A NEW COMPARISON CRITERION FOR HYBRID RULES

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Abstract: Searching techniques are used for retrieving records from a database. If linear search technique is used to search the data stored in an array and if the request probabilities are not known, the problem of reducing average search cost arises. If the records are somehow arranged according to their request probabilities, the cost can be reduced because search cost is directly proportional to the position of the record. To solve this problem, various statisticians suggested different Self Organizing Schemes (SOS) wherein, the requested records are brought forward after each request, so that they are arranged almost according to the descending order of their request probabilities in the long run.

Two well known SOS are Transposition Scheme (TR) and Move- to–Front Scheme (MTF). To improve the performance in terms of the average search cost, different hybrid rules are also developed. Generally, the SOS are compared on the basis of Average Search Cost (ASC). In [6], we have proposed a new criterion for comparing various SOS. The criterion is the expected number of records moved in the long run for the SOS under consideration. In this paper, we use the same criterion to compare some hybrid rules. We present the results in the form of theorems and give simulation results to support our theoretical findings.

Keywords: Hybrid rule, Linear Search, Move-to-Front Scheme, Optimal Ordering, Self Organizing Schemes, Transposition Scheme.

Introduction: The self organizing schemes (SOS) are developed, for faster and economical retrieval of records under the search technique adopted. During the last five decades this subject has been studied in detail and many SOS are devised by the researchers.

In his pioneering work McCabe [1] introduced the Move-to-Front and Transposition schemes. In MTF scheme, requested record is placed in the first position after use by moving other records backward to make a room for it. In TR scheme the requested record interchanges its position with the preceding record. If the requested record is at the first position it retains its position in MTF as well as TR scheme.

Assumptions:

In this paper we consider various SOS developed under the following assumptions.

1. There are 'n' records, where 'n' is a positive integer.
2. The records are requested independently of all other requests and only one record is requested at any instant of time.
3. The request probabilities of various records are unknown and non-zero but remain constant throughout the search.
4. No record of previous request is kept.

In section 2, we give notations needed for the analysis of SOS. In section 3, we give the motivation behind devising different hybrid schemes which are the combination of various SOS. We also describe the hybrid schemes proposed by Tenenbaum and Nemes [4], viz. SWITCH (k) and POS (k). In section 4, we describe our new criterion for comparison. In sections 5, 6 and 7 we derive the results regarding the new criterion for POS (k), SWITCH (k) and MMTF schemes respectively. In section 8, we give comparison of POS (k), SWITCH (k) and MMTF schemes on the basis of the comparison criterion defined in section 4. We also present the related results of computer programs. Finally, in section 9, we give conclusions.

Notations:

n: Total no of records.

R_i: Record number i (i = 1…n)

p_i: Request probability for R_i

$\underline{p}$: Request probability vector of order 1 x n

$\underline{p} = [p_1 \; p_2 \; \dots \; p_n]_{1xn}$ where $p_i > 0$ and $\sum_{i=1}^{n}(p_i) = 1$

ASC: Average Search Cost

ASC gives the expected position of requested record in the long run.

[E(X)]: Expected number of records moved for a scheme under consideration.

S_n: Sum of squares of first 'n' natural numbers

$S_n = n.(n+1).(2n+1)/6$

Need for Hybrid Scheme: In this paper, we consider the following special request probability vector-

$\underline{p} = [\, p \; r \; r \; \dots \; r]_{1xn}$ such that $0 < r < p < 1$ where $p + (n-1).r = 1$

We consider that 'R_1' is the most important record with request probability 'p' and the remaining (n – 1) records are less important than 'R_1' with request probability 'r'.

Let the stationary probability vector for 'R_1' be denoted by $\underline{\pi} = [\pi_1, \pi_2 \dots \pi_n]_{1xn}$

where π_j is the stationary probability that 'R_1' will be in the position j and $\sum_{j=1}^{n}(\pi_j) = 1$

The ASC in this case is given by

$$ASC = (p - r)\sum_{j=1}^{n} j\,\pi_j + \frac{rn(n+1)}{2}$$

[please refer to [5]]

In order to improve the performance of SOS as far as ASC is concerned different hybrid schemes are proposed. Hybrid schemes are the different combination of SOS.

Tenenbaum and Nemes [4] have suggested two classes of hybrid schemes. The first is SWITCH (k), which can be summarized as follows:

Consider a position 'k' such that $1 \le k \le n$.

(i) If the record is requested from the first position, then it will retain its position.

(ii) If the record is requested from the position 'j' (j= 2 to k) it will be placed at position '1'by shifting the records originally in the positions 1 to (j - 1) one position backward.

(iii) If the record is requested from the position 'j' (j = (k+1) to n) then it is interchanged with immediately preceding record in position 'j-1' and all other records remain unaltered.

The second scheme proposed by Tenenbaum and Nemes [4] is POS (k) which can be summarized as follows:

Consider a position 'k' such that $1 \le k \le n$.

(i) If the record is requested from the first position, then it will retain its position.

(ii) If the record is requested from the position 'j' (j= 2 to k) it will be interchanged with immediately preceding record and all other records will remain unaltered.

(iii) If the record is requested from the position 'j' (j = (k+1) to n) then it will be placed at k^{th} position by shifting the records originally in the position k to (j - 1) one position backward to make room for it.

Note that POS (1) is Move to front scheme while, POS (n - 1) is transposition scheme.

We have also devised the new scheme called Modified Move to Front Scheme (MMTF) [7] which can be summarized as follows:

Consider a position 'k' such that $1 \le k \le [n/2]$.

(i) If the record is requested from the first position, then it will retain its position.

(ii) If the record is requested from the position 'j' (j= 2 to k) it will be placed at position '1'by shifting the records originally in the positions 1 to (j - 1) one position backward to make room for it.

(iii) If the record is requested from the position 'j' (j= k+1 to n) it will be placed at position 'k' by shifting the records originally in the positions k to (j - 1) one position backward to make room for it.

New Comparison Criterion: In [6] a new comparison criterion was proposed for comparing different SOS. It is the expected number of records moved, in the scheme under consideration. Let $[E(X)]_j$ represent the expected number of records moved when record 'R_1' is in position 'j', (j = 1, 2, ..., n)

Therefore, $[E(X)] = \sum_{j=1}^{n} \pi_j [E(X)]_j$ ... (4.1)

Note that $[E(X)]_j$ will differ with the adopted scheme. In [6] we have compared various SOS like MTF, TR and MMTF on the basis of new comparison criterion. In this paper we compare various hybrid schemes on the basis of the new criterion.

We now find out [E(X)] for two hybrid schemes proposed by Tenenbaum and Nemes [4].

5. Expected Number of Records Moved under SWITCH (k) Scheme:

[E(X)] for SWITCH (k) Scheme is given by,

$[E(X)]_{SWITCH} = \sum_{j=1}^{n} \pi_j [E(X)]_j$

$= \sum_{j=1}^{k} \pi_j [E(X)]_j + \sum_{j=k+1}^{n} \pi_j [E(X)]$

5.1 Method to Find $[E(X)]_j$ for Different Values of 'j' Under SWITCH (k) Scheme:

The number of records moved under SWITCH (k) scheme when record 'R_1' is in first position is given by

$$[E(X)]_1 = \frac{k\,(k-1)r}{2} + (n-k).r$$

$$[E(X)]_2 = 1.p + \left[\frac{k\,(k-1)}{2} - 1\right].r + (n-k).r$$

In general, for j = 1, ..., k

$$[E(X)]_j = (j-1).p + \left[\frac{k\,(k-1)}{2} - (j-1)\right].r + (n-k).r \quad (5.1.1)$$

$[E(X)]_{k+1}$, the expected number of records moved when record R_1 is in $(k+1)^{th}$ position is given by:

$$[E(X)]_{k+1} = \left[\frac{k\,(k-1)}{2}.r + 1.p + (n-k-1).r\right]$$

In general, for j = k+1, ..., n

$$[E(X)]_j = \left[\frac{k\,(k-1)}{2}r + 1.p + (n-k-1).r\right] \quad \dots (5.1.2)$$

Now, we present the formula for $[E(X)]_{SWITCH}$ as a theorem.

Theorem 1:

Expected number of records moved under SWITCH (k) scheme is given by

$$[E(X)]_{SWITCH} = \left[\frac{k\,(k-1)}{2}.r + (n-k).r + (p-r)\left[\sum_{j=1}^{k}(j-1)\pi_j + \sum_{j=k+1}^{n}\pi_j\right]\right]$$

Proof:

[E(X)] under the SWITCH (k) scheme is given by

$[E(X)]_{SWITCH} = \sum_{j=1}^{n} \pi_j [E(X)]_j$

$= \sum_{j=1}^{k} \pi_j [E(X)]_j + \sum_{j=k+1}^{n} \pi_j [E(X)]_j$ ... (5.1.3)

From equation (5.1.1), (5.1.2) and we get,

For j = 1 to k

$[E(X)]_j = (j-1).p + \left[\frac{k\,(k-1)}{2} - (j-1)\right].r +$

$(n-k).r$

For j = k+1 to n

$[E(X)]_j = \left[\frac{k\,(k-1)}{2}.r + 1.p + (n-k-1).r\right]$

On simplification we get,

$[E(X)]_{SWITCH} = \left[\frac{k\,(k-1)}{2} r + (n-k).r + (p-r)\left[\sum_{j=1}^{k}(j-1)\pi_j + \sum_{j=k+1}^{n}\pi_j\right]\right]$ ... (5.1.4)

This proves the theorem.

Now, we present the formula for variance of expected number of records moved under SWITCH (k) scheme as a theorem.

Theorem 2:

Variance of the expected number of records moved under SWITCH (k) scheme is given by

$[V(X)]_{SWITCH} = \frac{k(k-1)(2k-1)}{6}.r + (n-k).r + (p-r)[\sum_{j=2}^{k}(j-1)^2\,\pi_j + \sum_{j=k+1}^{n}\pi_j] - \left[\frac{k\,(k-1)}{2}.r + (n-k).r + (p-r)\left[\sum_{j=2}^{k}(j-1)\pi_j + \sum_{j=k+1}^{n}\pi_j\right]\right]^2$

Proof:

Let $[E(X^2)]_j$ denotes the value of $E(X^2)$ when 'R_1' is in j^{th} position. for j = 1, ..., n

$[E(X^2)]_1 = 0.p + (1^2 + 2^2 + \cdots + (k-1)^2).r + (n-k).r$

$[E(X^2)]_1 = S_{(k-1)}.r + (n-k).r$

Also,

$[E(X^2)]_j = (j-1)^2.p + [S_{(k-1)} - (j-1)^2].r + (n-k).r$ for $j = 2, \ldots, k$

$[E(X^2)]_j = S_{(k-1)}.r + 1.p + (n-k-1).r$ for $j = (k+1), \ldots, n$

Thus,

$$[E(X^2)]_{SWITCH} = \sum_{j=1}^{n}\pi_j[E(X^2)]_j$$

$$= [S_{(k-1)}.r + (n-k).r].\pi_1 + \sum_{j=2}^{k}\pi_j\,[(j-1)^2.p + [S_{(k-1)} - (j-1)^2].r + (n-k).r] + \sum_{j=k+1}^{n}\pi_j[S_{(k-1)}.r + 1.p + (n-k-1).r]$$

On simplification we get,

$[E(X^2)]_{SWITCH} = \frac{k(k-1)(2k-1)}{6}.r + (n-k).r + (p-r)\left[\sum_{j=2}^{k}(j-1)^2\pi_j + \sum_{j=k+1}^{n}\pi_j\right]$

... (5.1.5)

The variance of expected number of records moved under SWITCH (k) scheme is given by

$[V(X)]_{SWITCH} = [E(X^2)]_{SWITCH} - [[E(X)]_{SWITCH}]^2$

From equations (5.1.4) and (5.1.5) we get,

$[V(X)]_{SWITCH} = \frac{k(k-1)(2k-1)}{6}.r + (n-k).r + (p-r)[\sum_{j=2}^{k}(j-1)^2\,\pi_j + \sum_{j=k+1}^{n}\pi_j] - \left[\frac{k\,(k-1)}{2}.r + (n-k).r + (p-r)\left[\sum_{j=2}^{k}(j-1)\pi_j + \sum_{j=k+1}^{n}\pi_j\right]\right]^2$

This proves the theorem.

6. Expected Number of Records Moved under POS (k) Scheme:

[E(X)] for POS (k) Scheme is given by,

$[E(X)]_{POS} = \sum_{j=1}^{n}\pi_j\,[E(X)]_j$

$= \sum_{j=1}^{k}\pi_j\,[E(X)]_j + \sum_{j=k+1}^{n}\pi_j\,[E(X)]$

6.1 Method to Find $[E(X)]_j$ for Different Values of j Under POS (k) Scheme

$[E(X)]_1$, the expected number of records moved when record 'R_1' is in first position is given by:

$$[E(X)]_1 = (k-1).r + \frac{(n-k)(n-k+1)}{2}.r$$

$[E(X)]_2$, the expected number of records moved when record 'R_1' is in second position is given by:

$$[E(X)]_2 = 1.p + (k-2).r + \frac{(n-k)(n-k+1)}{2}.r$$

$[E(X)]_{k+1}$, the expected number of records moved when record R_1 is in position (k+1) is given by:

$$[E(X)]_{k+1} = (k-1).r + 1.p + \left[\frac{(n-k)(n-k+1)}{2} - 1\right].r$$

In general,

$$[E(X)]_1 = (k-1).r + \frac{(n-k)(n-k+1)}{2}.r \quad \text{for } j = 1 \qquad \ldots (6.1.1)$$

$$[E(X)]_j = 1.p + (k-2).r + \frac{(n-k)(n-k+1)}{2}.r \quad \text{for } j = 2, \ldots, k \qquad \ldots (6.1.2)$$

and

$$[E(X)]_j = (k-1).r + (j-k).p + \left[\frac{(n-k)(n-k+1)}{2} - (j-k)\right].r \quad \text{for } j = (k+1), \ldots, n \qquad \ldots (6.1.3)$$

Now, we present the formula for $[E(X)]_{POS}$ as a theorem.

Theorem 3:

Expected number of records moved under POS (k) scheme is given by

$$[E(X)]_{POS} = (k-1).r\left[\pi_1 + \sum_{j=k+1}^{n} \pi_j\right] + \frac{(n-k)(n-k+1)}{2} r + \sum_{j=2}^{k} \pi_j[(k-2).r + p] + (p-r)\sum_{j=k+1}^{n} \pi_j(j-k)$$

Proof:

[E(X)] under the POS scheme is given by

$$[E(X)]_{POS} = \sum_{j=1}^{n} \pi_j \ [E(X)]_j$$

$$= \sum_{j=1}^{k} \pi_j [E(X)]_j + \sum_{j=k+1}^{n} \pi_j [E(X)]_j$$

… (6.1.4)

From equation (6.1.1), (6.1.2) and (6.1.3) we get,

$$[E(X)]_1 = (k-1).r + \frac{(n-k)(n-k+1)}{2}.r$$

For j = 2 to k

$$[E(X)]_j = 1.p + (k-2).r + \frac{(n-k)(n-k+1)}{2}.r$$

For j = k+1 to n

$$[E(X)]_j = (k-1).r + (j-k).p + \left[\frac{(n-k)(n-k+1)}{2} - (j-k)\right].r$$

Using (6.1.4) and On simplification we get,

$$[E(X)]_{POS} = (k-1).r\left[\pi_1 + \sum_{j=k+1}^{n} \pi_j\right] + \frac{(n-k)(n-k+1)}{2}.r + \sum_{j=2}^{k} \pi_j[(k-2).r + p] + (p-r)\sum_{j=k+1}^{n} \pi_j(j-k)$$

… (6.1.5)

This proves the theorem.

Now, we present the formula for Variance of expected number of records moved under POS (k) scheme as a theorem.

Theorem 4:

Variance of the expected number of records moved under POS (k) scheme is given by

$$[V(X)]_{POS} = (k-1)r\left[\pi_1 + \sum_{j=k+1}^{n} \pi_j\right] + \frac{(n-k)(n-k+1)(2n-2k+1)}{6} r + \sum_{j=2}^{k} \pi_j[(k-2)r + p] + (p-r)\sum_{j=k+1}^{n} \pi_j(j-k)^2 - \Big[(k-1)r\left[\pi_1 + \sum_{j=k+1}^{n} \pi_j\right] + \frac{(n-k)(n-k+1)}{2} r + \sum_{j=2}^{k} \pi_j[(k-2)r + p] + (p-r)\sum_{j=k+1}^{n} \pi_j(j-k) \Big]^2$$

Proof:

Let $[E(X^2)]_j$ denotes the value of $E(X^2)$ when 'R_1' is in j^{th} position for j = 1 to n

We get,

$$[E(X^2)]_1 = (k-1).r + S_{(n-k)}.r$$

For j = 2 to k

$$[E(X^2)]_j = 1.p + (k-2).r + S_{(n-k)}.r$$

Also for j = k+1 to n

$$[E(X^2)]_j = (k-1).r + (j-k)^2.p + \left[S_{(n-k)} - (j-k)^2\right].r$$

Thus,On simplification we get,

$[E(X^2)]_{POS} = (k-1).r[\pi_1 + \sum_{j=k+1}^{n} \pi_j] + S_{(n-k)}.r + \sum_{j=2}^{k}[p + (k-2).r]\pi_j + (p-r)\sum_{j=k+1}^{n}(j-k)^2\pi_j$ ... (6.1.6)

The variance of the expected number of records moved under POS (k) scheme is given by

$[V(X)]_{POS} = [E(X^2)]_{POS} - [[E(X)]_{POS}]^2$

By using equations (6.1.5) and (6.1.6) we get

$$[V(X)]_{POS} = (k-1)r\left[\pi_1 + \sum_{j=k+1}^{n}\pi_j\right] + \frac{(n-k)(n-k+1)(2n-2k+1)}{6}r + \sum_{j=2}^{k}\pi_j[(k-2)r+p] + (p-r)\sum_{j=k+1}^{n}\pi_j(j-k)^2 - [(k-1)r\left[\pi_1 + \sum_{j=k+1}^{n}\pi_j\right] + \frac{(n-k)(n-k+1)}{2}r + \sum_{j=2}^{k}\pi_j[(k-2)r+p] + (p-r)\sum_{j=k+1}^{n}\pi_j(j-k)]^2$$

This proves the theorem.

7. Expected Number of Records Moved under MMTF Scheme:

[E(X)] for MMTF Scheme is given by,

$[E(X)]_{MMTF} = \sum_{j=1}^{n} \pi_j [E(X)]_j$

$= \sum_{j=1}^{k} \pi_j [E(X)]_j + \sum_{j=k+1}^{n} \pi_j [E(X)]$

In [7] we have explained the method to find $[E(X)]_j$ for different values of 'j' under MMTF scheme. We give below the formulae derived in [7] for the expected number of records moved and its variance under MMTF scheme.

$$[E(X)]_{MMTF} = \frac{k(k-1)}{2}.r + \frac{(n-k)(n-k+1)}{6}.r + (p-r)[\sum_{j=1}^{k}(j-1)\pi_j + \sum_{j=k+1}^{n}(j-k)\pi_j] \quad ... (7.1)$$

and

$$[V(X)]_{MMTF} = \frac{k(k-1)(2k-1)}{6}.r + \frac{(n-k)(n-k+1)(2n-2k+1)}{6}.r + (p-r)[\sum_{j=1}^{k}(j-1)^2\pi_j + \sum_{j=k+1}^{n}(j-k)^2\pi_j] - [ASC_{MMTF} - (n-k+1).k.r - (p-r)[(k-1)\sum_{j=k+1}^{n}\pi_j + 1]]^2 \quad ... (7.2)$$

8. Comparison of Different Hybrid Schemes on the Basis of New Criterion:

Using C++ program written on the basis of all the formulae derived so far, we now compare various SOS.

Table I Comparison of E(X) for different Hybrid Schemes and MMTF Scheme (n = 10 and k=5)

SOS	p = 0.5			p = 0.6			p = 0.7		
	V(X)	E(X)	ASC	V(X)	E(X)	ASC	V(X)	E(X)	ASC
SWITCH	1.196782	1.011158	3.678145	1.095854	0.819935	3.153336	0.933594	0.621214	2.621225
POS	1.246845	1.012339	3.555694	1.119684	0.836215	3.044468	0.953853	0.64127	2.533337
MMTF	2.542065	1.567012	3.679503	2.391659	1.264444	3.153637	2.075002	0.954558	2.621275

Observations:

From the Table I it is observed that

1. As 'p' increases the value of E(X) decreases for all self organizing schemes. Thus, if request

probability of record 'R1' increases then the rate of convergence also decreases.

2. Also for the fixed value of 'n' and 'p' for any SOS we have, $[E(X)]_{SWITCH} < [E(X)]_{POS} < [E(X)]_{MMTF}$
3. As 'p' increases, the average search cost decreases for all SOS.
4. Also as 'p' increases variance for all the self organizing schemes decreases.
5. For fixed n, p and k, the relation between the variances for different hybrid schemes is given by

$[V(X)]_{SWITCH} < [V(X)]_{POS} < [V(X)]_{MMTF}$

Table II Comparison of E(X) for different Hybrid Schemes and MMTF Scheme (p = 0.7 and k = 5)

SOS	n = 10			n = 20			n = 25		
	V(X)	E(X)	ASC	V(X)	E(X)	ASC	V(X)	E(X)	ASC
SWITCH	0.933594	0.621214	2.621225	0.571592	0.455108	4.060372	0.4991	0.423246	4.798246
POS	0.953853	0.64127	2.533337	13.482995	1.93059	4.01579	25.540554	2.640402	4.7625
MMTF	2.075002	0.954558	2.621275	15.656035	2.113004	4.060378	28.473057	2.798246	4.798248

Observations:

From the table II it is observed that

1. As the number of records increases for the SWITCH (k) scheme, the value of [E(X)] decreases and for the POS (k) and MMTF scheme, the value of [E(X)] increases. Therefore for SWITCH (k) scheme there will be less number of records moved as number of records increases compared to other two schemes, which shows that SWITCH (k) scheme is slower in reaching optimal ordering.
2. Also notice that, for fixed 'n' and 'p' , $[E(X)]_{SWITCH} < [E(X)]_{POS} < [E(X)]_{MMTF}$
3. As number of records increases, the value of variance decreases for SWITCH (k) scheme, whereas it increases for POS (k) and MMTF schemes.

Table III Comparison of E(X) for different Hybrid Schemes and MMTF Scheme (p = 0.7 and n =20)

SOS	k = 3			k = 5			k = 7		
	V(X)	E(X)	ASC	V(X)	E(X)	ASC	V(X)	E(X)	ASC
SWITCH	0.259163	0.34599	4.030231	0.5715789	0.455108	4.060372	1.358577	0.627399	4.090557
POS	19.290686	2.408053	4.0162	13.482995	1.93059	4.01579	8.900361	1.514602	4.01579
MMTF	22.076813	2.49367	4.031012	15.656035	2.113004	4.060378	11.022722	1.858978	4.090558

From the Table III it is observed that

1. For the fixed value of 'n' and 'p' for any SOS we have, $[E(X)]_{SWITCH} < [E(X)]_{POS} < [E(X)]_{MMTF}$
2. As 'k' increases, the Average Search Cost increases for SWITCH (k) scheme and ASC decreases for POS (k) and MMTF scheme.
3. Also as 'k' increases variance for SWITCH (k) increases while variance decreases for POS (k) and MMTF schemes.
4. For fixed n, p and k, the relation between the variances for different hybrid schemes is given by $[V(X)]_{SWITCH} < [V(X)]_{POS} < [V(X)]_{MMTF}$

Conclusion: In [6] it was shown using new comparison criterion that although TR scheme has lesser average search cost, it has slow rate of convergence as compared to MTF and MMTF. In this paper we have applied the new comparison criterion to different hybrid schemes. We have theoretically derived expected number of records moved for each hybrid scheme and its variance. We further proved by simulation that SWITCH (K) scheme has slower rate of convergence than POS (k) and MMTF schemes when the sample size 'n' increases.

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