SBM DATA ENVELOPMENT ANALYSIS IN FUZZY ENVIRONMENT

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Abstract: Data envelopment analysis (DEA) is a methodology that has been widely used to evaluate the relative efficiency of a set of decision-making units (DMUs). Slack based measure (SBM) model of DEA deals with the directly input excess and output shortfall to assess the effect of slacks on efficiency with common crisp inputs and outputs. The uncertain theory has played an important role in DEA when input and output data of DMUs can't be precisely measured. Thus, the input and output variables can be represented by fuzzy numbers. This paper attempts to extend the DEA model to a fuzzy framework, thus proposing SBM DEA model in fuzzy environment based on α-cut approach to deal with the efficiency measuring problem with the given fuzzy input and output data. Finally, numerical examples are presented to illustrate the proposed fuzzy SBM model. Since the efficiency measures are expressed by membership functions rather than by crisp values, more information is provided for management. By extending to fuzzy environment, the DEA approach is made more powerful for application.

Keywords: Data Envelopment Analysis, Efficiency, Fuzzy LPP, SBM.

Introduction: Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the relative efficiency of decision making units (DMUs) such as bank branches, schools, transport sectors, hospitals, post offices etc. on the basis of multiple inputs and outputs. There exist many DEA models such as CCR (Charnes et al.(1978)), BCC (Banker et.al.(1984)), SBM (Tone (2001), and NSM (Agarwal et. al. (2011)) models. The existing DEA models are usually limited to common crisp inputs and outputs. In some cases, input and output data of DMUs can't be precisely measured, so, the uncertain theory has played an important role in DEA. In these cases, the data can be represented as linguistic variable characterized by fuzzy sets.

Some researchers have proposed several fuzzy models to evaluate DMUs with fuzzy data [Agarwal (2010), Ammar and Wright (2000), Guo and Tanaka (2001), Hougard (1999), Kao and Liu (2000), Liu and Chang (2009), Sengupta (1992), Saati and Memariani (2009), Wen and Li (2009)]. This study selected a SBM model introduced by Tone (2001), because this measure deals directly with the input excesses and the output shortfalls to assess the effect of slacks on efficiency of DMU concerned. Saati and Memariani (2009) have proposed SBM model with fuzzy input-output level. In their study only the most favorable or the upper bound, efficiency was calculated.

In this paper, SBM model is extended with fuzzy environment for evaluating efficiency and ranking of DMUs with fuzzy data. The proposed model is based on the α cuts of the objective and constraints of fuzzy SBM model. In most α cut based methods, the resulting model is solved by comparing two intervals, i.e., interval of LHS and interval of RHS of each equality/ inequality constraints. The efficiency measures by proposed model are expressed by membership functions rather than by crisp values.

The rest of the paper is organized as follows. In section 2, review of SBM model for crisp data is given. The proposed SBM model with fuzzy data is described in section 3. In section 4, numerical illustration of fuzzy SBM model is exhibited followed by conclusions in the last.

SBM Model with crisp data: Tone (2001) proposed a new measure of efficiency which deals directly with the slacks. This model is known as SBM model. In order to estimate the efficiency of k^{th} DMU, SBM model is given as

$$E_k = Min \quad \tau_k = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_{ik}^{-}}{x_{ik}}$$

$$subject \;\; to$$

$$t + \frac{1}{s}\sum_{r=1}^{s}\frac{S_{rk}^{+}}{y_{rk}} = 1$$

$$\sum_{j=1}^{n}\lambda_{jk} y_{rj} - S_{rk}^{+} = t y_{rk} \qquad \forall r = 1,\ldots,s \quad (1)$$

$$\sum_{j=1}^{n}\lambda_{jk} x_{ij} + S_{ik}^{-} = t x_{ik} \qquad \forall i = 1,\ldots,m$$

$$\lambda_{jk} \geq 0 \qquad \forall j = 1,\ldots,n$$

$$S_{rk}^{+}, S_{ik}^{-} \geq 0 \; ; r = 1,\ldots,s, \quad i = 1,\ldots,m$$

$$t > 0$$

Let an optimal solution of (1) be (τ^*, t^*, Λ^*, S^{-*}, S^{+*}).

The interpretation of the results of the model (1) can be summarized as follows:

- The DMU under reference is said to be Pareto efficient if all slacks are zero i.e. S_{rk}^{+*} and S_{ik}^{-*} = 0 for every r and i which is equivalent to $E_k = 1$.
- The non-zero slacks and (or) $E_k \leq 1$ identify the sources and amount of any inefficiency that may exist in the DMU_k.

SBM Model with fuzzy data

In a set of DMUs, suppose that the inputs $\tilde{X}_{ij}$ and outputs $\tilde{Y}_{rj}$ are approximately known and can be represented by convex fuzzy sets with membership functions $\mu_{\tilde{X}_{ij}}$ *and* $\mu_{\tilde{Y}_{rj}}$, respectively. Thus, fuzzy SBM model is as follows

$$\tilde{E}_k = Min \;\; \tilde{\tau}_k = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_{ik}^{-}}{\tilde{X}_{ik}}$$

$$subject \;\; to$$

$$t + \frac{1}{s}\sum_{r=1}^{s}\frac{S_{rk}^{+}}{\tilde{Y}_{rk}} = 1$$

$$\sum_{j=1}^{n}\lambda_{jk} \tilde{Y}_{rj} - S_{rk}^{+} = t \tilde{Y}_{rk} \qquad \forall r = 1,\ldots,s \quad (2)$$

$$\sum_{j=1}^{n}\lambda_{jk} \tilde{X}_{ij} + S_{ik}^{-} = t \tilde{X}_{ik} \qquad \forall i = 1,\ldots,m$$

$$\lambda_{jk} \geq 0 \qquad \forall j = 1,\ldots,n$$

$$S_{rk}^{+}, S_{ik}^{-} \geq 0 \; ; r = 1,\ldots,s, \quad i = 1,\ldots,m$$

$$t > 0$$

The efficiency score evaluated from the model should be fuzzy because this model contains fuzzy parameters. Let $S(\tilde{X}_{ij})$ *and* $S(\tilde{Y}_{rj})$ denote the support of $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$, respectively. The α-cuts of $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$ are defined as

$$(X_{ij})_\alpha = \{x_{ij} \in S(\tilde{X}_{ij}); \mu_{\tilde{X}_{ij}}(x_{ij}) \geq \alpha\}, \;\; \forall i, j$$
$$(Y_{rj})_\alpha = \{y_{rj} \in S(\tilde{Y}_{rj}); \mu_{\tilde{Y}_{rj}}(y_{rj}) \geq \alpha\}, \quad \forall r, j \qquad (3)$$

The inputs and outputs can be represented by different level of confidence interval by α-cuts. The fuzzy DEA model is transformed to a family of crisp DEA models with different α-cuts $\{(X_{ij})_\alpha \left| 0<\alpha\le 1\right.\}$ and $\{(Y_{rj})_\alpha \left| 0<\alpha\le 1\right.\}$ since $(X_{ij})_\alpha$ and $(Y_{rj})_\alpha$ are crisp sets.

Based on Zadeh's extension principle [Yager (1986), Zadeh (1978), Zimmermann (1991)], the membership function of the efficiency of k[th] DMU can be defined as

$$\mu_{\tilde{E}_k}(z)=\sup_{x,y}\min\{\mu_{\tilde{X}_{ij}}(x_{ij}),\mu_{\tilde{Y}_{rj}}(y_{rj})\,\forall i,j,r \left| z=E_k(x,y)\right.\} \quad (4)$$

where $E_k(x,y)$ is defined in (1). The membership function $\mu_{\tilde{E}_k}$ can be constructed by driving the α-cuts of $\tilde{E}_k$. According to (4), $\mu_{\tilde{E}_k}$ is the minimum of $\mu_{\tilde{X}_{ij}}(x_{ij})$ and $\mu_{\tilde{Y}_{rj}}(y_{rj})\,\forall i,j,r$ and $\mu_{\tilde{X}_{ij}}(x_{ij})\ge\alpha,\mu_{\tilde{Y}_{rj}}(y_{rj})\ge\alpha$ and at least one $\mu_{\tilde{X}_{ij}}(x_{ij})$ or $\mu_{\tilde{Y}_{rj}}(y_{rj})$ equal to α, $\forall i,j,r$, such that $z=E_k(x,y)$ to satisfy $\mu_{\tilde{E}_k}(z)=\alpha$. Furthermore, all α-cuts form a nested structure with respect to α [Zimmermann (1991)]; i.e.,

$$[(X_{ij})^L_{\alpha_1},(X_{ij})^U_{\alpha_1}]\subseteq[(X_{ij})^L_{\alpha_2},(X_{ij})^U_{\alpha_2}]\; and\; [(Y_{rj})^L_{\alpha_1},(Y_{rj})^U_{\alpha_1}]\subseteq[(Y_{rj})^L_{\alpha_2},(Y_{rj})^U_{\alpha_2}], \qquad for\; 0<\alpha_2<\alpha_1\le 1.$$

Therefore, $\mu_{\tilde{X}_{ij}}(x_{ij})\ge\alpha$ and $\mu_{\tilde{X}_{ij}}(x_{ij})=\alpha$ and $\mu_{\tilde{Y}_{rj}}(y_{rj})\ge\alpha$ and $\mu_{\tilde{Y}_{rj}}(y_{rj})=\alpha$, respectively, have the same smallest and largest elements. For $\mu_{\tilde{E}_k}$, it is sufficient to find the upper and lower bounds of α-cuts of $\tilde{E}_k$, which, based on (4), can be solved as

$$(E_k)^L_\alpha=\min_{\substack{(X_{ij})^L_\alpha\le x_{ij}\le(X_{ij})^U_\alpha\\(Y_{rj})^L_\alpha\le y_{rj}\le(Y_{rj})^U_\alpha\\ \forall i,r,j}}\left\{\begin{array}{l}Min \quad t-\frac{1}{m}\sum_{i=1}^{m}\frac{S^-_{ik}}{x_{ik}}\\ subject\;\; to\\ t+\frac{1}{s}\sum_{r=1}^{s}\frac{S^+_{rk}}{y_{rk}}=1\\ \sum_{j=1}^{n}\lambda_{jk}y_{rj}-S^+_{rk}=ty_{rk} \quad \forall r=1,\dots,s\\ \sum_{j=1}^{n}\lambda_{jk}x_{ij}+S^-_{ik}=tx_{ik} \quad \forall i=1,\dots,m\\ \lambda_{jk}\ge 0 \qquad \forall j=1,\dots,n\\ S^+_{rk},S^-_{ik}\ge 0\;;r=1,\dots,s,\; i=1,\dots,m\\ t>0\end{array}\right. ; \qquad (5a)$$

$$(E_k)^U_\alpha=\max_{\substack{(X_{ij})^L_\alpha\le x_{ij}\le(X_{ij})^U_\alpha\\(Y_{rj})^L_\alpha\le y_{rj}\le(Y_{rj})^U_\alpha\\ \forall i,r,j}}\left\{\begin{array}{l}Min \quad t-\frac{1}{m}\sum_{i=1}^{m}\frac{S^-_{ik}}{x_{ik}}\\ subject\;\; to\\ t+\frac{1}{s}\sum_{r=1}^{s}\frac{S^+_{rk}}{y_{rk}}=1\\ \sum_{j=1}^{n}\lambda_{jk}y_{rj}-S^+_{rk}=ty_{rk} \quad \forall r=1,\dots,s\\ \sum_{j=1}^{n}\lambda_{jk}x_{ij}+S^-_{ik}=tx_{ik} \quad \forall i=1,\dots,m\\ \lambda_{jk}\ge 0 \qquad \forall j=1,\dots,n\\ S^+_{rk},S^-_{ik}\ge 0\;;r=1,\dots,s,\; i=1,\dots,m\\ t>0\end{array}\right. \qquad (5b)$$

This is a two level mathematical model which can be solved by the conventional one level model based on the proposition given by Liu and Chuang (2009). According to the proposition, at a specific α level, the largest efficiency score for DMU k is reached by setting its fuzzy inputs as the lower bounds and the fuzzy outputs at the upper bounds; meanwhile; the fuzzy inputs of all other DMUs at their corresponding highest level and the fuzzy outputs at their lowest level. On the contrary, the smallest efficiency score for the DMU is reached by setting its fuzzy inputs as the upper bounds and the fuzzy outputs at the lower bounds; meanwhile; the fuzzy inputs of all other DMUs at their corresponding lowest level and the fuzzy outputs at their highest level. Therefore, the models (5a) and (5b) become (6a) and (6b), respectively as follows

$$(E_k)_\alpha^L = Min \quad t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_{ik}^-}{(X_{ik})_\alpha^U}$$

subject to

$$\begin{aligned}
& t + \frac{1}{s}\sum_{r=1}^{s}\frac{S_{rk}^+}{(Y_{rk})_\alpha^L} = 1 \\
& \lambda_{jk}(Y_{rj})_\alpha^L + \sum_{j=1}^{n}\lambda_{jk}(Y_{rj})_\alpha^U - S_{rk}^+ = t(Y_{rk})_\alpha^L \qquad \forall r = 1,\dots,s \\
& \lambda_{jk}(X_{ij})_\alpha^U + \sum_{j=1}^{n}\lambda_{jk}(X_{ij})_\alpha^L + S_{ik}^- = t(X_{ik})_\alpha^U \quad \forall i = 1,\dots,m \\
& \lambda_{jk} \geq 0 \qquad \forall j = 1,\dots,n \\
& S_{rk}^+, S_{ik}^- \geq 0 \;; r = 1,\dots,s,\; i = 1,\dots,m \\
& t > 0
\end{aligned} \tag{6a}$$

and

$$(E_k)_\alpha^U = Min \quad t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_{ik}^-}{(X_{ik})_\alpha^L}$$

subject to

$$\begin{aligned}
& t + \frac{1}{s}\sum_{r=1}^{s}\frac{S_{rk}^+}{(Y_{rk})_\alpha^U} = 1 \\
& \lambda_{jk}(Y_{rj})_\alpha^U + \sum_{j=1}^{n}\lambda_{jk}(Y_{rj})_\alpha^L - S_{rk}^+ = t(Y_{rk})_\alpha^U \qquad \forall r = 1,\dots,s \\
& \lambda_{jk}(X_{ij})_\alpha^L + \sum_{j=1}^{n}\lambda_{jk}(X_{ij})_\alpha^U + S_{ik}^- = t(X_{ik})_\alpha^L \quad \forall i = 1,\dots,m \\
& \lambda_{jk} \geq 0 \qquad \forall j = 1,\dots,n \\
& S_{rk}^+, S_{ik}^- \geq 0 \;; r = 1,\dots,s,\; i = 1,\dots,m \\
& t > 0
\end{aligned} \tag{6b}$$

The pair of mathematical programs falls into the category of parametric programming (1979). If both $(E_k)_\alpha^L$ and $(E_k)_\alpha^U$ are invertible with respect to α then $\mu_{\tilde{E}_k}$ can be constructed. Otherwise, the set of intervals $\{[(E_k)_\alpha^L,(E_k)_\alpha^U] | \alpha \in (0,1]\}$ reveals the shape of $\mu_{\tilde{E}_k}$, although the exact function form is not known explicitly. Model (6a) defines as the worst-best case where decision maker is pessimistic about the DMU_k and optimistic about the remaining DMUs. Model (6b) defines as the best-worst case where decision maker is optimistic about the DMU_k and pessimistic about the remaining DMUs. The other two cases i.e. best-best case and worst-worst case lie between the above mentioned cases.

Ranking of DMUs

Ranking of DMUs is an important part of DEA interpretations. The final efficiency of a DMU in DEA model with fuzzy data is no longer a crisp number; it is a fuzzy number. There exist many methods for ranking the fuzzy numbers [Chen (1985), Chen and Klein (1997), Chen and Hwang (1992), Liou and Wang (1992)]. These ranking methods are based on the membership functions of the fuzzy numbers. These methods include degree of optimality, hamming distance, comparison function, fuzzy mean and spread, α-cut, linguistic method etc. The method given by Chen and Klein (1997), is very suitable for this study since its is based on α-cut and it can handle a large quantity of fuzzy numbers. The ranking index for the j[th] DMU as

$$I_j = \frac{\sum_{i=0}^{n} ((E_j)_{\alpha_i}^U - c)}{[\sum_{i=0}^{n} ((E_j)_{\alpha_i}^U - c) - \sum_{i=0}^{n} ((E_j)_{\alpha_i}^L - d)]} ; n \to \infty \qquad (8)$$

$$where\ c = \min_{i,j} \{(E_{ji})_{\alpha_i}^L\};\ d = \max_{i,j} \{(E_{ji})_{\alpha_i}^U\}.$$

n is the number of α-cuts and $\alpha_i = ih / n;\ i = 0,\ldots,n$. h is the maximum height of $\mu_{\tilde{E}_j}$. The j[th] DMU with a higher index I_j is considered more efficient than the DMU with a lower index. The value of I_j is between 0 and 1, which is consistent with the efficiency in crisp DEA model. Theoretically, this method is valid when n approaches to infinite. Practically, this method requires only n is equal to 3 or 4 and uses the summation of each α-cuts which does not require normality to measure the summation for the ranking order of the fuzzy numbers. This allows for applicability to a wider range of fuzzy numbers.

Numerical Illustration

To illustrate the proposed fuzzy SBM model, consider the data taken by Kao and Liu (2000). Data consists of four DMUs with single input and single output which is shown in Table 1.

Table 1: Input and output data of four DMUs

DMU	Input	α- cut	Output	α- cut
A	(11,12,14)	[11+α, 14-2α]	10	[10,10]
B	30	[30,30]	(12,13,14,16)	[12+α, 16-2α]
C	40	[40,40]	11	[11,11]
D	(45,47, 52,55)	[45+2α, 55-3α]	(12,15,19,22)	[12+3α, 22-3α]

Source: Kao and Liu [12].

Analytic solutions are not obtainable in this example. Fig. 1 depicts the rough shape of $\mu_{\tilde{E}_A}, \mu_{\tilde{E}_B}, \mu_{\tilde{E}_C}$ and $\mu_{\tilde{E}_D}$, constructed from one hundred and one values of α: 0, 0.01,..., 1.00. The rough shape turns out rather than fine, looks like a continuous function.

The results evince that there is no direct correspondence between the membership function of the efficiency measures and the observations. For example, the input and output of DMU A are fuzzy but its efficiency measure is a crisp value and the input and output of DMU C are crisp but its efficiency measure is fuzzy.

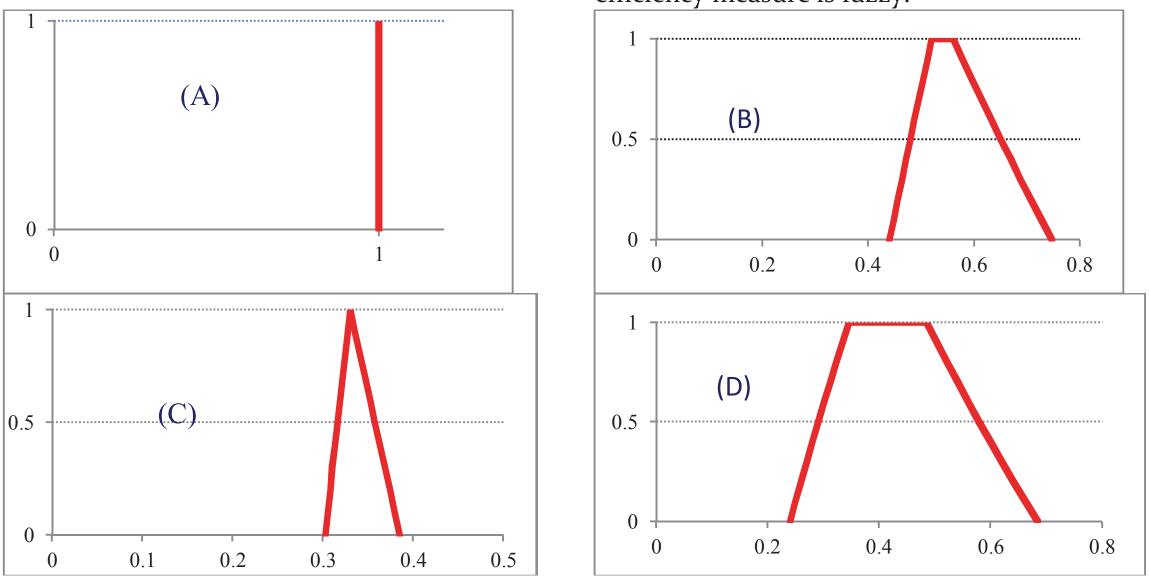

Fig. 1: Membership functions of (a) $\mu_{\tilde{E}_A}$ (b) $\mu_{\tilde{E}_B}$ (c) $\mu_{\tilde{E}_C}$ (d) $\mu_{\tilde{E}_D}$

Further, the fuzzy ranking method developed by Chen and Klein (1997) is utilised to rank the DMUs. The ranking indicies are calculated as I_A = 1.0, I_B = 0.44, I_C = 0.15, I_D = 0.33. The result shows that $I_A > I_B > I_D > I_C$, which conclude that the efficiency grade order being DMU A > DMU B > DMU D > DMU C.

Another numerical example is presented to compare the proposed approach with the existing methods.The example is taken from Guo and Tanaka (2001) with single fuzzy input and single fuzzy output. Data of five DMUs are listed in Table 2. The input and output have symmetrical triangular membership functions.

Table 2: Input and output data of five DMUs

DMU	A	B	C	D	E
Input	(1.5,2,2.5)	(2.5,3.0,3.5)	(2.4,3.0,3.6)	(4.0,5.0,6.0)	(4.5,5.0,5.5)
Output	(0.7,1.0,1.3)	(2.3,3.0,3.7)	(1.6,2.0,2.4)	(3.0,4.0,5.0)	(1.8,2.0,2.2)

Source: Guo and Tanaka [10]

We obtained the minimum and maximum efficiency [$[(E_k)_\alpha^L,(E_k)_\alpha^U]$ using model (6a) and (6b), respectively, for each α value. Table 3 depicts the results of efficiency when α value gradually increases from 0 to 1 by step 0.1. The results reveal that the efficiency obtained by method suggested by Saati and Memariani (2009) is same as the upper bound of the efficiency obtained by the proposed method. However, in contrast to the crisp value of efficiency measure by Saati and Memariani (2009), efficiency measures obtained by the proposed model are expressed by membership functions. Thus, more information is provided for management. The DEA approach is made more powerful for application by extending to fuzzy environment.

Table 3: The fuzzy efficiency measure of five DMUs

α	Eff. of A $[(E_A)_\alpha^L,(E_A)_\alpha^U]$	Eff. of B $[(E_B)_\alpha^L,(E_B)_\alpha^U]$	Eff. of C $[(E_C)_\alpha^L,(E_C)_\alpha^U]$	Eff. of D $[(E_D)_\alpha^L,(E_D)_\alpha^U]$	Eff. of E $[(E_E)_\alpha^L,(E_E)_\alpha^U]$
0.0	[0.189,1]	[0.526,1]	[0.300,1]	[0.338,1]	[0.221,0.744]
0.1	[0.209,1]	[0.575,1]	[0.325,1]	[0.369,1]	[0.235,0.698]
0.2	[0.231,1]	[0.628,1]	[0.353,1]	[0.403,1]	[0.249,0.654]
0.3	[0.255,0.979]	[0.686,1]	[0.382,1]	[0.440,1]	[0.264,0.614]
0.4	[0.282,0.888]	[0.748,1]	[0.414,1]	[0.479,1]	[0.280,0.577]
0.5	[0.310,0.806]	[0.815,1]	[0.448,0.999]	[0.522,1]	[0.297,0.542]
0.6	[0.342,0.732]	[0.889,1]	[0.485,0.921]	[0.569,1]	[0.315,0.510]
0.7	[0.376,0.665]	[0.968,1]	[0.525,0.849]	[0.620,1]	[0.335,0.480]
0.8	[0.413,0.605]	[1,1]	[0.568,0.783]	[0.675,0.948]	[0.355,0.451]
0.9	[0.455,0.550]	[1,1]	[0.616,0.722]	[0.735,0.871]	[0.377,0.425]
1.0	[0.500,0.500]	[1,1]	[0.667,0.667]	[0.800,0.800]	[0.400,0.400]

The fuzzy efficiency scores are more informative. The α-cut of E_k shows the spread of the efficiency score at specific α level. Specifically, α = 1 indicates the efficiency that is most likely to be and α = 0 indicates the range that the efficiency will definitely appear. For example, the efficiency measure of DMU A at α = 1 is 0.5. On the other hand, the range of efficiency scores of DMU A at α = 0 is [0.189, 1]. It is indicating that the efficiency score of DMU A will never exceed 1 or fall below 0.189. Obviously, narrower ranges of α-cuts imply less fuzzy numbers. When all α-cuts degenerate to the same point, one has a crisp number.

The ranking indicies are calculated as I_A = 0.472, I_B = 0.707, I_C = 0.529, I_D = 0.578, I_E = 0.430. The result shows that $I_B > I_D > I_C > I_A > I_E$, which conclude that the efficiency grade order being DMU B > DMU D > DMU C > DMU A > DMU E.

Conclusions:DEA has wide application to evaluate the relative efficiency of a set of DMUs using multiple inputs to produce multiple outputs. The existing DEA models are usually limited to common crisp inputs and outputs. In some cases, input and output data of DMUs can't be precisely measured, for example, quality of service, quality of input resource, degree of satisfaction etc. So, the uncertain theory has played an important role in DEA. In these cases, the data with crisp number will not satisfy the real needs and this restriction will diminish the practical flexibility of DEA in application. Thus, the data can be represented as linguistic variable characterized by fuzzy sets. This paper attempts to extend the traditional DEA model to a fuzzy framework, thus proposing a fuzzy SBM DEA model based on ?-cut approach and Zadeh's extension principle to deal with the efficiency measuring problem with the given fuzzy input and output data. The proposed method provides the ability to offer more objective measurement of efficiency of DMUs in vague environment. Chen and Klein's ranking method is applied to rank the DMUs in fuzzy DEA model. This method is efficient and effective for a large quantity of fuzzy numbers. Finally, numerical examples are presented to illustrate the fuzzy SBM model. The study reveals that there is no direct correspondence between the membership function of the efficiency measures and the observed data. Since the efficiency measures are expressed by membership functions rather than by crisp values, more information is provided for management. By extending to fuzzy environment, the DEA approach is made more powerful for application.

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