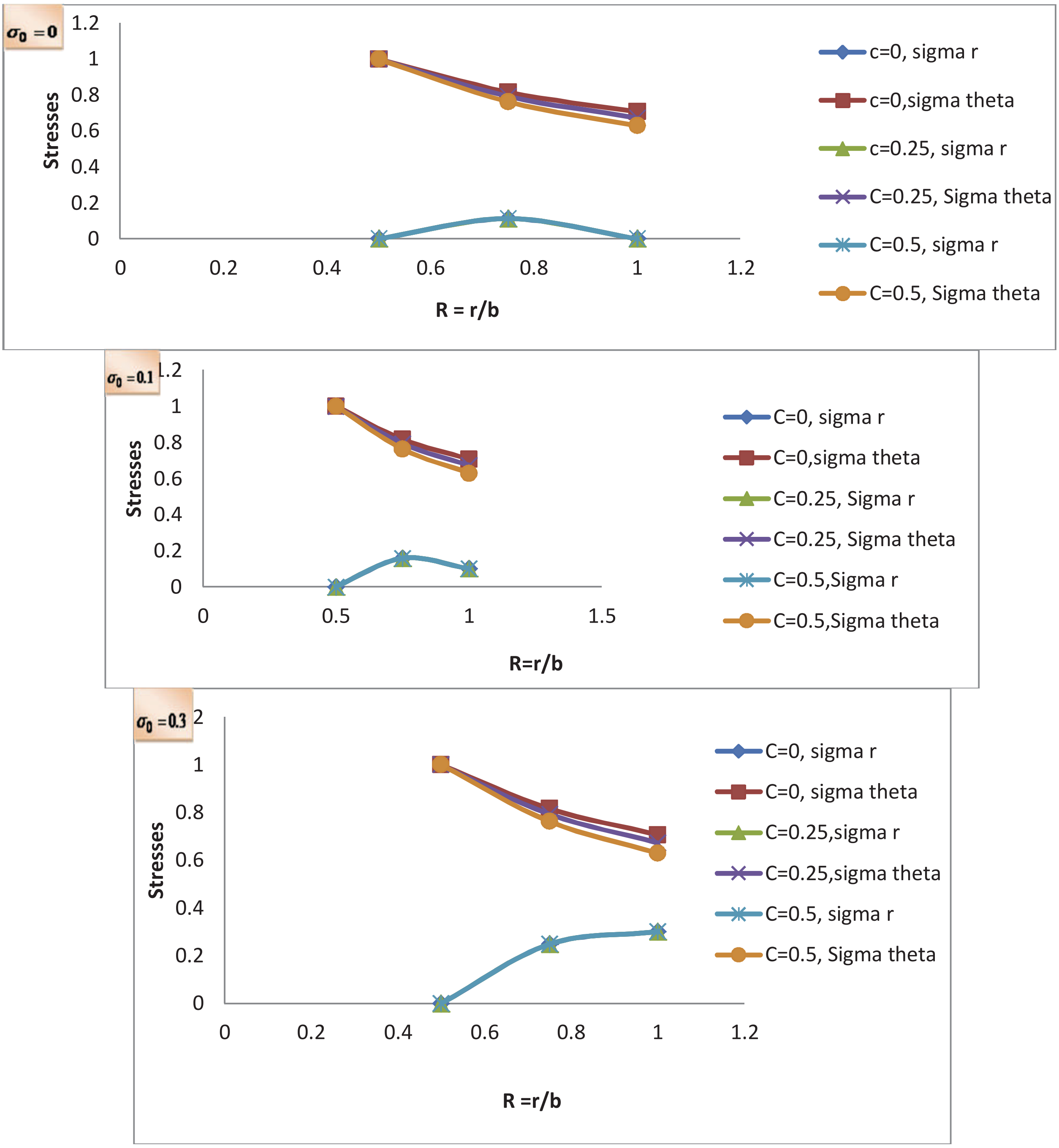



**Figure 3(a)**

Stresses at the elastic-plastic transition state without temperature $T_1 = 0$.

$T_1 = 0.125$

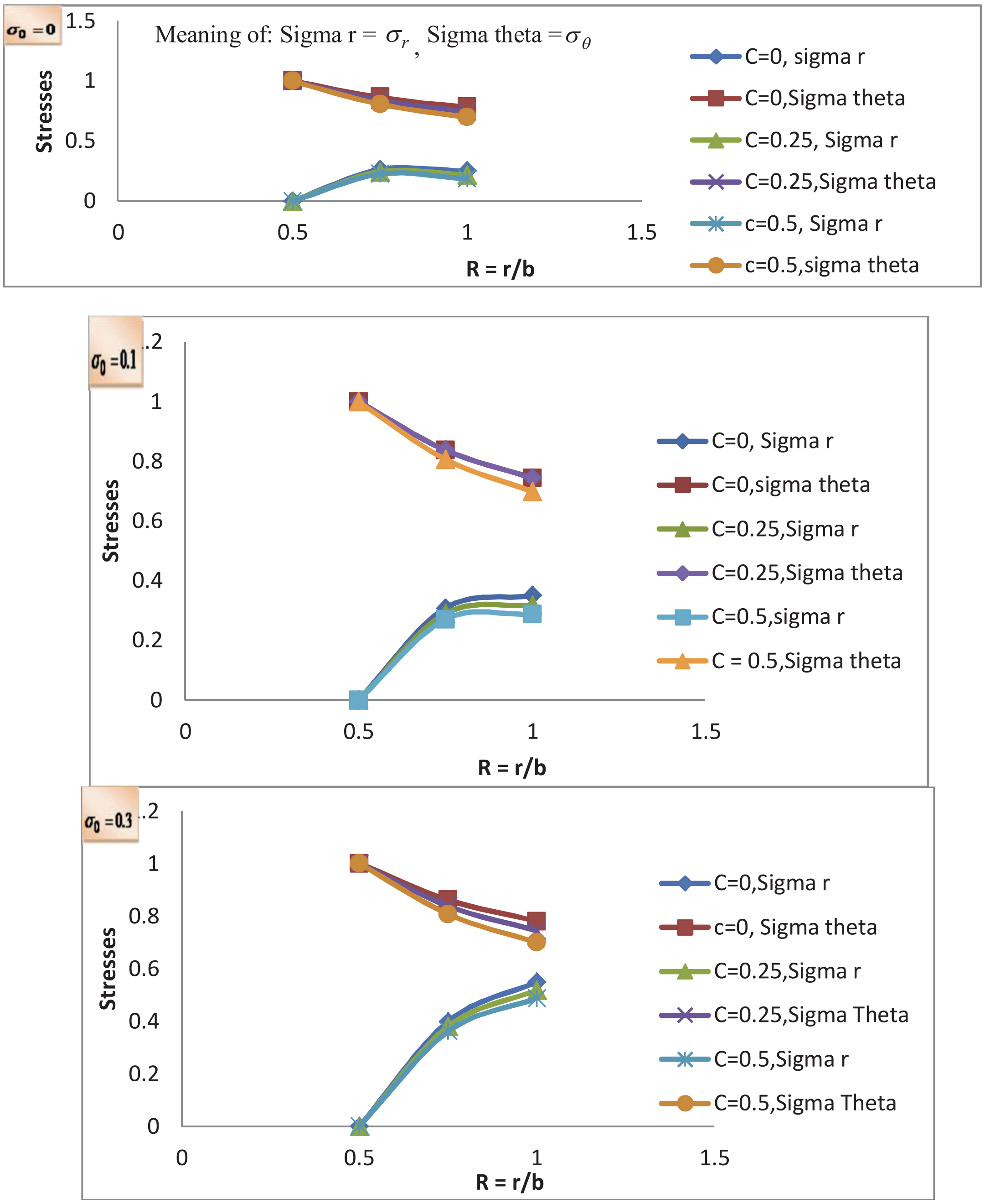



**Figure 3(b)**

Stresses at the elastic-plastic transition state with temperature $T_1 = 0.125$.

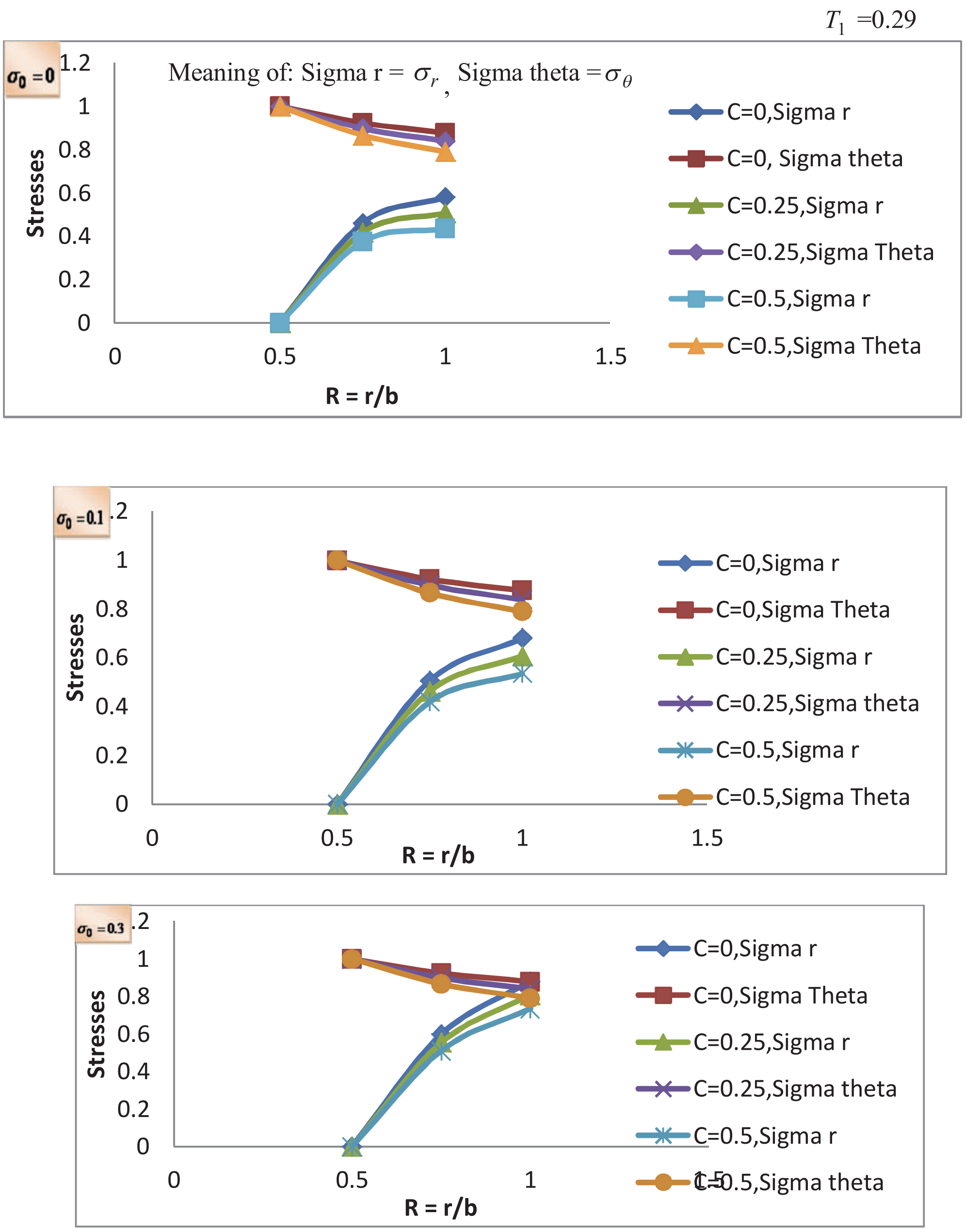
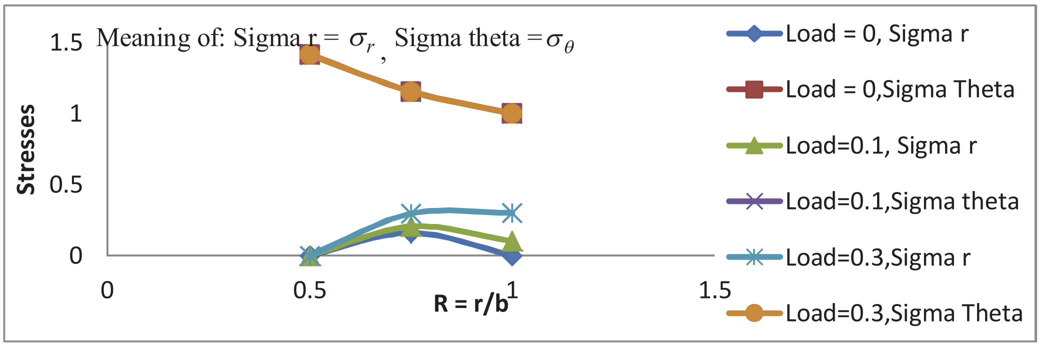


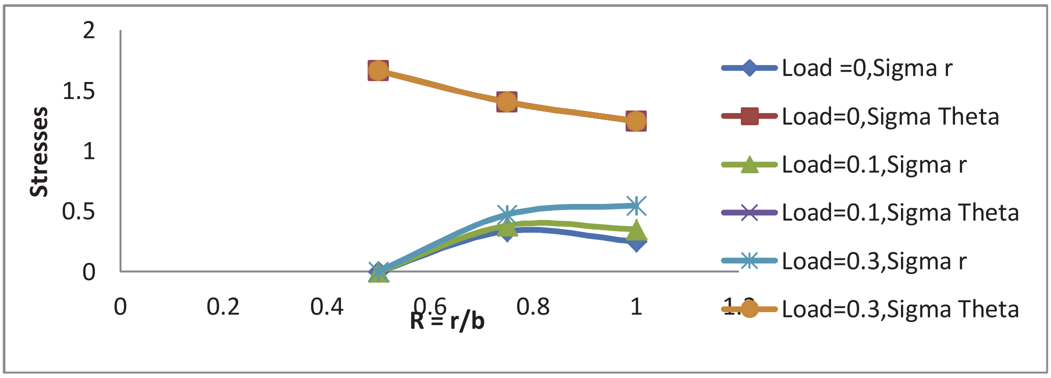

**Figure 3(c)**

**Stresses** at the elastic-plastic transition state with temperature $T_1 = 0.29$.

$T_1 = 0$




$T_1 = 0.125$




$T_1 = 0.29$

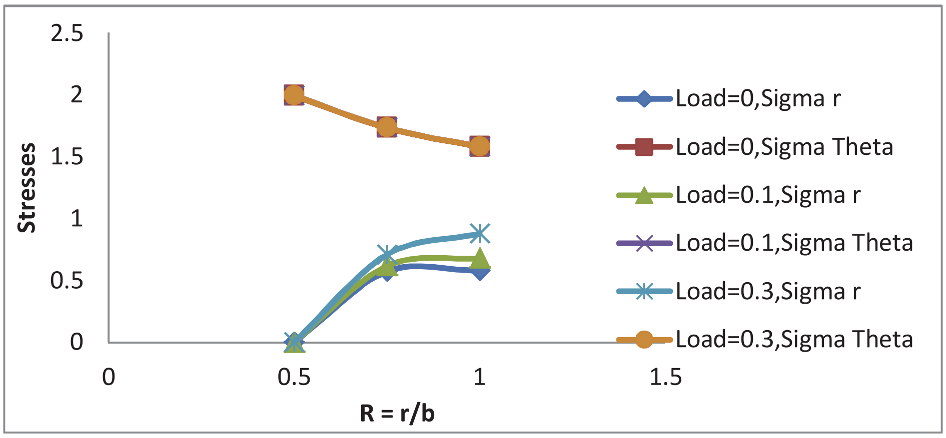



**Fig. 4**

**Stresses** for fully-plastic state at different values of Temperature.

* * *

Head Research and Development Cell, IEC University Baddi,
Himachal Pradesh, Solan, India-174103
Department of Mathematics, IEC University Baddi,
Solan, Himachal Pradesh, India-174103